Solving Dynamic Reliability Problems by means of Ordinary and Fluid Stochastic Petri Nets

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ABSTRACT: A benchmark on dynamic reliability taken from Marseguerra & Zio (1996) is considered; though the behaviour of this system is dynamic and is described by continuous variables, we show that the system is suitable to an analytical solution, instead of simulation. The system consists of a tank containing some liquid whose level is monitored by a controller acting on two pumps and one valve, in order to avoid the liquid dry out or overflow. The reliability evaluation of the system is obtained by resorting to *Generalized Stochastic Petri Nets* (GSPN) and *Fluid Stochastic Petri Nets* (FSPN). FSPNs are hybrid models and differ from GSPNs by the presence of both fluid places (modelling continuous variables) and discrete places (containing a discrete number of tokens). GSPNs are used for the analytical solution of the benchmark, while the simulation on the FSPNs, is run with the aim of validating the analytical results.

1 INTRODUCTION

Several models for the reliability analysis of complex systems have been proposed in the literature, but most of them are not suitable to represent the system when its behaviour needs to be expressed by means of continuous variables (temperature, pressure, etc.), or when the system changes its configuration during its life. In the first case, we talk about hybrid models (or systems), in the second case, of dynamic reliability. In both cases, simulation is typically the most utilized technique to evaluate the system behaviour, while the analytical approach is often unpractical. However, in some cases an analytical approach can be afforded by resorting to a peculiar use of ordinary *Generalized Stochastic Petri Nets* (GSPN) (Ajmone-Marsan et al. 1995) , or to a rather new extension called *Fluid Stochastic Petri Nets* (FSPN) (Bobbio et al. 2001, Bobbio et al. 2003, Gribaudo et al. 1999, Gribaudo et al. 2001). The advantages of using an analytical solution rather then a simulative approach are well known, and we show how to tackle a hybrid dynamic reliability problem by using the analytical approach on GSPNs. The example is a benchmark taken from the literature (Marseguerra & Zio 1996), and consists of a tank containing some liquid whose level is influenced by a controller acting on three components (two pumps and one valve); the controller orders the components to switch on or off, with the aim of avoiding the dry out or the overflow failure condition. Several configurations of the system have been proposed and simulated in Marseguerra & Zio (1996) starting from the usual case of time and state independent failure rates, and including the case of state dependent failure rates, and the case with repairable components.

FSPNs are a recently developed evolution of the Petri Nets that allow to augment a standard Petri Net by accommodating continuous variables, by means of new primitives called fluid places and fluid arcs. Fluid places contain a continuous level of fluid (instead of a discrete number of tokens) that flows in and out through fluid arcs (pipes). This extension increases the modelling power of Stochastic Petri Nets by providing a modelling framework in which discrete variables can be combined with continuous variables and the properties of the former depends on the latter and vice-versa. This new framework has proved to be useful to model systems where physical continuous quantities, such as the liquid level or temperature, need to be represented (Marseguerra & Zio 1996).

In this paper, the reliability evaluation of the benchmark proposed in Marseguerra & Zio (1996), is first obtained by modelling and analyzing the system as a GSPN. Moreover, it is shown that by means of a suitable discretization procedure to be applied to continuous variables, GSPNs can cope with the problem. Then the analytical results are validated by means of simulation on the FSPN. The obtained results are also quite similar to those returned by applying the Monte Carlo simulation, and reported in Marseguerra & Zio (1996).

2 THE CASE STUDY

The system (Fig. 1) is composed by a tank containing some liquid, two pumps (P1 and P2) to fill the tank, a valve (V) to remove liquid from the tank, and a controller monitoring the liquid level (H) and acting on P1, P2 and V.

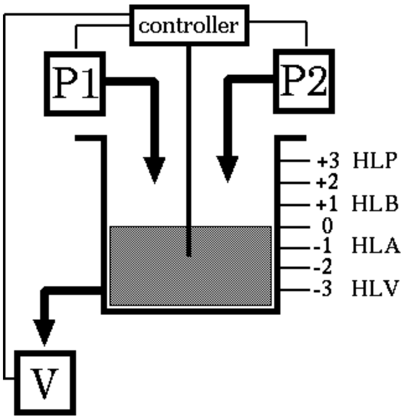

Figure 1. System scheme.

Initially H is equal to 0, with P1 and V in state ON, and P2 in state OFF; since both pumps and the valve have the same rate of level variation (0.6 m/h), while the initial configuration holds, the liquid level does not change. The cause of a variation of H is the occurrence of a failure of one of the components; a failure consists of turning to the states stuck ON or stuck OFF. The failure probability obeys the negative exponential distribution; the failure rate does not depend on the current state of the component, so the effect of the failure is the stuck condition, while the state transitions towards the stuck ON or the stuck OFF state, are uniformly distributed (Fig. 2). Table 1 shows the failure rate for each component.

