Business Process Verification with Constraint Temporal Answer Set Programming ∗

Laura Giordano
DISIT, Università del Piemonte Orientale, Italy

Alberto Martelli, Matteo Spiotta
Dipartimento di Informatica, Università di Torino, Italy

Daniele Theseider Dupré
DISIT, Università del Piemonte Orientale, Italy

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Abstract

The paper provides a framework for the verification of business processes, based on an extension of answer set programming (ASP) with temporal logic and constraints. The framework allows to capture expressive fluent annotations as well as data awareness in a uniform way. It allows for a declarative specification of a business process but also for encoding processes specified in conventional workflow languages. Verification of temporal properties of a business process, including verification of compliance to business rules, is performed by bounded model checking techniques in Answer Set Programming, extended with constraint solving for dealing with conditions on numeric data.

1 Introduction

The verification of business process compliance to business rules and regulations has gained a lot of interest in recent years, leading to the development of a process annotation approach (Governatori and Sadiq 2009; Weber et al. 2010; Hoffmann et al. 2009): information relevant for compliance verification is added, capturing the semantics of atomic tasks execution through preconditions and effects. The treatment of data in process verification has also attracted growing interest, with the definition of *artifact-centric* and *data-centric* process models (Nigam and Caswell 2003; Deutsch et al. 2009).

In this paper we combine the two perspectives and propose a framework, based on Answer Set Programming (ASP) (Gelfond 2007), for the specification and verification of business processes, which integrates the treatment of data in business processes with an expressive treatment of annotations, that allows for the specification of conditional effects of atomic tasks as well as for the definition of causal dependencies among annotations. This semantic specification provides background knowledge common to the process and to the compliance rules to be verified. Following (D'Aprile et al. 2010), annotations are specified by an action theory which defines the direct and indirect effects and the

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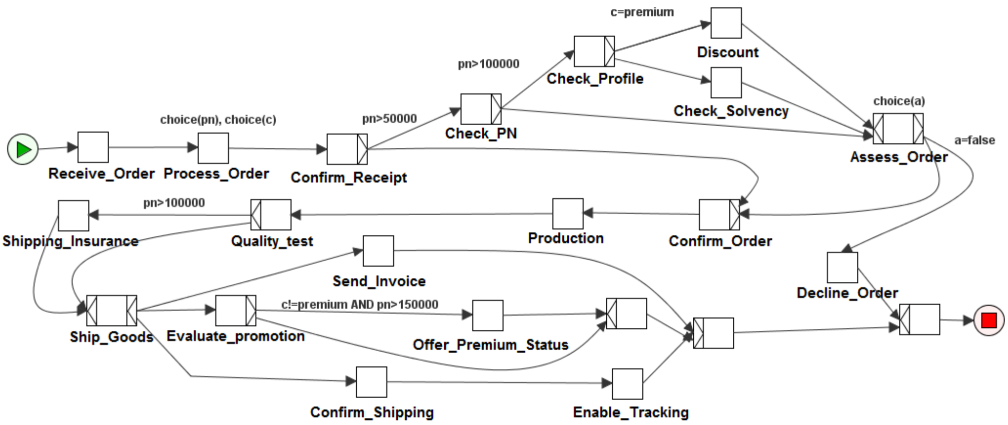

Fig. 1. Example business process in YAWL

preconditions of atomic tasks; effects are fluents representing properties of the world that are affected by the execution of the tasks and are subject to the compliance rules. We adopt a similar approach to capture data properties. Formulas involving conditions on such data may be verified, and to deal with numeric data we rely on constraint solving.

In particular, we combine two extensions of ASP: Temporal ASP in (Giordano et al. 2013a), which allows for the specification of temporal action domains, and the Constraint ASP approach in (Gebser et al. 2009) where ASP is enriched with constraint solving in a modular way: constraints are labeled as true or false by the ASP solver, and a constraint solver performs propagation on the constraint store possibly detecting inconsistency.

This approach is well suited for a declarative specification of the business process, which has been advocated by many authors in the literature (van der Aalst and Pesic 2006; Singh 2000; Montali et al. 2010), but it also allows for encoding processes specified in current business process management systems. In the paper we concentrate on the latter option, showing how a model in a subset of the YAWL language (van der Aalst and ter Hofstede 2005), and its annotations, can be mapped into the framework.

Verification of business process properties, including compliance to business rules, is done through Bounded Model Checking (Biere et al. 2003) and exploits the approach in (Giordano et al. 2013a) for DLTL bounded model checking in ASP, which extends Bounded LTL Model Checking with Stable Models in (Heljanko and Niemelä 2003).

2 Example

As a motivating example, let us consider a variation of the order-production-delivery business process model in (Knuplesch et al. 2010) whose model in YAWL is in Figure 1. It involves *net variables* (i.e., global variables) like the piece number of an order, an integer *pn*; the model provides information on which atomic task has the variable as output ("Process Order", in the example) and XOR splits include conditions on such a variable (Appendix A illustrates the basic control flow elements used). The process in Figure 1 should be checked to be compliant with respect to business rules such as:

(a) After confirming an order, goods have to be shipped eventually.

(b) An order shall either be confirmed or declined.
(c) Orders with a piece number beyond 50000 shall be approved before they are confirmed.
(d) For orders of a non-premium customer with a piece number beyond 80000 a solvency check is necessary before assessing the order.

Such rules hold, except for the last one: for orders of a non-premium customer with a piece number between 80000 and 100000, flow goes from "Check PN" to "Assess Order" without executing "Check Solvency".

Rule verification requires establishing a link between atomic tasks in the business process and domain properties in rules, especially in case different tasks have effect on the same property, or tasks have effects on properties that are related to the ones occurring in the rules. This issue motivates the idea of a semantic *annotation* of the atomic tasks, in terms of their effects and preconditions (Governatori and Sadiq 2009). As background knowledge, in (Weber et al. 2010; Hoffmann et al. 2009) process annotation includes a domain ontology, namely a theory in clausal form. Our approach to process annotation builds on work in reasoning about actions and change (Reiter 2001) and, specifically, is based on the temporal action language in (Giordano et al. 2013a) enriched with constraint atoms (Gebser et al. 2009) to express conditions on process variables both in the process model and in rules to be verified, as detailed in the next section.

3 A Constraint Temporal Action Language

In this section we recall the temporal logic DLTL (Henriksen and Thiagarajan 1999) and the temporal ASP language in (Giordano et al. 2013a), extending them with constraints in the line of (Gebser et al. 2009).

DLTL extends LTL with temporal operators enriched with program expressions; the next state modality can be indexed by actions, and the until operator $\mathcal{U}^\pi$ can be indexed by a program π which, as in PDL (Harel 1984), can be a regular expression built from atomic actions using sequence (;), nondeterministic choice (+) and finite iteration ($*$).

We further extend DLTL allowing, as a base case of formulae, constraints in a language $\mathcal{C}$. Such constraints involve a set $\mathcal{V}$ of variables, and suitable function and relation symbols, but we will refer to them as *constraint atoms* since, as in (Gebser et al. 2009), they will be treated as atoms at the answer set level, and at the temporal logic level. Similarly to (Gebser et al. 2009), a function $\gamma : \mathcal{C} \to C$ maps constraint atoms to constraints in the sense of (Dechter 2003): each $x \in \mathcal{V}$ has a domain $dom(x)$ (which in this paper we assume to be finite), and a constraint $c \in C$ is a pair (S, R) where R is a k-ary relation on a vector S of variables in $\mathcal{V}$: for $S = (x_1, \ldots, x_k)$, $R \subseteq dom(x_1) \times \ldots \times dom(x_k)$. For a constraint $c = (S, R)$, let $S(c) = S$ and $R(c) = R$. Given an assignment $A : \mathcal{V} \to \bigcup_{x\in\mathcal{V}} dom(x)$, where $A(x) \in dom(x)$, for a constraint (S, R), with $S = (x_1, \ldots, x_k)$, let $A(S) = (A(x_1), \ldots, A(x_k))$. The mapping γ provides a predefined interpretation of the constraint language, i.e., it defines whether an assignment A satisfies a $g \in \mathcal{C}$, which we write $A \models_\gamma g$ and holds iff $A(S(\gamma(g))) \in R(\gamma(g))$. E.g., for the constraint language with variables $\{x, y, z, \ldots\}$, all with the integer domain, and the usual arithmetic operators and relations, for which γ provides the usual interpretation, for the assignment $A((x, y, z)) = (10, 20, 15)$ we have that $A \models_\gamma x + y > z$.

Let $\Sigma = \{a_1, \ldots, a_n\}$ be a finite non-empty alphabet of actions. Let Σ^* and Σ^ω be the set of finite and infinite words on Σ. Let $\Sigma^\infty = \Sigma^* \cup \Sigma^\omega$. We denote by σ, σ' the words over Σ^ω and by τ, τ' the words over Σ^*. Moreover, we denote by $\leq$ the usual prefix ordering over Σ^*, namely, $\tau \leq \tau'$ iff $\exists \tau''$ such that $\tau\tau'' = \tau'$, and $\tau < \tau'$ iff $\tau \leq \tau'$ and $\tau \neq \tau'$. For $u \in \Sigma^\infty$, we denote by *prf(u)* the set of finite prefixes of u.

