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Procedural Approaches for Allocation Problems in Insurance

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Fairness in Co-Insurance

A risk may be too high for a single insurance company but not for a pool (if an adequate premium is paid)
The companies have different evaluations of the risks, depending on risk-aversion, information, size of the company etc.

<u>Co-insurance problem</u>

How to divide the risk and the premium among the companies, satisfying some fairness criteria

$\mathcal{X}$	set of real valued random variables on a probability space
$R \in \mathcal{X}$	risk (non-negative random variable)
$\pi \in \mathbb{R}$	premium
$N = \{1, ..., n\}$	set of companies

<u>Quota share allocation</u>

$(q_i \pi, q_i R)_{i \in N}$

where $(q_i)_{i \in N}$ is an n-dimensional real vector, s.t. $q_i > 0$ and $\sum_{i \in N} q_i = 1$

Is this solution fair?

- efficiency (weak Pareto optimality)
  depends only on an optimal risk decomposition (Deprez and Gerber 1985)
- individual rationality
- envy-freeness
- proportionality
- equitability

(see also Brams and Taylor, 1996 and 1999)

The Aims

- analyze some possible fair divisions
- give some step-by-step procedures

Brams and Taylor (1999) deal "*with disputes ... in which everybody can win. For such disputes ... (they) describe and illustrate step-by-step procedures that help the disputants resolve their differences, capture the mutual gain, and reach a fair settlement*"

Haake, Raith and Su (2002) "*view a procedure as featuring the following characteristics: it is* intuitive, *meaning that each step must be easy to understand; it is* plausible, *meaning that each step must be simple to argue; and it is* manageable, *meaning that each step must be straightforward to compute ... see these subjective criteria as relevant for the practical implementation of a fair division outcome, in particular when parties in real life prefer to establish fairness by themselves, rather than trust the 'magic' of a computer algorithm*"

<u>Procedural approach to fair division</u>

Knaster (1946)
Steinhaus (1948)
Brams and Taylor (1996, 1999)
Raith (2000)
Haake, Raith and Su (2002)

The Problem

Decomposition of R $(X_i)_{i\in N}$ s.t. $X_i \in \mathcal{X},\ \forall\ i \in N$ and $\sum_{i\in N} X_i = R$

Division of π $(c_i)_{i\in N}$ s.t. $c_i \in \mathbb{R},\ \forall\ i \in N$ and $\sum_{i\in N} c_i = \pi$

The allocation $(c_i, X_i)_{i\in N}$ of (π, R) means that company $i \in N$ gets the stochastic amount $c_i - X_i$
Company $i \in N$ assigns to $X \in \mathcal{X}$ a value $m_i(X) \in [-\infty, +\infty]$ and let $\mathcal{L}_i = \{X \in \mathcal{X}\ s.t.\ m_i(X) \in \mathbb{R}\}$

The preferences $\succeq_i$ of company $i \in N$ over the set $\mathcal{L}_i$ are such that:

$$X \succeq_i Y \iff m_i(X) \geq m_i(Y) \qquad \forall\ X, Y \in \mathcal{L}_i$$

Hypothesis 1 *For each company* $i \in N$*:*

$$m_i(w) = w \qquad \forall\ w \in \mathbb{R}$$

Hypothesis 2 *For each company* $i \in N$*:*

$$m_i(w + X) = w + m_i(X) \qquad \forall\ w \in \mathbb{R}, \forall\ X \in \mathcal{L}_i$$

By Hypotheses 1 and 2 company $i \in N$ is indifferent between X and $m_i(X)$ and between $X - m_i(X)$ and a null amount, $\forall\ X \in \mathcal{L}_i$

Company $i \in N$ may insure a risk Y only if $-Y \in \mathcal{L}_i$

$H_i(Y) = -m_i(-Y)$ can be interpreted as the minimal premium amount that company i requires for covering the risk Y without suffering losses (not including commercial loading for commissions and expenses)