Table 1. Failure rates.

Component	failure rate
P1	0.004566 1/h
P2	0.005714 1/h
V	0.003125 1/h

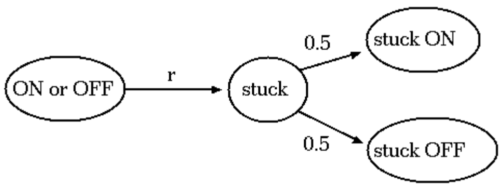

Figure 2. The state of a component; r is the failure rate.

Table 2 shows how H changes with respect to the current configuration of the component states; the controller believes that the system is functioning correctly while H is inside the region between the levels denoted by HLA (-1) and HLB (+1). If H reaches HLB there is the risk of the liquid overflow; this event occurs when H exceeds the level denoted as HLP (+3). To avoid this undesired situation, the controller orders to both pumps to switch OFF and to the valve to switch ON, with the aim of decreasing H. If a component is stuck, it does not obey the controller order and maintains its current state.

Table 2. H variation for every system configuration.

P1	P2	V	effect on H	variation rate
ON	OFF	OFF	↑	0.6 m/h
ON	ON	OFF	↑↑	1.2 m/h
ON	OFF	ON	=	
ON	ON	ON	↑	0.6 m/h
OFF	OFF	OFF	=	
OFF	ON	OFF	↑	0.6 m/h
OFF	OFF	ON	↓	0.6 m/h
OFF	ON	ON	=	

The other undesired situation is the tank dryout; this happens when H is below HLV (-3); to avoid the dryout, when H reaches HLA, the controller orders to both pumps to switch ON and to the valve to switch OFF, with the aim of increasing H. Table 3 shows the control laws with respect to H.

The failure of the whole system happens when the dry out or the overflow occurs.

Table 3. Control laws.

Boundary	P1	P2	V
H ≤ HLA	ON	ON	OFF
H ≥ HLB	OFF	OFF	ON

2.1 *Some notions on GSPNs*

GSPNs are an extension of Petri nets including timed transitions whose firing delay is a random variable.

Their composing elements are places, timed transitions, immediate transitions, directed arcs and inhibitor arcs. Places (graphically denoted by circles) can contain a discrete number of tokens; immediate transitions (black rectangles) fire as soon as they are enabled, while timed transitions (white rectangles) fire after a random period of time which is ruled by a negative exponential distribution. Directed arcs are used to move tokens when a transition fires, while inhibitor arcs (ending with a small circle) can connect a place to a transition in order to disable the transition if the place is not empty.

The first step of the analysis of a GSPN consists of generating the reachability graph, i. e. all the possible GSPN markings and transitions between markings. From the reachability graph, the corresponding *Continuous Time Markov Chain* (CTMC) (Sahner et al. 1996) can be obtained and analyzed.

2.2 *Modelling the system as a GSPN*

The system has been modelled as a GSPN with the aim of performing analytically the reliability evaluation of the system. Figure 3 shows the GSPN modelling the system behaviour. The state of a component, for instance P1, is modelled by three places: *P1on*, *P1off* and *P1stuck*; when *P1on* contains one token, P1 is ON; when *P1off* contains one token, P1 is OFF; if *P1stuck* contains one token, P1 is also stuck. The component state variations due to a failure, are modelled by four timed transitions: *P1failONON*, *P1failONOFF*, *P1failOFFON*, *P1failOFFOFF*. The transition *P1failONOFF* for instance, models the transition from the state ON to the state stuck OFF by moving the token from *P1on* to *P1off* and putting one token in *P1stuck*; since the failure is ruled by an exponential distribution and the states stuck ON and stuck OFF are uniformly distributed, all the four timed transitions have the same firing rate, equal to the half of the failure rate of P1. The failure of P2 and V is modelled in the same way.