Let the set of programs (regular expressions) generated by Σ be $Prg(\Sigma) ::= a \mid \pi_1 + \pi_2 \mid \pi_1;\pi_2 \mid \pi^*$, where $a \in \Sigma$ and π_1, π_2, π range over $Prg(\Sigma)$. A set of finite words can be associated with each program by the mapping $[[]] : Prg(\Sigma) \to 2^{\Sigma^*}$ in the usual way.

Let $\mathcal{P} = \{p_1, p_2, \ldots\}$ be a countable set of atomic propositions, let $\mathcal{C}$ be a constraint language, such that, for $g \in \mathcal{C}$, $\gamma(g) \in C$ is a constraint on a subset of variables $\mathcal{V}$, with domains $dom(x)$ for $x \in \mathcal{V}$, where $D = \bigcup_{x \in \mathcal{V}} dom(x)$. The set of formulas of DLTL$(\Sigma, \mathcal{C})$ is:

$$\text{DLTL}(\Sigma, \mathcal{C}) ::= \top \mid \bot \mid p \mid g \mid \neg\alpha \mid \alpha \vee \beta \mid \alpha \mathcal{U}^\pi \beta$$

where $\top$ and $\bot$ stand for *true* and *false*, $p \in \mathcal{P}$, $g \in \mathcal{C}$, $\pi \in Prg(\Sigma)$ and α, β range over DLTL$(\Sigma, \mathcal{C})$. From the until operator, the derived modalities $\langle\pi\rangle$, $[\pi]$, $\bigcirc$ (next), $\mathcal{U}$, $\Diamond$ and $\Box$ can be defined as follows: $\langle\pi\rangle\alpha \equiv \top\mathcal{U}^\pi\alpha$, $[\pi]\alpha \equiv \neg\langle\pi\rangle\neg\alpha$, $\bigcirc\alpha \equiv \bigvee_{a\in\Sigma}\langle a\rangle\alpha$, $\alpha\mathcal{U}\beta \equiv \alpha\mathcal{U}^{\Sigma^*}\beta$, $\Diamond\alpha \equiv \top\mathcal{U}\alpha$, $\Box\alpha \equiv \neg\Diamond\neg\alpha$, where, in $\mathcal{U}^{\Sigma^*}$, Σ is taken to be a shorthand for the program $a_1 + \ldots + a_n$.

A model of DLTL$(\Sigma, \mathcal{C})$ is a triple $M = (\sigma, V, v)$ where $\sigma \in \Sigma^\omega$, $V : prf(\sigma) \to 2^{\mathcal{P}}$ is a valuation function for propositions, and $v : prf(\sigma) \to (\mathcal{V} \to D)$ provides, for each $\tau \in prf(\sigma)$, an assignment $v(\tau)$ for constraint variables. A prefix τ of σ identifies the state reached after executing τ. Given a model $M = (\sigma, V, v)$, a prefix $\tau \in prf(\sigma)$ and a formula α, the satisfiability of a formula α at τ in M, written $M, \tau \models \alpha$, is defined as usual for boolean formulas, and as follows for $p \in \mathcal{P}$, $g \in \mathcal{C}$ and until formulas:

- $M, \tau \models p$ iff $p \in V(\tau)$;
- $M, \tau \models g$ iff $v(\tau) \models_\gamma g$;
- $M, \tau \models \alpha\mathcal{U}^\pi\beta$ iff there exists $\tau' \in [[\pi]]$ such that $\tau\tau' \in prf(\sigma)$ and $M, \tau\tau' \models \beta$. Moreover, for every τ'' such that $\varepsilon \leq \tau'' < \tau'$, $M, \tau\tau'' \models \alpha$.

A formula α is satisfiable iff there is a model $M = (\sigma, V, v)$ and a finite word $\tau \in prf(\sigma)$ such that $M, \tau \models \alpha$.

Observe that, although the extended DLTL language includes (constraint) variables, we assume that such variables have finite domains; hence, the language is, essentially, a propositional language, and its decidability follows from the decidability of DLTL (Henriksen and Thiagarajan 1999).

A *domain description* is a pair $(P, \mathcal{Q})$, where P is a set of laws describing the effects and executability preconditions of actions, and $\mathcal{Q}$ is a set of DLTL formulas.

Laws in P are defined as follows[1]. A *fluent literal* l is a $p \in \mathcal{P}$ or its negation $\neg p$. A *constraint literal* is a $g \in \mathcal{C}$. If l is a fluent literal, or $\bot$, representing the inconsistency, then $[a]l, \bigcirc l$ (for $a \in \Sigma$) are *temporal fluent literals*. If l is a constraint literal, then $[a]l, \bigcirc l$ (for $a \in \Sigma$) are *temporal constraint literals*. We use *temporal literal* as a shorthand for temporal (fluent or constraint) literal. *Dynamic constraint literals* are constraint literals

[1] To avoid confusion with constraint variables, we do not define laws with variables, which, as usual in ASP, would be a shorthand for the set of their ground instances.

in the language $\mathcal{C}$ extended with variables $\mathcal{V}^\circ = \{x^\circ, \text{ for } x \in \mathcal{V}\}$; variable x° represents "x in the next state". Given a literal l of any of the types above, *not* l represents the default negation of l. A literal, possibly preceded by a default negation, will be called an *extended literal.* Laws in P have the form

$$l_0 \leftarrow l_1, \ldots, l_m, not\ l_{m+1}, \ldots, not\ l_n \quad (1)$$

where l_0 is a fluent literal or temporal fluent literal, and $l_1, \ldots, l_n$ are fluent literals, constraint literals, temporal literals or dynamic constraint literals, with the following restrictions: (i) If l_0 is a fluent literal, then the body cannot contain temporal literals or dynamic constraint literals; (ii) If $l_0 = [a]l$, then the temporal literals in the body must have the form $[a]l'$; (iii) If $l_0 = \bigcirc l$, then the temporal literals in the body must have the form $\bigcirc l'$.

A *state* is a set of fluent literals and an assignment to constraint variables. A state is said to be *consistent* if it is not the case that both f and $\neg f$ belong to the state, or that $\perp$ belongs to the state. The execution of an action in a state may possibly change the values of fluents in the state through its direct and indirect effects, as well as the values of constraint variables, thus giving rise to a new state. We assume that a law as (1) can be applied in all states while, when prefixed with **Init**, it only applies to the initial state. With the restrictions (i) - (iii) on (1), possible successors of a state S only depend on S.

In (Giordano et al. 2013a) the semantics of a domain description is defined by extending the notion of *answer set* (Gelfond 2007) to *temporal answer sets*, so to capture the linear structure of temporal models. In particular, the notion of temporal literals is extended to those of the form $[a_1; \ldots; a_k]l$ for an action sequence $a_1; \ldots; a_k$ and a fluent literal l; a *partial temporal interpretation over* σ, where $\sigma \in \Sigma^\omega$, is defined as a pair (σ, S) where S is a set of temporal literals $[a_1; \ldots; a_k]l$, such that $a_1 \ldots a_k$ is a prefix of σ, and S is required not to contain $[a_1; \ldots; a_k]\bar{l}$ where $\bar{l}$ is the complementary literal of l. Membership of $[a_1; \ldots; a_k]l$ in S means that l is true in the state after executing $a_1 \ldots a_k$.

Temporal answer sets of the first component P of a domain description are defined in (Giordano et al. 2013a) as partial temporal interpretations, extending the notion of answer set to compute a different reduct for each prefix $a_1 \ldots a_k$ of a σ. Such a reduct is a set of rules of the form $[a_1; \ldots; a_k](H \leftarrow Body)$.

In order to extend temporal answer sets to take constraint literals into account, we follow (Gebser et al. 2009), where a constraint answer set is defined with respect to an assignment to constraint variables. We then define a constraint temporal answer set with respect to a $v : prf(\sigma) \rightarrow (V[P] \rightarrow D)$ which provides, for each prefix of σ, an assignment for variables $V[P] \subseteq \mathcal{V}$ occurring in constraint atoms in P.

Satisfiability of literals is defined extending (Giordano et al. 2013a). Note that while for temporal (fluent and constraint) literals we take into account the next state, for dynamic constraint literals the assignments at both the current state and the next one have to be considered. To simplify the definition, we extend v to $\hat{v}$ which, for each prefix $a_1 \ldots a_k$ of σ, provides an assignment also to variables $x^\circ \in \mathcal{V}^\circ$ as follows: $\hat{v}(a_1 \ldots a_k)(x^\circ) = v(a_1 \ldots a_k a_{k+1})(x)$, where $a_1 \ldots a_k a_{k+1}$ is a prefix of σ. Then, satisfiability of literals in (σ, S) at k (where $\sigma = a_1 \ldots a_k, \ldots$), wrt to a v, is defined as follows, where l is a fluent literal, g is a constraint or dynamic constraint literal, h is a fluent or constraint literal, *not* m is an extended literal:

$(\sigma, S), v, k \not\models \bot$
$(\sigma, S), v, k \models l$ *iff* $[a_1; \ldots; a_k]l \in S$,
$(\sigma, S), v, k \models g$ *iff* $\hat{v}(a_1 \ldots a_k) \models_\gamma g$
$(\sigma, S), v, k \models [a]h$ *iff* $(\sigma, S), v, k+1 \models h$, or $a \neq a_{k+1}$
$(\sigma, S), v, k \models \bigcirc h$ *iff* $(\sigma, S), v, k+1 \models h$
$(\sigma, S), v, k \models \mathit{not}\ m$ *iff* $(\sigma, S), v, k \not\models m$

The following definitions are similar to the definition of constraint reduct in (Gebser et al. 2009), simplifying or eliminating rules based on the interpretation of constraint literals; we start by defining the reduct of a single rule for a sequence $a_1, \ldots, a_k$.