Hypotheses 1 and 2 hold for many of the premium calculation principles:

- *Net premium principle*
  $H(X) = E(X)$
- *Variance principle*
  $H(X) = E(X) + \alpha Var(X), \qquad \alpha > 0$
- *Standard deviation principle*
  $H(X) = E(X) + \beta\sqrt{Var(X)}, \qquad \beta > 0$

(see Goovaerts, De Vylder and Haezendonck, 1984)

Let

$\Delta = \left\{ (q_i)_{i \in N} \in \mathbb{R}^N s.t.\ q_i > 0, \sum_{i \in N} q_i = 1 \right\}$

$\mathcal{A} = \{ (q_i R)_{i \in N} \in \mathcal{X}^n, s.t.\ (q_i)_{i \in N} \in \Delta, -q_i R \in \mathcal{L}_i\ \forall\ i \in N \}$

To reduce computational difficulties, consider a non-empty set of risk decompositions, $\mathcal{B} \subseteq \mathcal{A}$, s.t. if $(q_i R)_{i \in N} \in \mathcal{B}$ then $(q_{\sigma(i)} R)_{i \in N} \in \mathcal{B}$ for each $\sigma : N \to N,$ permutation of $\{1, ..., n\}$

Hypothesis 3 *There exists a vector* $(q_i^*)_{i \in N} \in \Delta$ *such that* $(q_i^* R)_{i \in N} \in \mathcal{B}$ *and:*

$$\sum_{i \in N} H_i(q_i^* R) = \min_{(q_i R)_{i \in N} \in \mathcal{B}} \sum_{i \in N} H_i(q_i R)$$

$(q_i^* R)_{i \in N}$ is an optimal decomposition of the risk in $\mathcal{B}$

$Q_{\mathcal{B}}^*(R) = \sum_{i \in N} H_i(q_i^* R)$

Remark 1 *In practice the companies consider a set* $\mathcal{B}$ *containing a finite number of elements (Hypothesis 3 trivially holds). For an* a priori *fixed decomposition of the risk* $(\bar{q}_i R)_{i \in N}$ *then*

$$\mathcal{B} = \{ (\bar{q}_{\sigma(i)} R)_{i \in N}, \sigma : N \to N, \textit{permutation of } \{1, ..., n\} \}$$

An optimal allocation of the risk may be obtained via the solution of a 0-1 linear assignment problem

Let

$$\mathcal{Z}(\mathcal{B}) = C \times \mathcal{B} = \text{set of the feasible allocations of } (\pi, R)$$

$$\begin{aligned}\mathcal{O}(\mathcal{B}) &= \{(c_i, q_i R)_{i \in N} \in \mathcal{Z}(\mathcal{B}) \mid \nexists\ (c_i', q_i' R)_{i \in N} \in \mathcal{Z}(\mathcal{B})\ s.t.\ (c_i', q_i' R) \sqsupset_i (c_i, q_i R)\ \forall\ i \in N\} \\ &= \text{set of the feasible Pareto optimal allocations of } (\pi, R)\end{aligned}$$

Proposition 1

$$(c_i, q_i R)_{i \in N} \in \mathcal{O}(\mathcal{B}) \iff (c_i, q_i R)_{i \in N} \in \mathcal{Z}(\mathcal{B})\ and\ \sum_{i \in N} H_i(q_i R) = Q^*_{\mathcal{B}}(R)$$

Let $(c_i, q_i^* R)_{i \in N}$ be a Pareto optimal allocation

Company $i \in N$ assigns to the pair $(c_j, q_j^* R)$ received by company $j \in N$ the value $m_i(c_j, q_j^* R) = c_j - H_i(q_j^* R)$

Let $P_i = \pi - \sum_{j \in N} H_i(q_j^* R), i \in N$ be the value of the estate to divide (for i), after that each company $j \in N$ received $H_i(q_j^* R)$