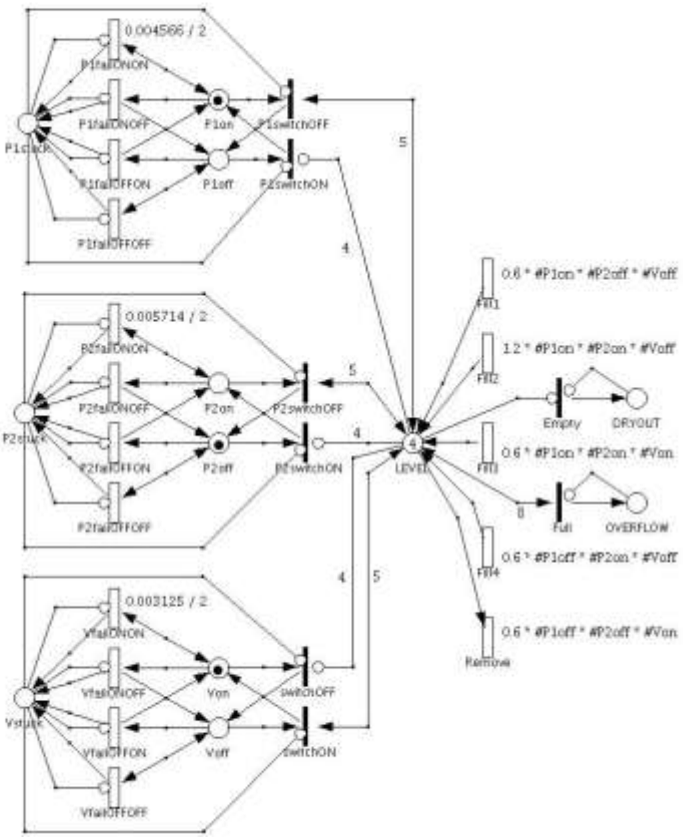

Figure 3. GSPN model of the system.

The liquid level has been discretized: nine intermediate levels have been modelled by a set of tokens inside the place named *LEVEL*; Table 4 shows the correspondence between the number of tokens in *LEVEL* and the liquid level H. The controller action on the component states with respect to H, is modelled by two immediate transitions for each component, connected to place *LEVEL*. In the case of P1, we have *P1switchOFF* and *P1switchON*; the first one fires when *LEVEL* contains at least five tokens (H ≥ HLB), and moves the token from *P1on* to *P1off* if P1 is currently ON. Analogously, *P1switchON* fires when *LEVEL* contains less than four tokens (H ≤ HLA), with the effect of moving the token from *P1off* to *P1on* if P1 is currently OFF. Both transitions are disabled if P1 is stuck.

The liquid level variations are modelled by five timed transitions (*Fill1*, *Fill2*, *Fill3*, *Fill4*, *Remove*) which correspond to the component state configurations leading to a liquid level variation (Table 2). Each of these transitions can fire only while the relative state configuration holds; in this period, its firing rate is equal to the level variation rate of the relative state configuration indicated in Table 2. The effect ot the firing is the addition (or the removal) of one token in *LEVEL*; in this way, we model the increase (or the decrease) of H.

Table 4. Correspondence between H and the number of tokens inside *LEVEL*.

H	#tokens	Condition
> +3	8	overflow
+3	7	HLP
+2	6	
+1	5	HLB
0	4	correct functioning
-1	3	HLA
-2	2	
-3	1	HLV
< -3	0	dry out

The dryout and overflow conditions are detected by two specific immediate transitions: *Empty* and *Full* respectively; the first one fires when *LEVEL* contains no tokens (H<HLV, Table 4), and puts one token inside the place *DRYOUT* meaning that the dry out has occurred; the second one fires when *LEVEL* contains eight tokens (H>HLV), and puts one token inside *OVERFLOW*.

To model the initial configuration, *P1on*, *P2off* and *Von* are marked with one token, while *LEVEL* contains four tokens, corresponding to H=0.

2.3 *Some notions on FSPNs*

FSPNs are a new extension of Petri nets including the same elements of GSPNs with the addition of fluid places and arcs; fluid places contain a continuous fluid level and are suitable to represent continuous variables such as the temperature and the pressure. A fluid place can be connected to a timed transition by means of a fluid arc (pipe); while the timed transition is enabled, some fluid is moved through the fluid arc, from or to the fluid place with respect to the flow rate associated to the fluid arc. Moreover, the firing of a timed transition may depend on the fluid level inside a fluid place: the *Dirac* delta function is used to make a transition fire when the fluid level reaches a certain value.

The *Dirac* delta function returns 0 if its argument differs from 0, while it returns infinite if its argument is equal to 0. So if we want a transition to fire when the level *l* is equal to *x*, we have to set the firing rate of the transition as the function *Dirac*(*l*-*x*).

2.4 *Modelling the system as a FSPN*

In order to verify the correctness of the results (reported in section 2.5) obtained through the liquid level discretization in the GSPN model, we built also the FSPN model relative to the same system.

Figure 4 shows the FSPN model of the system, where some new elements appear: a fluid place named *L* modelling the liquid level in the tank and three fluid arcs with the shape of a pipe, modelling the action of the two pumps and of the valve on *L*; if we consider for instance P1, the fluid variation rate of the relative fluid arc is $0.6 \cdot \#P1on$, where *#P1on* is the number of tokens inside *P1on*; in other words, some fluid is moved to *L* only while P1 is ON. Table 5 shows the correspondence between the liquid level in the tank (H) and the fluid level in *L*.

Table 5. Correspondence between H and the level in the fluid place *L*.