Definition 1
The *constraint reduct* of a rule $\Box(H \leftarrow \mathit{Body})$ in P, wrt to (σ, S), to $v : \mathit{prf}(\sigma) \rightarrow (V[P] \rightarrow D)$ and to the prefix $a_1, \ldots, a_k$ of σ, is:

- none, if for some (temporal, extended, dynamic) constraint literal l in *Body*, $(\sigma, S), v, a_1 \ldots a_k \not\models l$
- $[a_1, \ldots, a_k](H \leftarrow \mathit{Body}')$ otherwise, where Body' is *Body* with (temporal, extended, dynamic) constraint literals removed.

Definition 2
The *constraint reduct,* $P^{(\sigma,S),v}_{a_1,\ldots,a_k}$*, of* P *relative to* (σ, S)*, to the prefix* $a_1, \ldots, a_k$ *of* σ *and to* $v : \mathit{prf}(\sigma) \rightarrow (V[P] \rightarrow D)$, is the set of all the rules

$$[a_1; \ldots; a_k](H \leftarrow t_1, \ldots, t_m)$$

such that $[a_1; \ldots; a_k](H \leftarrow t_1, \ldots, t_m, \mathit{not}\ t_{m+1}, \ldots, \mathit{not}\ t_n)$ is the constraint reduct of a rule in P wrt (σ, S), v and $a_1, \ldots, a_k$; moreover, $(\sigma, S), a_1, \ldots, a_k \not\models t_i$, for all $i = m+1, \ldots, n$. The *constraint reduct* $P^{(\sigma,S),v}$ *of* P *relative to* (σ, S) *and* v is the union of all reducts $P^{(\sigma,S),v}_{a_1,\ldots,a_k}$ for all prefixes $a_1, \ldots, a_k$ of σ.

Definition 3
A partial temporal interpretation (σ, S) is a *constraint temporal answer set of* P wrt v if (σ, S) is a temporal answer set (Giordano et al. 2013a) of the constraint reduct $P^{(\sigma,S),v}$, i.e., S is minimal among the R such that (σ, R) satisfies the rules in $P^{(\sigma,S),v}$.

Well-defined domain descriptions are defined in (Giordano et al. 2013a) as the ones for which P has total temporal answer sets, and it is observed that total temporal answer sets correspond to temporal models (as in the semantics of DLTL). The same holds for the case with constraints, given a v which provides, for the temporal answer set and the temporal model, the same interpretation for constraint atoms. I.e., given a constraint temporal answer set (σ, S) of P wrt v, the corresponding temporal model is (σ, V_S, v) where $p \in V_S(a_1, \ldots, a_h)$ if and only if $[a_1; \ldots; a_h]p \in S$, for all propositions $p \in \mathcal{P}$. This leads to the following definition of extension of a domain description.

Definition 4
An *extension of a well-defined domain domain description* $(P, \mathcal{Q})$ *wrt* v is a (total) constraint temporal answer set (σ, S) of P wrt v such that the corresponding temporal model (σ, V_S, v) satisfies the formulae in $\mathcal{Q}$.

Verifying that a DLTL$(\Sigma, \mathcal{C})$ formula α is valid in a well-defined domain description $(P, \mathcal{Q})$ means checking that there is no extension of $(P, \mathcal{Q})$ satisfying $\neg\alpha$, i.e., there are no σ, S, v such that (σ, S) is an extension of $(P, \mathcal{Q} \cup \{\neg\alpha\})$ wrt v.

4 Process Modeling and Annotation

In (D'Aprile et al. 2010) we proposed to specify annotations in the action theory in (Giordano et al. 2013a) to define direct and indirect effects of atomic tasks, as well as their preconditions. In the following we describe how the extended action language with constraints introduced in section 3 can be used as an annotation language.

An *action law* has a temporal literal $[a]l$ as head:

$$[a]l \leftarrow l_1, \ldots, l_m not\ l_{m+1}, \ldots, not\ l_n$$

In case l is a fluent literal, it specifies that, under the conditions in the body, an action a (a task in the process) has l as a direct effect. In case l is a constraint literal (e.g., $0 \leq x \leq 1000$ to represent that a provides a value to x in its domain, or in a subset of a common domain for all variables) it specifies an effect on process data[2].

Nonmonotonic persistence of fluents can also be specified with action laws such as:

$$[a]f \leftarrow f, not\ [a]\neg f$$

A *precondition law* is an action law with $[a]\bot$ as head and it specifies that action a cannot be executed under the conditions in the body, which can be conditions on fluents, or constraints on process variables, e.g.:

$$[a]\bot \leftarrow x \geq 10000$$

means that a cannot be executed if $x \geq 10000$, i.e., it has $x < 10000$ as precondition.

Causal laws are used, in reasoning about actions and change, to specify relations between fluents, and therefore side effects of actions (*ramifications*). In the action language in (Giordano et al. 2013a), *static* causal laws involve fluent literals, while *dynamic* causal laws involve temporal fluent literals. For example, the static law $f \leftarrow g$ makes f true whenever g is true, while the dynamic law $\bigcirc f \leftarrow \neg g, \bigcirc g$ makes f true when g becomes true; but f may later be made false while g is still true.

Dynamic causal laws are useful, among other things, for a flexible modeling of processes with respect to rules they should be compliant with: in particular, to allow for activities that trigger obligations (e.g., confirming an order triggers the obligation to ship goods) and activities that cancel them (e.g., if the customer is allowed to cancel the order after confirmation, this cancels the obligation). Then, rather than verifying that obligations, once triggered, are eventually fulfilled (in the example, goods are shipped), one should verify that they are eventually either fulfilled or canceled. In (Giordano et al. 2013b) we present a general treatment, in our framework, of the types of obligations that in (Governatori 2010) are pointed out to be relevant for business process verification.

4.1 Declarative vs workflow specification of business processes

In this section we shortly describe how our language can be used for a declarative specification of business processes as well as for encoding a workflow representation of such processes.

[2] As in (Gebser et al. 2009), a rule $c \leftarrow Body$ with a constraint in the head is intended as the (less readable) $\bot \leftarrow not\ c, Body$.

The declarative specification of business processes has been advocated by many authors (van der Aalst and Pesic 2006; Singh 2000; Montali et al. 2010) as opposed to the more rigid transition based approach, which can be more constraining than necessary as regards the order of actions. A declarative specification of a business process can be given by exploiting the action theory above to define the effects of atomic tasks as well as their executability preconditions and, possibly, temporal logic constraints.

For instance, the fact than an order can be processed only after it has been received can be modeled by a precondition law: $[process\_order]\bot \leftarrow not\ order\_received$ To guarantee that it will be eventually executed, we can add in $\mathcal{C}$ the DLTL formula: $\Box[receive\_order]\Diamond\langle process\_order\rangle\top$.

Observe that, as DLTL is an extension of LTL, it is possible to provide an encoding in our language of all constraints in the ConDec language (Pesic and van der Aalst 2006). The additional expressivity, which comes from the presence of program expressions in DLTL, allows for a very compact encoding of certain declarative properties of the domain dealing with finite iterations. For instance, the property "action b must be executed immediately after any even occurrence of action a in a run" can be expressed by the temporal constraint: $\Box[(a; \Sigma^*; a)^*]\langle b\rangle\top)$, where Σ^* represents any finite action sequence[3].

On the other hand, workflow representation is widely used in process management systems and it represents a natural choice for modeling processes.

A translation from a workflow language to the action theory (or directly to ASP) can be defined. In particular, the basic constructs of the YAWL language[4] can be represented in the action language as follows, based on the enabling of arcs and tasks (details are provided in Appendix A). A precondition for the execution of a task a is its being enabled (represented by a fluent $enabled\_a$). Causal laws define that it is enabled when its only incoming arc is enabled (another fluent), or it is an AND-join and all incoming arcs are enabled, or it is a XOR-join and one incoming arc is enabled. The execution of a task disables (in action laws) the incoming arcs and enables the outgoing arcs; in case there is a nondeterministic XOR-split (named deferred choice in YAWL), the execution of one of the next tasks also disables arcs corresponding to other choices. For XOR-splits, which in YAWL have conditions on net variables associated with alternative arcs, such conditions are included in the action laws for arc enabling. In the example in section 2, the enabling for outgoing arcs from Confirm_Receipt is as follows (an implicit default is intended in YAWL on the arc to Confirm_Order):

$$[Confirm\_Receipt]en\_Confirm\_Receipt\_Check\_PN \leftarrow pn > 50000$$
$$[Confirm\_Receipt]en\_Confirm\_Receipt\_Confirm\_Order \leftarrow not\ pn > 50000$$

Values of a process variable x persist, except in case of tasks a which have x as output (a piece of information which is in the YAWL model); we represent this as the task having as effect the fluent $change\_x$ which is non persistent, and false by default:

$$x^\circ = x \leftarrow \bigcirc\neg change\_x$$
$$[a]change\_x$$

[3] In (D'Aprile et al. 2010) it has been shown that program expressions can be used to model the control flow of a business process in a rigid way. However, the solution in (D'Aprile et al. 2010) does not allow to deal with parallelism and non-structured workflows.