Hypothesis 4 $P_i \geq 0$, *for each* $i \in N$

Let $P^* = \pi - Q_{\mathcal{B}}^*(R)$ be the net profit to be shared after that each company $i \in N$ received $H_i(q_i^* R)$

Proposition 2 $P^* \geq \frac{1}{n} \sum_{i \in N} P_i$

If the companies reach an agreement on an optimal decomposition of the risk $(q_i^* R)_{i \in N}$ for which Hypothesis 4 holds, then by Proposition 2 $P^* \geq 0$ and there exist allocations $(c_i, q_i^* R)_{i \in N} \in \mathcal{O}(\mathcal{B})$ that are:

- *individually rational*: $m_i(c_i - q_i^* R) = c_i - H_i(q_i^* R) \geq 0, \forall\, i \in N$
- *proportional*: $c_i - H_i(q_i^* R) \geq \frac{1}{n} P_i, \forall\, i \in N$
- *equitable*: $\frac{c_i - H_i(q_i^* R)}{P_i} = \frac{c_j - H_j(q_j^* R)}{P_j}, \forall\, i, j \in N$
- *envy-free*: $m_i(c_i - q_i^* R) \geq m_i(c_j - q_j^* R), \forall\, i, j \in N$

A simple proportional allocation

$$(c_i^p, q_i^* R)_{i \in N} = \left(H_i(q_i^* R) + \frac{1}{n} P_i + \frac{1}{n} \left(P^* - \sum_{j \in N} \frac{1}{n} P_j \right), q_i^* R \right)_{i \in N} \tag{1}$$

The unique equitable allocation

$$(c_i^e, q_i^* R)_{i \in N} = \left(H_i(q_i^* R) + \frac{P_i}{\sum_{h \in N} P_h} P^*, q_i^* R \right)_{i \in N} \tag{2}$$

This allocation is also proportional:

$$c_i^e - H_i(q_i^* R) = \frac{P^*}{\sum_{h \in N} P_h} P_i \geq \frac{1}{n} P_i$$

where the inequality holds by Proposition 2

The Bargaining Approach

The bargaining problem was introduced by Nash (1950) in the form of a two-player problem

An n-person bargaining problem (see [9]) is a pair $B = (F, d)$

where	$F \subseteq \mathbb{R}^N$	feasible set, closed, convex
	$d = (d_i)_{i \in N} \in \mathbb{R}^N$	disagreement point

$\hat{F} = F \cap \left\{ (x_i)_{i \in N} \in \mathbb{R}^N \text{ s.t. } x_i \geq d_i, \forall\, i \in N \right\}$ is nonempty and bounded

Nash Solution

The unique point in $\hat{F}$ such that $x^N = argmax\ \left(\prod_{i \in N} (x_i - d_i), x \in \hat{F} \right)$

λ-Egalitarian Solution

The unique weakly efficient point in $\hat{F}$ such that $\lambda_i(x_i^\lambda - d_i) = \lambda_j(x_j^\lambda - d_j)$ for each $i, j \in N$

In our situation the feasible set is:

$$F = \left\{ (m_i(c_i - q_i R))_{i \in N} \in \mathbb{R}^N s.t.\ (q_i R)_{i \in N} \in \mathcal{B}, (c_i)_{i \in N} \in \mathbb{R}^N, \sum_{i \in N} c_i \leq \pi \right\}$$

or in our hypotheses:

$$F = \left\{ (x_i)_{i \in N} \in \mathbb{R}^N s.t.\ \sum_{i \in N} x_i \leq P^* \right\}$$

d depends on an agreement of the companies on an optimal risk decomposition $(q_i^* R)_{i \in N}$ and on their minimal rights on the premium, i.e. to select an allocation $(\bar{c}_i, q_i^* R)_{i \in N}$ such that:

$$d_i = m_i(\bar{c}_i - q_i^* R)), \qquad \forall\ i \in N$$

Preliminary agreement	company $i \in N$ gets the risk $q_i^* R$ and receives the refund of the purchased risk plus one n-th of the estate that in its opinion has to be divided among the companies
Allocation	$\left(H_i(q_i^* R) + \frac{1}{n} P_i^*, q_i^* R\right)_{i \in N}$
Disagreement point	$d_i^P = \frac{1}{n} P_i^*, i \in N$
Nash solution	$x_i^P = \frac{1}{n} P_i^* + \frac{1}{n}\left(P^* - \sum_{j \in N} \frac{1}{n} P_j^*\right), \forall\, i \in N$