H	*L*	Condition
+3	6	HLP
+2	5	
+1	4	HLB
0	3	initial level
-1	2	HLA
-2	1	
-3	0	HLV

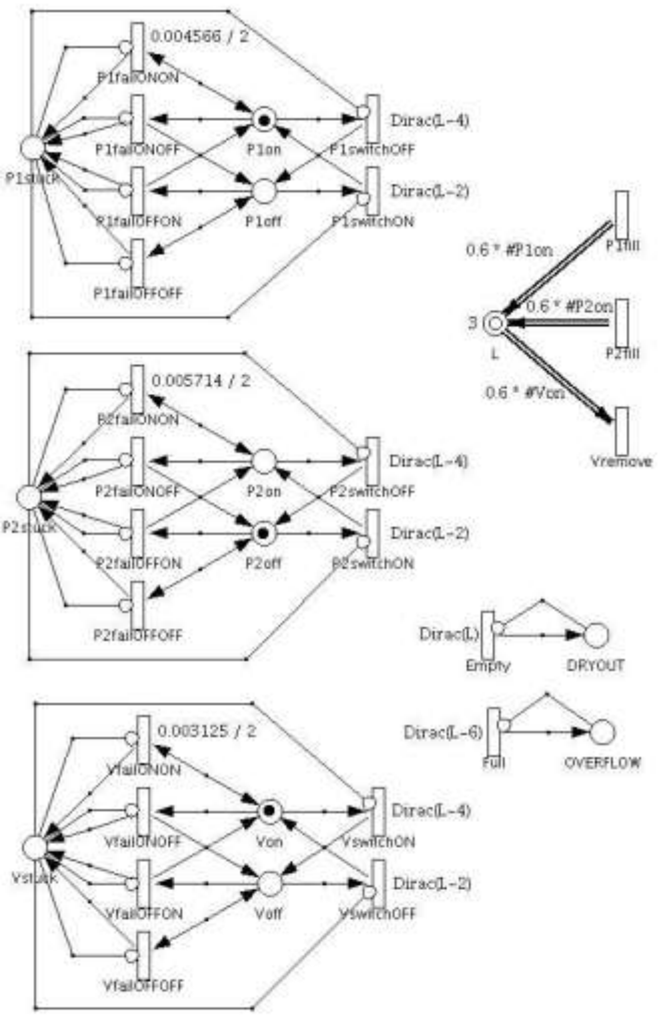

Figure 4. FSPN model of the system

The current state and the failure of a component are modelled in the same way as in the GSPN; the controller action on the components is now modelled by two timed transitions for each component. In the case of P1, they are *P1switchOFF* and *P1switchON*; the first one must fire when H reaches HLB, so its firing rate is the function *Dirac*(*L*-4) and it switches P1 to OFF if P1 is currently ON. The second transition must fire when H reaches HLA, so its firing rate is *Dirac*(*L*-2) and it switches P1 to ON if P1 is currently OFF. Both transitions are disabled if P1 is stuck.

Two timed transitions named *Empty* and *Full*, detect the dry out and the overflow condition respectively; *Empty* must fire when H reaches HLV; this happens when *L* is equal to 0, so the firing rate of this transition is *Dirac*(*L*). *Full* must fire when H reaches HLP, so its firing rate is *Dirac*(*L*-6).

2.5 *Comparison of the results obtained on both models*

In order to evaluate the reliability of the system, we calculated analytically the dry out and overflow *cumulative distribution function* (cdf) on the GSPN of Figure 3; this means computing the probability that the system is in the dry out or in the overflow condition respectively, as a function of the time. The cdf of the dry out is calculated as the probability of the presence of one token in the place *DRYOUT*; the cdf of the overflow is the probability to have one token in *OVERFLOW*. The GSPN model has been drawn and analyzed by means of the *GreatSPN* tool (Chiola et al. 1995).

On the FSPN model of Figure 4 instead, we performed by means of a specific simulator, 10000 cycles of simulation returning the lower and the upper bounds for the dry out cdf and the overflow cdf at the same mission times adopted for the analytical approach on the GSPN.

The results obtained on both the GSPN and the FSPN are reported in Table 6 (dry out) and in Table 7 (overflow); comparing the analytical results of the GSPN and the simulation results of the FSPN, we can observe that each probability value computed on the GSPN belongs to the range of values between the lower and the upper bound returned by the simulation on the FSPN for the same mission time; this situation is graphically shown in Figure 5 (dry out) and in Figure 6 (overflow).

Our analytical results are also quite similar to those reported in Marseguerra & Zio (1996), obtained via Monte Carlo simulation. This means that the analytical approach based on liquid level discretization and GSPN, returns acceptable results, considering that the simulation of the FSPN can deal with continuous variables, while in the GSPN we can deal only with discrete quantities.