[4] We refer to YAWL as it is an open system, but our goal is not to reproduce its full semantics, since it includes constructs like the OR-join whose semantics is computationally complex.

$$\neg change\_x \leftarrow not\ change\_x$$

5 Translation to ASP

In (Giordano et al. 2013a), a translation of domain descriptions to ASP is defined, as well as an encoding in ASP of Bounded Model Checking (BMC) (Biere et al. 2003). BMC looks for paths representing possible executions of a system, and satisfying the required temporal formulae (including the negation of the formula to be proved valid). It can be shown that the search can be restricted to infinite paths which can be finitely represented as paths of length k with a back loop from state k to a previous state in the path. BMC then searches paths whose finite representation has a length bounded by an integer k, iteratively increasing k until a run satisfying the required temporal formulae is found (if one exists). The translation in (Giordano et al. 2013a) is defined so that extensions of the domain description correspond to answer sets of its ASP translation; in this way, an ASP solver can be used for finding extensions of $(P, \mathcal{Q} \cup \{\neg\alpha\})$, i.e., counterexamples to the validity of α in $(P, \mathcal{Q})$.

The translation is based on a representation of states as integers, starting from 0, so that state i is the state after a prefix $a_1, \ldots, a_i$ of a σ; it uses the predicates: $occurs(Action, State)$, which means that $Action$ occurs in $State$, $holds(Literal, State)$, meaning that $Literal$ is satisfied at $State$, and $sat(Formula, State)$ meaning that a DLTL $Formula$ is satisfied at $State$.

For example, an action law $\Box([a](\neg)f_0 \leftarrow t_1, \ldots, t_m, not\ t_{m+1}, \ldots, not\ t_n)$ is translated to

$$(\neg)holds(f_0, S') \leftarrow state(S), S' = S+1, occurs(a, S), h_1 \ldots h_m, not\ h_{m+1} \ldots not\ h_n$$

where $h_i = (\neg)holds(f_i, S')$ if $t_i = [a](\neg)f_i$ and $h_i = (\neg)holds(f_i, S)$ if $t_i = (\neg)f_i$.

The translation can be extended to a domain description $(P, \mathcal{Q})$ with constraints in section 3 as follows, using $value(x, S)$ to represent the value of process variable x in state S. Following (Gebser et al. 2009), instances of $value(x, S)$ for different instances of S will be CSP variables in the Constraint ASP program. Each constraint literal g in $(P, \mathcal{Q})$ is given a unique name $name\_g$, and the following rule is added to the definition of sat:

$$sat(name\_g, S) \leftarrow state(S), g_S$$

where g_S is g with each variable x replaced with $value(x, S)$. Similarly, for a dynamic constraint literal d named $name\_d$, the rule:

$$sat(name\_d, S) \leftarrow state(S), S' = S+1, d_{S,S'}$$

is added, where $d_{S,S'}$ is d with variables $x \in \mathcal{V}$ replaced with $value(x, S)$ and variables $x^\circ \in \mathcal{V}^\circ$ replaced with $value(x, S')$.

For constraint literals g, $[a]g$, $\bigcirc g$ occurring in laws in P, the translation (exemplified above for action laws) is similar to the one for non-constraint literals, except that g_S and $g_{S'}$ are used in place of $holds(f, S)$ and $holds(f, S')$. For dynamic constraint literals, $d_{S,S'}$ is used[5].

[5] Again, as in (Gebser et al. 2009), a rule $c \leftarrow Body$ with a constraint atom in the head is intended as $\leftarrow not\ c, Body$.

For example, the translations of the enabling condition for the XOR-split example in section 4.1, and of the persistence of the value for variable pn are:

$$holds(en\_Confirm\_Receipt\_Check\_PN, S') \leftarrow state(S), S' = S + 1,$$
$$occurs(Confirm\_Receipt, S), value(pn, S)\ \$ > 50000$$
$$value(pn, S')\ \$ = value(pn, S) \leftarrow state(S), S' = S + 1, not\ holds(change\_pn, S')$$

where, as in (Gebser et al. 2009), we emphasize with "\$" relations in constraint atoms[6].

The BMC encoding considers two states different if they differ on fluents (as in (Giordano et al. 2013a)), or on constraint literals in the domain description. In this sense, a state corresponds to a set of states in the sense of section 3, which assign single values to variables.

The correspondence results in (Giordano et al. 2013a) still hold. In particular:

Proposition 1
There is a one to one correspondence between the extensions (σ, S) of a domain description, such that σ can be finitely represented as a finite path with a back loop, and constraint answer sets (Gebser et al. 2009) of its translation.

The extension and the corresponding constraint answer set R are associated with the same temporal model (σ, V, v), in particular, an extension wrt v corresponds to an answer set R wrt a v_R, where $v_R(value(x, s)) = v(a_1, \ldots, a_s)(x)$, with $a_1, \ldots, a_s \in prf(\sigma)$.

6 Business process verification

The approach in (Giordano et al. 2013a) is well suited for reasoning about systems with infinite computations. However, in a YAWL model only executions that reach the end are considered sound (van der Aalst et al. 2008) (soundness includes other issues and its verification is a problem by itself, but we do not address it in this paper). To model this we include the formula $\Diamond end$ in the domain description, where end is a fluent which becomes true when the end condition is reached; we also introduce a *dummy* action, which does not change the state, and it is only executable (and is the only executable action) when the end condition is reached: its infinite repetition at the end of a finite execution transforms it in an infinite one with a loop (given by the *dummy* action). In practice, as an optimization of the ASP representation of BMC, we can avoid looking for arbitrary models with loops and restrict to computations corresponding to finite traces.

The basic iterative procedure of BMC is a partial decision procedure for checking validity: if no model exists, it would never stop. Completeness can be obtained for general formulae or for special classes of formulae with the techniques described in (Biere et al. 2003; Clarke et al. 2004; Biere et al. 2006; Giordano et al. 2012), and involves identifying a *completeness threshold*, i.e. a value t such that validity of a formula α can be checked without using a value $> t$ for the bound k (i.e., if a model satisfying $\neg\alpha$ exists, one can be found using bounds up to t). For example, the approach to completeness in (Giordano

[6] Since our implementation is based on *clingcon*, the system in (Gebser et al. 2009; Ostrowski and Schaub 2012), which deals only with the integer domain, the translation of variables with enumerated type (like c in the example, for the customer type) departs from the one described above, being based on parametric fluents $var(name, value)$, so that, e.g., a condition $c = premium$ becomes $holds(var(c, V), S), V = premium$.

et al. 2012) could be extended to the language with constraints in this paper. Computing thresholds, however, is in general challenging from a computational point of view even for special types of formulae; therefore, existing approaches would not necessarily provide a feasible solution in a wide range of practical cases. For loop-free workflows, like the one in the example in section 2, there is a longest run, whose length can be used as threshold.

We experimented the approach on a machine with Intel Xeon E5520 processors (2.26 Ghz) using *clingcon* (Gebser et al. 2009; Ostrowski and Schaub 2012), and an encoding with an optimization for the persistence of variable values.

For the example in section 2, rules (c) and (d) can be encoded, respectively, as:

$$\Box(pn > 50000 \wedge \langle Confirm\_Order \rangle \top \rightarrow a = true) \tag{2}$$

$$\Box(pn > 80000 \wedge c \neq premium \wedge \langle Assess\_Order \rangle \top \rightarrow solvency\_check\_done) \tag{3}$$

where $solvency\_check\_done$ is an effect of $Check\_Solvency$.

The length of the longest run is 21 and can be computed in 1.35 s using the translated domain description (searching for a run of length k, not necessarily reaching the end, for increasing values of k, and stopping when no such run is found). Using such a threshold, validity of (2) can be proved in 0.12 s, and a counterexample for (3) can be found in 0.10 s. If *clingcon* is not asked to return an assignment of single values to constraint variables, it computes a *weak* constraint answer set (Ostrowski 2012), where an approximation of the restricted domain for variables is given; in this case, in the computed answer set, the value of pn after $Process\_Order$, i.e., $value(pn, s)$ for $s \geq 3$, is given the domain [80001..100000], because the ASP solver assigns true to $value(pn, s) > 80000$ and false to $value(pn, s) > 100000$, and this is consistent for the constraint solver.

Similar running times were obtained for a variation of the example with run length up to 30 and more complex constraints, where branching is conditioned on the total cost tc of the order, given by $pn * uc$ (a unit cost, known to be in a given range), possibly modified by a discount.