$(x_i^P + H_i(q_i^* R), q_i^* R)_{i \in N}$ is the proportional allocation defined in (1)

Preliminary agreement	companies can guarantee to themselves just the refund of the bought risk
Allocation	$(H_i(q_i^* R), q_i^* R)_{i \in N}$
Disagreement point	$d^E = (0, ..., 0)$
λ-Egalitarian solution	$x_i^E = \frac{P_i^*}{\sum_{h \in N} P_h^*} P^*, \forall\, i \in N$ with $\lambda_i = \frac{1}{P_i^*}, i \in N$

$(x_i^E + H_i(q_i^* R), q_i^* R)_{i \in N}$ is the equitable allocation defined in (2)

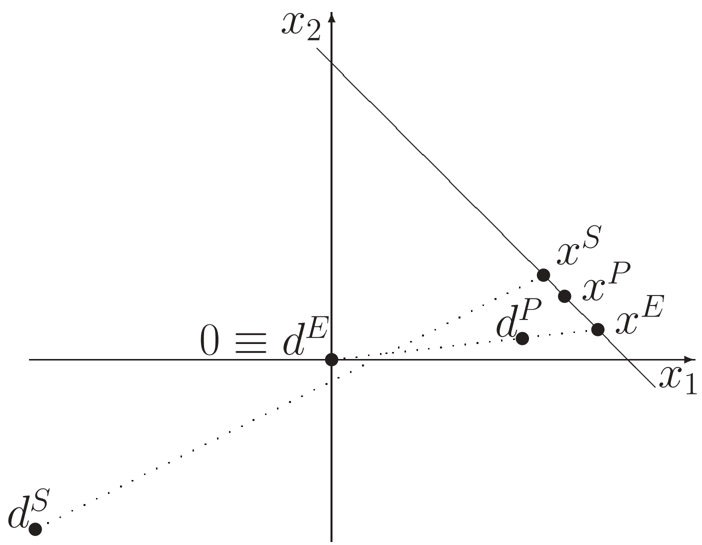
Preliminary agreement	no part of the premium is guaranteed to any of the companies
Allocation	$(0, q_i^* R)_{i \in N}$
Disagreement point	$d_i^S = -H_i(q_i^* R), i \in N$
λ-Egalitarian solution	$x_i^S = q_i^* \pi - H_i(q_i^* R), \forall\, i \in N$ with $\lambda_i = \frac{1}{q_i^*}, i \in N$

$(x_i^S + H_i(q_i^* R), q_i^* R)_{i \in N}$ is the quota share allocation

If $d \geq 0$ then for each solution x^* the allocation $(x_i^* + H_i(q_i^* R), q_i^* R)_{i \in N}$ is individually rational

Example 1 *Consider the co-insurance situation in which two companies, 1 and 2, face the fixed division of a risk* R *as* $\frac{1}{3}R$ *and* $\frac{2}{3}R$*, so* $\mathcal{B} = \{(\frac{1}{3}R, \frac{2}{3}R), (\frac{2}{3}R, \frac{1}{3}R)\}$*. If the evaluations are* $H_1(\frac{1}{3}R) = 2, H_1(\frac{2}{3}R) = 7, H_2(\frac{1}{3}R) = 4, H_2(\frac{2}{3}R) = 13$ *the optimal decomposition is to assign* $\frac{2}{3}R$ *to company 1 and* $\frac{1}{3}R$ *to company 2. Let* $\pi = 18$*, so* $Q^*_{\mathcal{B}}(R) = 11, P^* = 7, P^*_1 = 9, P^*_2 = 1$*. Finally* $F = \left\{(x_1, x_2) \in \mathbb{R}^2 s.t.\ x_1 + x_2 \leq 7\right\}$.