Table 6. Dry out cdf. In the second column the GSPN analytical results are reported; the third and the fourth column show the bounds returned by the simulation on the FSPN.

hours	dry out cdf	min	max
100	0.004463	0.002845	0.005355
200	0.022077	0.020963	0.027037
300	0.044846	0.041510	0.049890
400	0.065827	0.062215	0.072385
500	0.082568	0.076387	0.087613
600	0.095014	0.087603	0.099597
700	0.103939	0.095643	0.108157
800	0.110227	0.101947	0.114853
900	0.114622	0.105926	0.119074
1000	0.117689	0.109810	0.123190

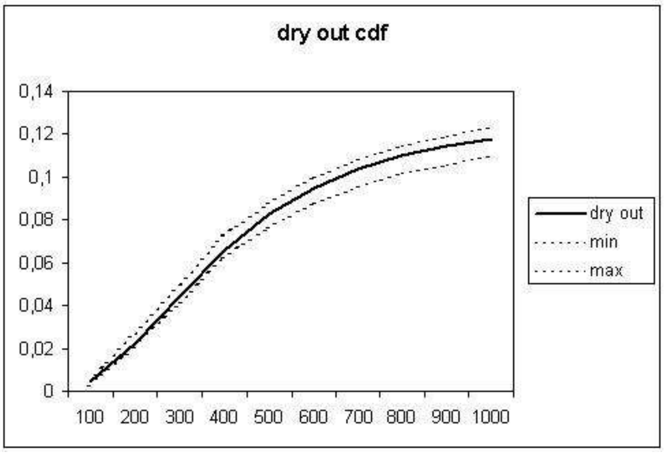

Figure 5. Dry out cdf. The solid line shows the GSPN analytical results; the dashed lines show the FSPN simulation results.

Table 7. Overflow cdf. In the second column the GSPN analytical results are reported; the third and the fourth column show the bounds returned by the simulation on the FSPN.

hours	overflow cdf	min	max
100	0.074208	0.068572	0.079228
200	0.195182	0.191723	0.209277
300	0.292146	0.284943	0.306257
400	0.359876	0.350404	0.373996
500	0.405374	0.397253	0.422347
600	0.435689	0.428575	0.454625
700	0.455953	0.448382	0.475018
800	0.469595	0.462773	0.489827
900	0.478857	0.471251	0.498549
1000	0.485200	0.477266	0.504734

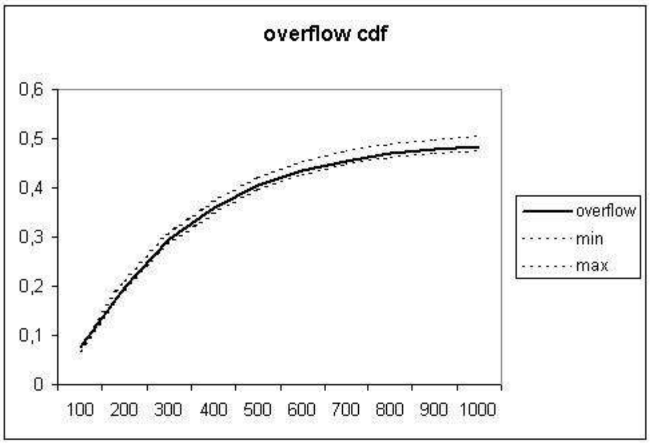

Figure 6. Overflow cdf. The solid line shows the GSPN analytical results; the dashed lines show the FSPN simulation results.

3 ADDING MODELLING COMPLEXITY

In Marseguerra & Zio (1996) some variations to the initial version of the benchmark are also proposed and simulated. In this paragraph, we concentrate on two of them: in the first case, the value of a component failure rate depends on the current state of the component; in the second case, the controller has a 10% probability of failing on demand, so the controller may not act on the components state even if the liquid level is outside the region of correct functioning (HLA<H<HLB).

In order to evaluate the reliability of the system in these cases, we applied the same method used for the initial version of the benchmark. In both cases, the analytical results obtained through the discretization of the liquid level in the GSPN, are still coherent with the results given by the simulation on the corresponding FSPN, and with the results reported in Marseguerra & Zio (1996) obtained by Monte Carlo simulation.

3.1 *The case with state dependent failure rates*

In this version of the system, the failure rates depend on the current state of the component; Table 8 shows this situation with the indication of the new rates. The probability of the dry out and overflow conditions vs time are reported in Table 9 and in Table 10 respectively.

Table 8. Failure rates for each state of the components.