In appendix B we report more experiments on the feasibility and scalability of the approach with current Constraint ASP technology. For the case where the process structure is a sequence of blocks similar to the one in the running example, we show problems where verification runs in seconds for processes with up to 200 activities (a size which is in line with the one of real-world processes in (Fahland et al. 2011)) and run length of more than 100 activities. The additional cost of constraint solving is acceptable, even though this does not hold when constraints involve expressions with several variables.

When there is significant parallelism in the process, all the interleavings are considered, however, verification is shown to run in a minute or less for problems where the number of different executions is up to 10^{28}.

7 Conclusions and related work

The paper presents an approach to the verification of the compliance of business processes, which allows to capture expressive fluent annotations as well as data awareness in a uniform way. The business process, its semantic annotation and the norms are encoded using temporal ASP rules, and temporal logic formulae. Compliance verification can be performed using BMC techniques in Constraint ASP, based on the approach developed in (Giordano et al. 2013a) for DLTL bounded model checking in ASP, which extends the approach for bounded LTL model checking with Stable Models in (Heljanko and Niemelä 2003). This paper enhances the approach to process compliance verification in (D'Aprile et al. 2010) by taking into consideration the data perspective, and especially arithmetic constraints, and alternative ways of modeling business processes. The paper concentrates on business process verification, but the proposed approach can also be used for the verification of clinical guidelines in the line of (Bottrighi et al. 2010), where, however, conditions on (numerical) data are not explicit modeled.

In (Knuplesch et al. 2010) a constraint solver is used to compute in advance consistent sets of conditions on numeric data (occurring in the model and in the formula to be verified), then performing model checking using such sets as abstractions of actual values. In our approach, the Constraint ASP solver performs both tasks at once.

The idea of introducing annotations on business processes for compliance verification has been proposed in (Ghose and Koliadis 2007; Governatori and Sadiq 2009; Weber et al. 2010). In particular, in (Governatori and Sadiq 2009) annotations and normative specifications are provided in the same logical language, the Formal Contract Language, combining Defeasible Logic (Antoniou et al. 2001) and Deontic Logic of Violations (Governatori and Rotolo 2006). Compliance is verified by traversing the graph describing the process and identifying the effects of tasks and the obligations triggered by task execution. Algorithms for propagating obligations through the process graph are defined.

In (Hoffmann et al. 2009) effects and preconditions of atomic tasks are sets of atomic formulas, and background knowledge is a theory in clausal form; I-Propagation (Weber et al. 2010) is exploited for computing annotations. In our approach the domain theory contains, rather than general clauses, directional causal rules (which avoid unintended conclusions when reasoning about side effects), and domain annotations are combined with data properties in a uniform approach.

An annotation approach is also used in (Lu et al. 2006) for the automatic verification of correctness of workflow models based on Hoare semantics.

In (Deutsch et al. 2009) a service over an artifact schema is defined as a triple: a precondition, a post-condition and a set of rules in a first-order temporal logic, which define changes on state relations. The authors identify a class of *guarded* artifacts for which verification of properties in a (guarded) first-order extension of LTL is decidable. In (Damaggio et al. 2011) decidability results are provided for verification with arithmetic constraints on infinite domains. Our (decidable) language is less expressive, not allowing for explicit quantification and assuming finite domains for variables.

In (Montali et al. 2010) the Abductive Logic Programming framework SCIFF (Alberti et al. 2008) is exploited in the declarative specification of business processes as well as in the verification of their properties. CLP is used for constraints on the timing of events. In (Alberti et al. 2005) expectations are used for modelling obligations and prohibitions and norms are formalized by abductive integrity constraints.

In (Roman and Kifer 2008) Concurrent Transaction Logic (CTR) is used to model and reason about general service choreographies. Service choreographies and contract requirements are represented in CTR. The paper addresses the problem of deciding if there is an execution of the service choreography that complies both with the service policies and the client contract requirements.

References

ALBERTI, M., CHESANI, F., GAVANELLI, M., LAMMA, E., MELLO, P., AND TORRONI, P. 2008. Verifiable agent interaction in abductive logic programming: the SCIFF framework. *ACM Trans. Comput. Log. 9,* 4.

ALBERTI, M., GAVANELLI, M., LAMMA, E., MELLO, P., TORRONI, P., AND SARTOR, G. 2005. Mapping of Deontic Operators to Abductive Expectations. *NORMAS*, 126–136.

ANTONIOU, G., BILLINGTON, D., GOVERNATORI, G., AND MAHER, M. J. 2001. Representation results for defeasible logic. *ACM Trans. on Computational Logic 2*, 255–287.

BIERE, A., CIMATTI, A., CLARKE, E. M., STRICHMAN, O., AND ZHU, Y. 2003. Bounded model checking. *Advances in Computers 58*, 118–149.

BIERE, A., HELJANKO, K., JUNTILA, T., LATVALA, T., AND SCHUPPAN, V. 2006. Linear Encodings of Bounded LTL model checking. *Logical Methods in Computer Science 2*, 1–64.

BOTTRIGHI, A., GIORDANO, L., MOLINO, G., MONTANI, S., TERENZIANI, P., AND TORCHIO, M. 2010. Adopting model checking techniques for clinical guidelines verification. *Artificial Intelligence in Medicine 48,* 1, 1–19.

CLARKE, E., KROENING, D., OUAKNINE, J., AND STRICHMAN, O. 2004. Completeness and complexity of bounded model checking. In *VMCAI*. 85–96.

DAMAGGIO, E., DEUTSCH, A., AND VIANU, V. 2011. Artifact systems with data dependencies and arithmetic. In *ICDT*.

D'APRILE, D., GIORDANO, L., GLIOZZI, V., MARTELLI, A., POZZATO, G. L., AND THESEIDER DUPRÉ, D. 2010. Verifying business process compliance by reasoning about actions. In *CLIMA XI*. 99–116.

DECHTER, R. 2003. *Constraint processing*. Elsevier Morgan Kaufmann.

DEUTSCH, A., HULL, R., PATRIZI, F., AND VIANU, V. 2009. Automatic verification of data-centric business processes. In *ICDT*. 252–267.

FAHLAND, D., FAVRE, C., KOEHLER, J., LOHMANN, N., VÖLZER, H., AND WOLF, K. 2011. Analysis on demand: Instantaneous soundness checking of industrial business process models. *Data Knowl. Eng. 70,* 5, 448–466.

GEBSER, M., OSTROWSKI, M., AND SCHAUB, T. 2009. Constraint answer set solving. In *ICLP*. 235–249.

GELFOND, M. 2007. Answer Sets. *Handbook of Knowledge Representation, chapter 7, Elsevier*.

GHOSE, A. AND KOLIADIS, G. 2007. Auditing business process compliance. *ICSOC, LNCS 4749*, 169–180.

GIORDANO, L., MARTELLI, A., AND THESEIDER DUPRÉ, D. 2012. Achieving completeness in bounded model checking of action theories in ASP. In *Proc. KR 2012*.

GIORDANO, L., MARTELLI, A., AND THESEIDER DUPRÉ, D. 2013a. Reasoning about actions with temporal answer sets. *Theory and Practice of Logic Programming 13*, 201–225.

GIORDANO, L., MARTELLI, A., AND THESEIDER DUPRÉ, D. 2013b. Temporal deontic action logic for the verification of compliance to norms in ASP. In *Proc. ICAIL 2013*.

GOVERNATORI, G. 2010. Law, logic and business processes. In *Third International Workshop on Requirements Engineering and Law*. IEEE.

GOVERNATORI, G. AND ROTOLO, A. 2006. Logic of Violations: A Gentzen System for Reasoning with Contrary-To-Duty Obligations. *Australasian Journal of Logic 4*, 193–215.

GOVERNATORI, G. AND SADIQ, S. 2009. The journey to business process compliance. *Handbook of Research on BPM, IGI Global*, 426–454.

HAREL, D. 1984. Dynamic logic. In *Handbook of Philosophical Logic, vol. 2*. 497–604.

HELJANKO, K. AND NIEMELÄ, I. 2003. Bounded LTL model checking with stable models. *Theory and Practice of Logic Programming 3,* 4-5, 519–550.

Henriksen, J. and Thiagarajan, P. 1999. Dynamic Linear Time Temporal Logic. *Annals of Pure and Applied logic 96,* 1-3, 187–207.

Hoffmann, J., Weber, I., and Governatori, G. 2009. On compliance checking for clausal constraints in annotated process models. *Information Systems Frontiers*.

Knuplesch, D., Ly, L. T., Rinderle-Ma, S., Pfeifer, H., and Dadam, P. 2010. On enabling data-aware compliance checking of business process models. In *Proc. ER 2010, 29th International Conference on Conceptual Modeling.* 332–346.

Lu, S., Bernstein, A. J., and Lewis, P. M. 2006. Automatic workflow verification and generation. *Theor. Comput. Sci. 353,* 1-3, 71–92.

Montali, M., Torroni, P., Chesani, F., Mello, P., Alberti, M., and Lamma, E. 2010. Abductive logic programming as an effective technology for the static verification of declarative business processes. *Fundam. Inform. 102,* 3-4, 325–361.

Nigam, A. and Caswell, N. S. 2003. Business artifacts: An approach to operational specification. *IBM Systems Journal 42,* 3, 428445.

Ostrowski, M. 2012. What is this thing called "clingcon"? A language description. Available at potassco.sourceforge.net.

Ostrowski, M. and Schaub, T. 2012. ASP modulo CSP: The clingcon system. *TPLP 12,* 4-5, 485–503.

Pesic, M. and van der Aalst, W. M. P. 2006. A declarative approach for flexible business processes management. In *Business Process Management Workshops, LNCS 4103*. Springer, 169–180.

Reiter, R. 2001. Knowledge in action. MIT Press.

Roman, D. and Kifer, M. 2008. Semantic web service choreography: Contracting and enactment. In *International Semantic Web Conference, LNCS 5318.* 550–566.

Singh, M. P. 2000. A social semantics for Agent Communication Languages. *Issues in Agent Communication, LNCS(LNAI) 1916*, 31–45.

van der Aalst, W. and ter Hofstede, A. 2005. YAWL: Yet Another Workflow Language. *Information Systems 30,* 4, 245–275.

van der Aalst, W., van Hee, K., ter Hofstede, A., Sidorova, N., Verbeek, H., Voorhoeve, M., and Wynn, M. 2008. Soundness of Workflow Nets: Classification, Decidability, and Analysiss. *BPM Center Report BPM-08-02, BPMcenter.org*.

van der Aalst, W. M. P. and Pesic, M. 2006. Decserflow: Towards a truly declarative service flow language. In *The Role of Business Processes in Service Oriented Architectures*. Dagstuhl Seminar Proceedings, vol. 06291.

Weber, I., Hoffmann, J., and Mendling, J. 2010. Beyond soundness: On the verification of semantic business process models. *Distributed and Parallel Databases (DAPD).*

Online appendix for the paper

Business Process Verification with Constraint Temporal Answer Set Programming

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Laura Giordano
DISIT, Università del Piemonte Orientale, Italy

Alberto Martelli, Matteo Spiotta
Dipartimento di Informatica, Università di Torino, Italy

Daniele Theseider Dupré
DISIT, Università del Piemonte Orientale, Italy

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Appendix A Representation of control flow

In this appendix we describe the action language representation of the basic control flow elements in YAWL (figure A 1).

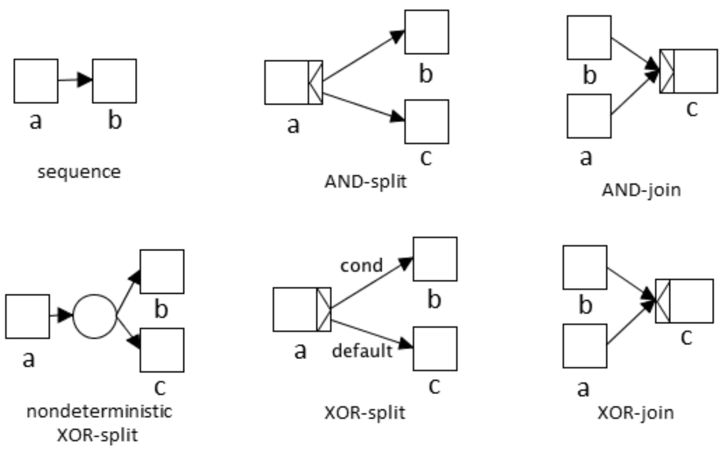


Fig. A 1. Basic control flow elements in YAWL

Fluents $enabled\_a$ are used to represent that a task named a is enabled, fluents $en\_arc\_a\_b$ are used to represent that the arc from a to b is enabled. The rules are as follows:

Sequence:

$[a]en\_arc\_a\_b$
$[b]\neg en\_arc\_a\_b$

$enabled\_b \leftarrow en\_arc\_a\_b$
$\neg enabled\_b \leftarrow \neg en\_arc\_a\_b$

AND-split:

$[a]en\_arc\_a\_b$ $\quad$ $[a]en\_arc\_a\_c$
$[b]\neg en\_arc\_a\_b$ $\quad$ $[c]\neg en\_arc\_a\_c$
$enabled\_b \leftarrow en\_arc\_a\_b$ $\quad$ $\neg enabled\_b \leftarrow \neg en\_arc\_a\_b$
$enabled\_c \leftarrow en\_arc\_a\_c$ $\quad$ $\neg enabled\_c \leftarrow \neg en\_arc\_a\_c$

AND-join:

$[b]en\_arc\_b\_c$ $\quad$ $[a]en\_arc\_a\_c$
$[c]\neg en\_arc\_b\_c$ $\quad$ $[c]\neg en\_arc\_a\_c$
$enabled\_c \leftarrow en\_arc\_b\_c, en\_arc\_a\_c$
$\neg enabled\_c \leftarrow \neg en\_arc\_b\_c$ $\quad$ $\neg enabled\_c \leftarrow \neg en\_arc\_a\_c$

Nondeterministic XOR-split:

$[a]en\_arc\_a\_b$ $\quad$ $[a]en\_arc\_a\_c$
$[b]\neg en\_arc\_a\_b$ $\quad$ $[b]\neg en\_arc\_a\_c$
$[c]\neg en\_arc\_a\_b$ $\quad$ $[c]\neg en\_arc\_a\_c$
$enabled\_b \leftarrow en\_arc\_a\_b$ $\quad$ $\neg enabled\_b \leftarrow \neg en\_arc\_a\_b$
$enabled\_c \leftarrow en\_arc\_a\_c$ $\quad$ $\neg enabled\_c \leftarrow \neg en\_arc\_a\_c$

XOR-split:

Analogous to the nondeterministic one, except for the first two rules which become:

$[a]en\_arc\_a\_b \leftarrow Cond$ $\quad$ $[a]en\_arc\_a\_c \leftarrow not\ Cond$

XOR-join:

$[b]en\_arc\_b\_a$ $\quad$ $[c]en\_arc\_c\_a$
$[a]\neg en\_arc\_b\_a$ $\quad$ $[a]\neg en\_arc\_c\_a$
$enabled\_a \leftarrow en\_arc\_b\_a$ $\quad$ $enabled\_a \leftarrow en\_arc\_c\_a$
$\neg enabled\_a \leftarrow \neg en\_arc\_b\_a, \neg en\_arc\_c\_a$

Appendix B Experiments

In this appendix we report some experiments on the feasibility and scalability of the approach in the paper with current Constraint ASP technology. In particular, we run in *clingcon* (Gebser et al. 2009; Ostrowski and Schaub 2012) version 2.0.2 (and in some cases *clingo*, version 3.0.3), on a machine with Intel Xeon E5520 processors (2.26Ghz) and 32 GB RAM, an encoding with an optimization for the persistence of variable values: rather than propagating values of variables across tasks which do not set their value, a fluent $lastchange(Var, S, S1)$ is propagated from S to the next state, representing that $S1$ is the last state where the value of Var was set; therefore, the value of a variable in a state is actually evaluated in the state where it last changed. This eliminates a number of equalities on constraint variables (e.g. $value(pn, S')\ \$ = value(pn, S)$ in section 5) that should be processed by the constraint solver.

Experiment 1 is based on the process structure in figure B 1 as a basic block, which has a structure similar to the one of the running example; in particular, process models are as follows:

- The process structure is (see figure B 2) a sequence of r blocks with the structure in figure B 1 (i.e., for $i < r$, task 19 of the i-th block is followed by task *set* of the $i+1$-th block). We use l_i to denote the instance, in the i-th repetition, of a task or condition labeled l in figure B 1.
- Conditions in different elements of the sequence vary; e.g., condition $c1_i$ ($c1$ in block i) is different from $c1_j$ for $i \neq j$.

Different classes of process models are checked; each class has r as a parameter and classes differ in branching conditions. Depending on the logical relations among such conditions, there can be up to 2^{4r} alternative runs of the process (since there are 2^4 in a single block).