Comp.	from state	to state	failure rate
P1	ON	stuck (ON or OFF)	0.004566
P1	OFF	stuck ON	0.045662
P1	OFF	stuck OFF	0.456621
P2	ON	stuck ON	0.057142
P2	ON	stuck OFF	0.571429
P2	OFF	stuck (ON or OFF)	0.005714
V	ON	stuck (ON or OFF)	0.003125
V	OFF	stuck ON	0.031250
V	OFF	stuck OFF	0.312500

Table 9. Dry out cdf in the case with state dependent failure rates.

hours	dry out cdf	min	max
100	0.019585	0.016763	0.022237
200	0.039143	0.038078	0.046122
300	0.053337	0.048871	0.057929
400	0.063036	0.058176	0.068024
500	0.069508	0.064140	0.074460
600	0.073770	0.068957	0.079643
700	0.076553	0.071754	0.082646
800	0.078357	0.073877	0.084923
900	0.079520	0.074649	0.085751
1000	0.080267	0.075518	0.086682

Table 10. Overflow cdf in the case with state dependent failure rates.

hours	overflow cdf	min	max
100	0.077879	0.070307	0.081093
200	0.167562	0.160161	0.176239
300	0.234353	0.221677	0.240523
400	0.280056	0.265212	0.285788
500	0.310778	0.295452	0.317148
600	0.331513	0.316283	0.338717
700	0.345659	0.331619	0.354581
800	0.355426	0.340273	0.363527
900	0.362243	0.345387	0.368813
1000	0.367048	0.350502	0.374098

3.2 *The case with failure on demand of the controller*

The results of the analysis of the GSPN and of the simulation on the FSPN in this case, are reported in Table 11 (dry out) and in Table 12 (overflow). Figure 7 and Figure 8 show the dry out and the overflow cdf curves respectively, in all the cases dealt so far.

Table 11. Dry out cdf in the case with failure on demand of the controller.

hours	dry out cdf	min	max
100	0.008493	0.006782	0.010418
200	0.030211	0.028210	0.035190
300	0.054819	0.051457	0.060743
400	0.076268	0.072526	0.083474
500	0.092903	0.088668	0.100732
600	0.105077	0.100297	0.113103
700	0.113728	0.110004	0.123396
800	0.119789	0.113890	0.127510
900	0.124012	0.118265	0.132135
1000	0.126952	0.121279	0.135321

Table 12. Overflow cdf in the case with failure on demand of the controller.

hours	overflow cdf	min	max
100	0.080069	0.072622	0.083578
200	0.205031	0.189180	0.206620
300	0.303499	0.285827	0.307173
400	0.371438	0.349814	0.373386
500	0.416654	0.397450	0.422550
600	0.446577	0.424338	0.450262
700	0.466482	0.441680	0.468120
800	0.479840	0.455281	0.482119
900	0.488890	0.464153	0.491247
1000	0.495079	0.470265	0.497535

4 MODELLING THE COMPONENTS REPAIR

4.1 *Repair policy*

Another variation proposed in Marseguerra & Zio (1996) to the initial version of the benchmark, deals with the possibility of repairing the failed (stuck) components. The reliability of the system can be improved in a considerable way by introducing the repair process. In the case of the benchmark, the controller discovers a failure by observing the liquid level H: if H is not inside the region of correct functioning (HLA<H<HLB), the controller suspects the occurrence of a failure, so it enables the repair process for the stuck components. The time to repair of a component is a random variable obeying a negative exponential distribution with the repair rate equal to 0.2 1/h.

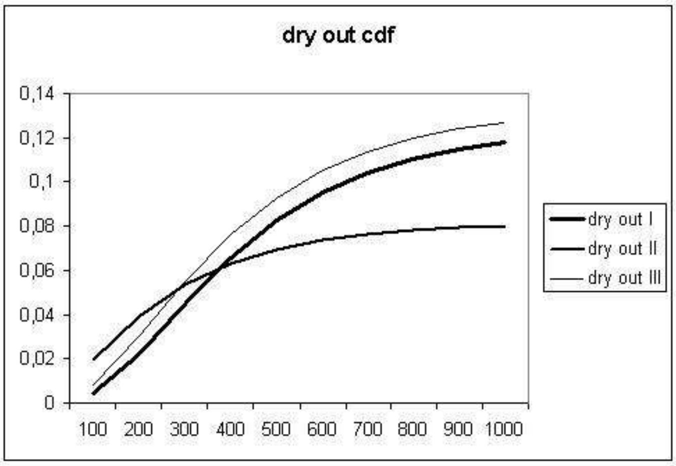

Figure 7. Dry out cdf for the initial version of the benchmark (I), for the version with state dependent failure rates (II), and for the version with failure on demand of the controller (III).