In **problem class 1.1** we further have:

- The task set_1 sets two integer variables v, v';

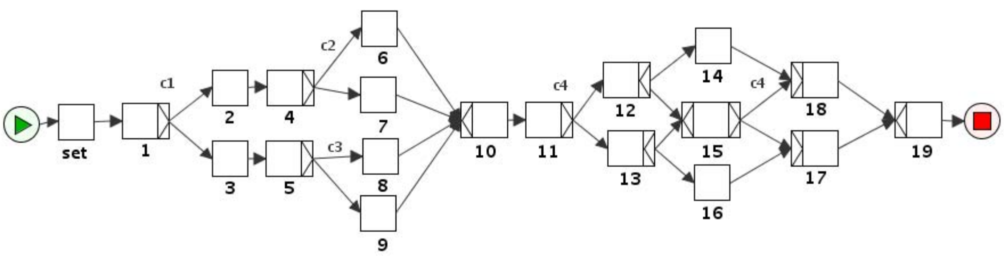


Fig. B 1. Basic process model

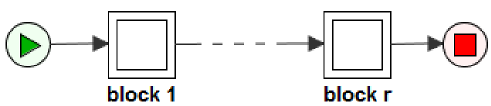
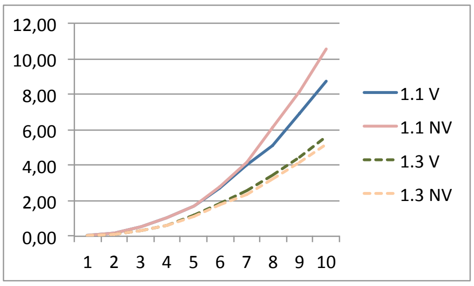


Fig. B 2. Models for experiment 1; each block has the structure in figure B 1

Problem class	1	2	3	4	5	6	7	8	9	10
1.1 V	0.05	0.19	0.51	1.03	1.71	2.70	4.01	5.08	6.91	8.74
1.1 NV	0.05	0.20	0.51	1.03	1.73	2.83	4.17	6.10	8.13	10.53
1.2 V	0.05	0.23	0.61	1.29	2.38	4.14	8.12	12.98	21.57	25.61
1.2 NV	0.05	0.21	0.60	1.31	2.50	4.10	6.67	11.68	17.38	24.57
1.3 V	0.04	0.13	0.30	0.58	1.19	1.82	2.58	3.42	4.44	5.65
1.3 NV	0.04	0.13	0.31	0.59	1.13	1.75	2.38	3.25	4.13	5.18
1.4 V	0.03	0.16	0.41	0.87	1.26	2.06	2.36	4.60	6.79	11.02
1.4 NV	0.05	0.16	0.38	0.73	1.18	1.78	2.47	3.33	4.64	5.39

Table B 1. *Running times for experiment 1*



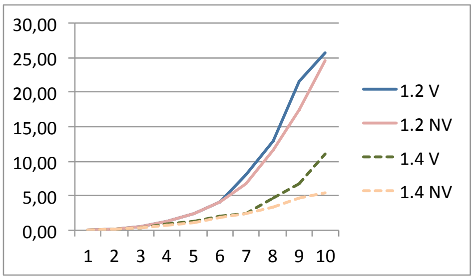


Fig. B 3. Running times compared for experiment 1

- Conditions $c1_i$ are $v > k_i$, where k_i, for $i = 1, \ldots, r$ are constants such that $k_i > k_{i+1}$.
- Conditions $c4_i$ are $v > k_i/2$.
- Conditions $c2_i$ and $c3_i$ are $v' < k'$ (so they are independent from the other conditions)
- Task 4_i has fluent a_i as effect, task 5_i has effect d_i, task 14_i has effect b_i.

Note that $c1_i$ implies $c4_i$ which in turn implies $c1_{i+1}$. This makes branching quite constrained, in particular there are only $O(r)$ different runs.

The following formulae are checked for validity:

- $\Box(a_1 \rightarrow \Diamond b_r)$, which is valid, because $c1_1$ implies $c4_r$.
- $\Box(d_1 \rightarrow \Diamond b_r)$, which is not valid.

Table B 1, provides, in lines labeled **1.1 V** (resp. **1.1 NV**) the running times in seconds to verify the validity (resp., non-validity) of the two formulae for values of r up to 10, using the length of the longest run as completeness threshold (for $r = 10$ it is 112 and it requires 567 seconds to be computed, with more than half time spent to check that there are no runs of length 113).

In **problem class 1.2** we have more complex dependencies among branching conditions; in particular, we have the following differences wrt 1.1:

- The task set_i sets an integer variable v_i;
- Conditions $c1_i$ are $v_i > k_i$.
- Conditions $c2_i, c3_i$ are as in 1.1 (and v' is set by set_1).
- Conditions $c4_i$ are $v_i > k_i/2$ for $i \neq r$
- Condition $c4_r$ is $\bigwedge_{i=1}^{r} v_i > k'_i/2$ where $k'_i < k_i$ (and then $v_i > k_i$ implies $v_i > k'_i$)

In this case, for $i \neq r$, $c1_i$ implies $c4_i$, while $\bigwedge_{i=1}^{r} c1_i$ implies $c4_r$. The number of different runs is exponential in r (12^r).

The following formulae are checked for validity:

- $\Box(\bigwedge_{i=1}^{r} a_i \rightarrow \Diamond b_r)$, which is valid, because $\bigwedge_{i=1}^{r} c1_i$ implies $c4_r$.
- $\Box(\bigwedge_{i=1}^{r} d_i \rightarrow \Diamond b_r)$, which is not valid.

and running times are in lines **1.2 V** and **1.2 NV** of table B 1.

The approach is sensitive to the way a branching condition is implied by other ones. For example, if in problem class 1.2 we change $c4_r$ to:

$$\sum_{i=1}^{r} v_i > \sum_{i=1}^{r} k_i$$

which is implied by $\bigwedge_{i=1}^{r} c1_i$ as before, the verification is only feasible up to $r = 3$.

This behavior is apparently due to the solver integration and the constraint solver itself (running times increase significantly with the actual size of the numerical domain).

We then further measure the cost of relying, in general, on an integrated solver that calls a constraint solver for constraint atoms. In two further problem classes, in fact, we use branching conditions that do not involve constraint atoms, but conditions on variables with enumerated type; as stated in the paper, their translation does not involve constraint atoms but just ASP atoms. Therefore, the encoding can be run in *clingo*.

In particular, **problem class 1.3** should be compared with 1.1, in the sense that the same logical relations hold between branching conditions, but in 1.1 the constraint solver is responsible for detecting such logical relations.

- set_1 sets $r + 1$ variables $v_1, \ldots, v_{r+1}$ with domain $\{false, true\}$;
- Conditions $c1_i$ are $\bigwedge_{j=i*2-1}^{r} v_j = true$.
- Conditions $c2_i, c3_i$ are as in 1.1 (and v' is set by set_1).
- Conditions $c4_i$ are $\bigwedge_{j=i*2}^{r} v_j = true$.

As in 1.1, $c1_i$ implies $c4_i$ which implies $c1_{i+1}$. The formulae to be verified are as in 1.1 and running times are in lines **1.3 V** and **1.3 NV** of table B 1; a graphical comparison for 1.1 and 1.3 is in figure B 3 (left).

Then, in **problem class 1.4** (to be compared with 1.2), we have

- set_i sets three variables va_i, vb_i, vc_i with domain $\{false,\ true\}$;
- Conditions $c1_i$ are $va_i = true \wedge vb_i = true \wedge vc_i = true$.
- Conditions $c2_i, c3_i$ are as in 1.1 (and v' is set by set_1).
- Condition $c4_r$ is $\bigwedge_{i=1}^{r} va_i = true$ and is implied by $\bigwedge_{i=1}^{r} c1_i$.

Conditions are related as in 1.2. The formulae to be verified are as in 1.2 and results are in lines **1.4 V** and **1.4 NV** of table B 1; a graphical comparison for 1.2 and 1.4 is in figure B 3 (right).

The results show that the additional cost of relying on a constraint solver is acceptable for 1.1 and 1.2.

In **experiment 2** we tested the case where r blocks, each with the structure in figure B 1, are in parallel (using AND-split and AND-join), with a single *set* task that assigns variables before the split (see figure B 4). Parallel execution means that all interleavings of every possible execution of each block should in general be considered (no reduction technique is introduced in our current approach), then the number of possible executions becomes extremely high even for low values of r.

In **problem class 2.1**, *set* assigns variables v_1, v_2 and for block i we have:

- Conditions $c1_i$ are $v_1 > k_1$.
- Conditions $c2_i$ and $c3_i$ are $v_2 > k_2$.
- Conditions $c4_i$ are $v_1 > k_1/2$.
- Task 4_i has fluent a_i as effect, task 5_i has effect d_i, task 14_i has effect b_i.

and the following formulae are checked for validity:

- $\Box \bigwedge_{i=1}^{r} (a_i \rightarrow \Diamond b_i)$, which is valid.
- $\Box \bigwedge_{i=1}^{r} (d_i \rightarrow \Diamond b_i)$, which is not valid.

In **problem class 2.2**, *set* assigns variables $v_{1,i}, v_{2,i}$ for $i = 1, \ldots, r$ and conditions in block i are as in 2.1, but on $v_{1,i}, v_{2,i}$.

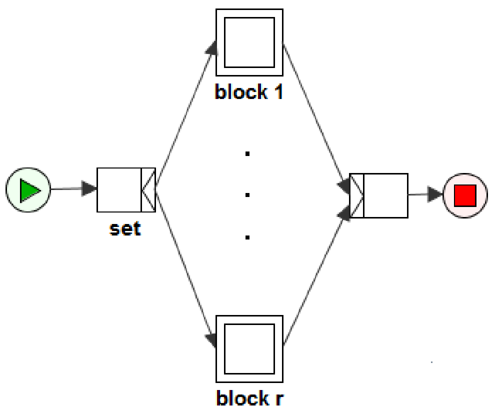


Fig. B 4. Models for experiment 2; block are as in Figure B 1 with no initial “set” task

Problem class	2	3	4	5
2.1	$8.4 \cdot 10^6$	$1.6 \cdot 10^{15}$	$1.3 \cdot 10^{25}$	$1.5 \cdot 10^{36}$
2.2	$1.0 \cdot 10^8$	$2.4 \cdot 10^{17}$	$2.2 \cdot 10^{28}$	$3.1 \cdot 10^{40}$

Table B 2. *Number of different runs for process models in experiment 2*

Problem class	2	3	4	5
2.1 V	0.34	2.01	39.48	269.66
2.1 NV	0.23	0.63	72.01	94.24
2.2 V	0.28	2.12	22.79	1239.76
2.2 NV	0.26	0.86	17.46	11594.07

Table B 3. *Running times for experiment 2*

The number of different executions for r up to 5 is in table B 2.