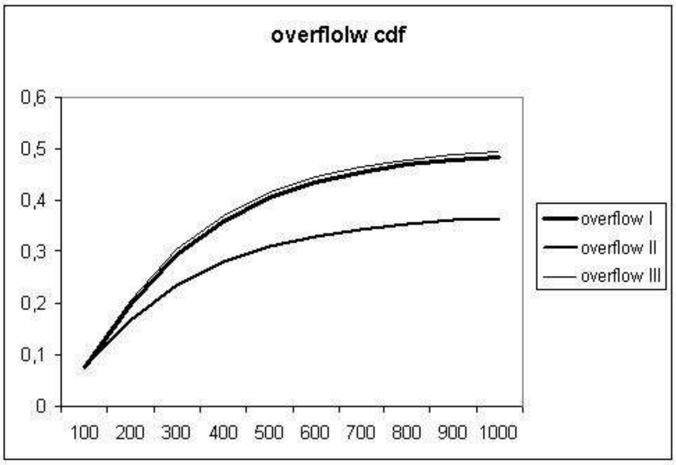

Figure 8. Overflow cdf for the initial version of the benchmark (I), for the version with state dependent failure rates (II), and for the version with failure on demand of the controller (III).

The repair is only allowed while the liquid level is outside the region of correct functioning (grace period); the effect of the repair consists of removing the stuck condition of a component. So, after the repair, a component can respond again and immediately to the controller orders changing its state if necessary. In order to have a significant grace period with respect to the repair time, the thresholds for the dry out and the overflow conditions, are now –5 and +5 respectively.

4.2 *GSPN model of the repairable system*

With the aim of modelling the presence of the repair process in the system, and the new thresholds, we changed the discretization of the liquid level H, and added some new places and transitions in the GSPN model (Fig. 9); Table 13 shows the new correspondence between H and the number of tokens inside the place called *LEVEL*.

In the current GSPN, we have some new elements: the transition *tooHIGH* fires when *LEVEL* contains at least seven tokens (H ≥ HLB), and puts one token in the place *dangerHIGH* (enabling the immediate transitions modelling both pumps stop and the activation of the valve). In a similar way, the transition *tooLOW* fires when *LEVEL* contains less than six tokens (H ≤ HLA) and puts one token in the place *dangerLOW* (enabling the activation of both pumps and the valve stop).

Table 13. Correspondence between H and the number of tokens in *LEVEL*.

H	#tokens	Condition
> +5	12	overflow
+5	11	HLP
+4	10	
+3	9	
+2	8	
+1	7	HLB
0	6	correct functioning
-1	5	HLA
-2	4	
-3	3	
-4	2	
-5	1	HLV
< -5	0	dry out

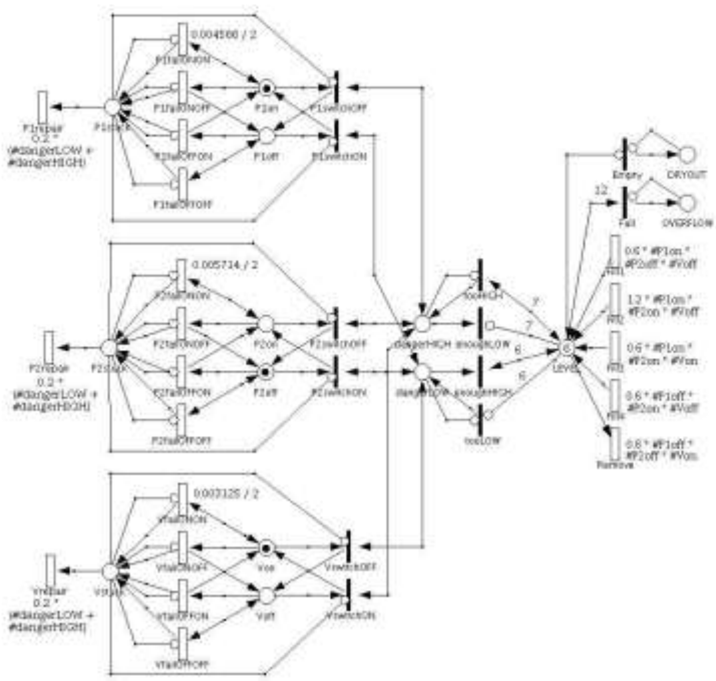

Figure 9. GSPN model of the repairable system.

The presence of one token inside the place *dangerLOW* or *dangerHIGH*, means we are in the grace period, so a timed transition modelling the repair, is enabled for each component; the firing rate of such transitions is the repair rate. In the case of P1, the transition *P1repair* removes the token from the place *P1stuck*. The repair is allowed only during the grace period, so if H is back to the correct functioning region (six tokens inside *LEVEL*) as a consequence of a repair, the transition *enoughLOW* or *enoughHIGH* fires removing the token from *dangerHIGH* or *dangerLOW* respectively, and disabling the repairs.

4.3 *FSPN model of the repairable system*

As in the initial case, we verify the correctness of the analytical results on the GSPN by comparison with the simulation results computed on the FSPN model of the system (such results are reported in section 4.4). Figure 10 shows the FSPN including the repair process; Table 14 shows the correspondence between the level in *L* and the new thresholds.

Table 14. Correspondence between H and the level in the fluid place *L*.

H	*L*	Condition
+5	10	HLP
+4	9	
+3	8	
+2	7	
+1	6	HLB
0	5	initial level
-1	4	HLA
-2	3	
-3	2	
-4	1	
-5	0	HLV

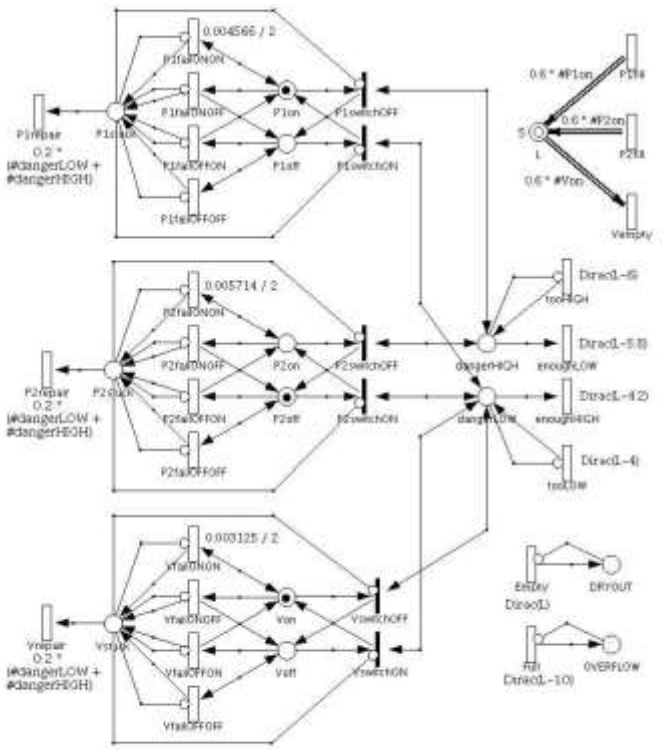

Figure 10. FSPN model for the repairable system.

With respect to the initial FSPN, we added some new elements in order to model the repair in the grace period: the firing rate of the timed transition *tooHIGH* is the function *Dirac*(*L*-6), so it fires when H reaches HLB putting one token in the place *dangerHIGH* which enables the immediate transitions modelling the relative control law on the components. The timed transition *tooLOW* instead, fires when H reaches HLA putting one token in *dangerLOW*, enabling the immediate transitions modelling the control law in this condition.

The repair is modelled in the same way as in the GSPN, by three timed transitions enabled by the presence of one token inside the place *dangerHIGH* or *dangerLOW*. The grace period ends when H is back to the correct functioning region; in this case, the transition *enoughLOW* or the transition *enoughHIGH* fires removing the token from *dangerHIGH* or *dangerLOW* respectively.

The dry out occurs when the level in *L* is equal to 0 (transition *Empty)* or when it reaches 10 (transition *Full*).

4.4 *Comparison of the results obtained on both models.*

The probabilities of the dry out and of the overflow conditions, computed in the analytical way on the GSPN and by simulation on FSPN, are reported in Table 15 (dry out) and in Table 16 (overflow) for a mission time varying from 50 to 500 hours. As in the previous system configurations, the analytical results are coherent with those returned by the simulation.

Table 15. Dry out cdf in the case with repair.

hours	dry out cdf	min	max
50	0.000010	0.000000	0.000015
100	0.000062	0.000000	0.000070
150	0.000156	0.000000	0.000296
200	0.000280	0.000008	0.000792
250	0.000419	0.000120	0.001080
300	0.000561	0.000246	0.001354
350	0.000697	0.000246	0.001354
400	0.000825	0.000312	0.001488
450	0.000941	0.000312	0.001488
500	0.001045	0.000521	0.001879

Table 16. Overflow cdf in the case with repair.

hours	overflow cdf	min	max
50	0.000967	0.000380	0.001620
100	0.003168	0.001844	0.003956
150	0.006015	0.003442	0.006158
200	0.009238	0.005891	0.009309
250	0.012682	0.009762	0.014038
300	0.016254	0.014259	0.019341
350	0.019893	0.017787	0.023413
400	0.023558	0.021807	0.027993
450	0.027221	0.025474	0.032126
500	0.030860	0.028588	0.035612

5 CONCLUSIONS

Though simulation is the typical method to evaluate the reliability of hybrid and dynamic systems, in this paper we showed the way to adapt a form of discrete modelling and analysis such as GSPNs, to the benchmark proposed in Marseguerra and Zio (1996) and its variations. The success of our approach has been verified by comparing the analytical results we obtained in the several cases using GSPNs, with those returned by simulation on FSPNs (a model specifically studied to deal with continuous quantities) and with the results reported in Marseguerra and Zio (1996). So, we can conclude that the use of GSPNs is suitable in several cases of dynamic reliability analysis.

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