In spite of such a large search space, verification is feasible (running times are in table B 3) if the length of the longest run is used as a bound. What is not feasible is computing such a bound with the approach used in other cases. Of course, if a process model is hierarchical, e.g. using composite tasks in YAWL, the bound (or an overestimate of it) can be computed by separately computing the longest activity sequence for component blocks (composite tasks).

In **experiment 3** the process structure is fixed, the one in figure B 5, while branching conditions of increasing complexity are used, however each variable expression in constraint atoms involves a constant and low (3) number of variables. In this case running times do not explode.

In particular, we have the following conditions which depend on a parameter r and involve $3r$ variables set by activity 0:

- Condition $c1$ is $\bigwedge_{i=1}^{r} v_{1,i} > k_1$.
- Condition $c2$ is $\bigwedge_{i=1}^{r} v_{2,i} > k_2$.
- Condition $c6$ is $\bigwedge_{i=1}^{r} v_{3,i} > k_3$.
- Condition $c12$ is $\bigwedge_{i=1}^{r} v_{1,i} + v_{2,i} + v_{3,i} > k_1 + k_2 + k_3$.
- Task 10 has fluent d as effect, task 11 has effect b, task 34 has effect a.

The following formulae are checked for validity:

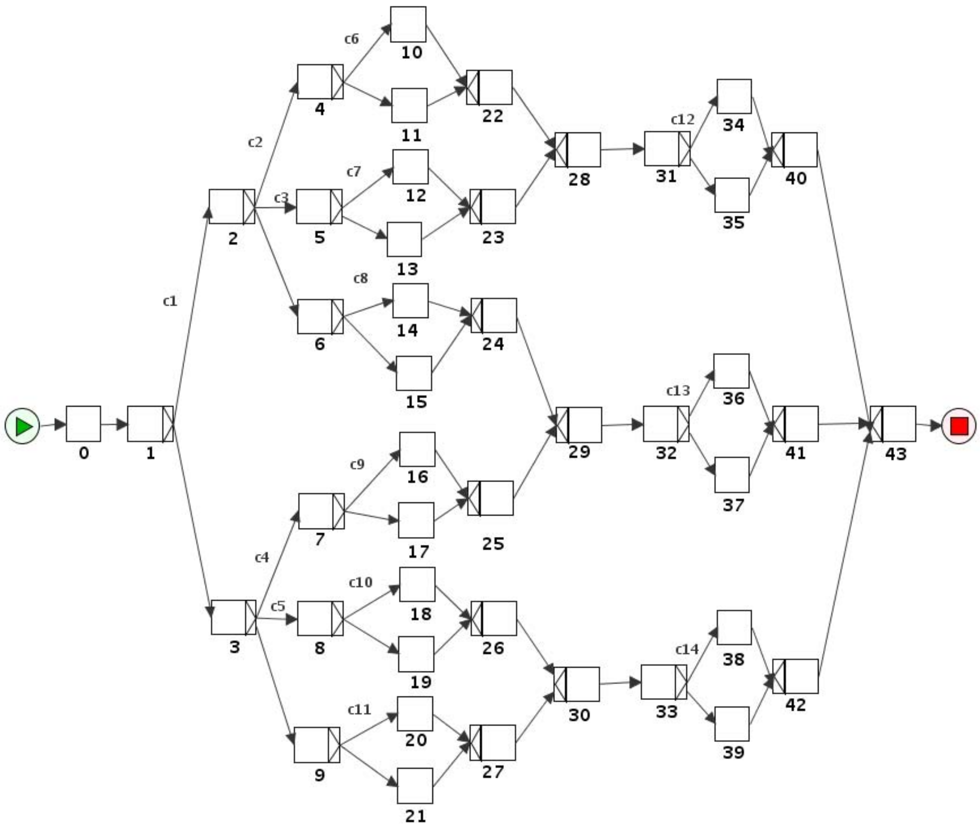


Fig. B 5. Process model

r	1	2	3	4	5	6	7	8	9	10
valid formula	0.81	1.61	2.46	3.28	4.01	4.82	5.68	6.59	8.67	8.45
non-valid formula	0.82	1.64	2.49	3.28	4.21	4.97	6.00	6.59	7.63	8.49

Table B 4. *Running times for experiment 3*

- $\Box(d \rightarrow \Diamond a)$, which is valid, because $c1 \wedge c2 \wedge c6$ implies $c12$.
- $\Box(b \rightarrow \Diamond a)$, which is not valid.

Table B 4 provides the results for this experiment.

As stated in the paper, completeness of BMC is a problem, since, while solutions for computing completeness thresholds for general formulae exist, as well as for formulae of a given form, the actual computation of such thresholds tends to be unfeasible.

In **Experiment 4** we report some results on a variation, with a **non-loop-free workflow**, of the running example in the paper. The process is in figure B 6: if the assessment

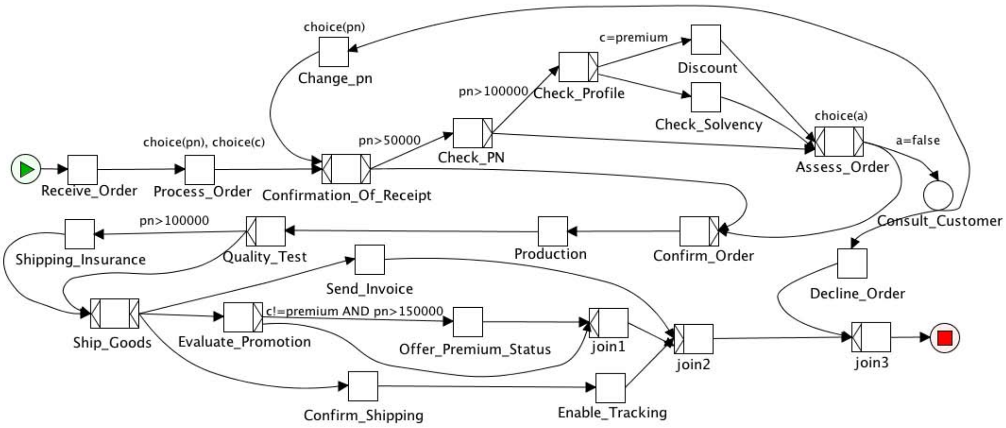


Fig. B 6. Process model with loop

of the order is negative, then, nondeterministically (abstracting input from the customer), either the order is declined or the piece number is changed, restarting evaluation of the modified order. In a single execution, the state (in terms of fluents and constraint atoms) S' reached after "Change_pn" is in general different from the state S reached after "Process_order" (because of a different boolean value of constraint atoms such as $pn > 50000$), even though S' is reachable (in a different run) from the initial state in the same number of steps as S.

For checking the validity of formulae $\Box p$, where p is propositional (or, more generally, p can be evaluated in a single state), i.e. searching counterexamples satisfying $\Diamond\neg p$, the completeness threshold is ((Biere et al. 2003), section 5.1) the number of steps to reach all states (reachability diameter, rd) which can be overapproximated by the length of the longest loop-free path starting from an initial state. In our case, the reachability diameter cannot be computed in a single answer set with the ASP encoding, since different executions correspond to different answer sets. The length of the longest loop-free path can indeed be computed, searching, for increasing values of k, for a simple run of length k, i.e., a run made by different states, where two states differ if they assign a different value to a fluent or a constraint atom in the process model or in the formula to be checked. Such a threshold can be used for checking formulae $\Box p$, relaxing the requirement $\Diamond end$ (e.g., checking later that an end state can be reached from the state satisfying $\Box\neg p$, if found); before knowing such a threshold, empirically "large" bounds can be used, possibly finding a counterexample in a time smaller than the one to compute the threshold (of course, validity is not guaranteed if no counterexample is found).

For the non-valid formula (3) in section 6, such a threshold (51) can be found in 108 seconds. Using it as the bound, a counterexample can be found in 0.32 seconds. Using 100, 200 or 300 as bounds, the counterexample can be found in 1, 3.1 and 6.7 seconds respectively.

For the valid formula (2) in section 6, the threshold (36) can be found in 20 seconds; using it, 0.95 seconds are needed to check the validity. In this case, using larger bounds is only feasible up to 60.

References

BIERE, A., CIMATTI, A., CLARKE, E. M., STRICHMAN, O., AND ZHU, Y. 2003. Bounded model checking. *Advances in Computers 58*, 118–149.

GEBSER, M., OSTROWSKI, M., AND SCHAUB, T. 2009. Constraint answer set solving. In *ICLP*. 235–249.

OSTROWSKI, M. AND SCHAUB, T. 2012. ASP modulo CSP: The clingcon system. *TPLP 12,* 4-5, 485–503.