	(P)	(E)	(S)
d	$(\frac{9}{2}, \frac{1}{2})$	$(0, 0)$	$(-7, -4)$
λ	$- -$	$(\frac{1}{9}, 1)$	$(\frac{3}{2}, 3)$
x	$(\frac{11}{2}, \frac{3}{2})$	$(\frac{63}{10}, \frac{7}{10})$	$(5, 2)$



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The Knaster's Procedure

Input: m indivisible objects $b_1, ..., b_m$ ($M = \{1, ..., m\}$) to divide among the players of the set $N = \{1, ..., n\}$

Output: assignment of each object to just one player and monetary compensations

Step 1 each player $i \in N$ evaluates each object $b_k, k \in M$ as v_{ik}; let $E_i = \frac{1}{n} \sum_{k \in M} v_{ik}$ (E_i is the initial proportional share according to the evaluations of i);

Step 2 each object $b_k, k \in M$ is assigned to the player $j(k)$ that gives the maximal evaluation ($j(k) = argmax\ (v_{ik}, i \in N)$); let $v_k = v_{j(k),k}$ (if more players give the same evaluation of an object it is assigned randomly);

Step 3 let $G_i = \sum_{k:j(k)=i} v_k$ (G_i is the value of the objects received by player i);

Step 4 let $s = \sum_{i \in N} (G_i - E_i)$ (s is the surplus);

Step 5 let $V_i = E_i + \frac{s}{n}$ (V_i is the adjusted fair share);

Step 6 for each player $i \in N$, if the monetary amount $V_i - G_i$ is positive the player i receives it in addition to his objects;

otherwise he has to pay $G_i - V_i$; STOP.

Remark 2

- *The surplus s is non-negative*
- *The division is proportional*
- *The division is equitable* $\Leftrightarrow E_i = E_j,\ \forall\ i, j \in N$
- *The sum of the compensations is zero (no money is required or produced)*

Our Procedures

The proportional procedure

Each company $i \in N$ submits, independently, its function H_i to a mediator, which selects an optimal decomposition of the risk $(q_j^* R)_{j \in N}$ for which the Hypothesis 4 holds.

Step 1 each company $i \in N$ receives $\frac{1}{n}\pi$;

Step 2 each company $i \in N$ pays $\frac{1}{n}\sum_{j \in N} H_i(q_j^* R)$;

Step 3 each company $i \in N$ receives the quota of risk $q_i^* R$ and the amount $H_i(q_i^* R)$;

Step 4 the surplus $s = \frac{1}{n}\sum_{j \in N}\sum_{i \in N} H_i(q_j^* R) - \sum_{i \in N} H_i(q_i^* R)$ is equally shared among all the companies; STOP.

The output of the procedure is the proportional allocation (1):

$$\left(\frac{1}{n}\pi - \frac{1}{n}\sum_{j \in N} H_i(q_j^* R) + H_i(q_i^* R) + \frac{1}{n}s, q_i^* R\right)_{i \in N} = \left(H_i(q_i^* R) + \frac{1}{n}P_i^* + \frac{1}{n}\left(P^* - \frac{1}{n}\sum_{j \in N} P_j^*\right), q_i^* R\right)_{i \in N}$$

Remark 3

- *The surplus s is non negative by Proposition 2*
- *In Step 2 each company pays one n-th of its evaluation of each quota of risk $q_j^* R, j \in N$*
- *After Step 2 each company has received the amount $\frac{1}{n} P_i^*$ that is non negative by Hypothesis 4*
- *In Step 3 each company receives its quota of risk $q_i^* R$ and its evaluation of it. Note that the sum of the amounts received by the companies in Step 3 is not greater than the sum of the amounts paid in Step 2 by Proposition 2*

The equitable procedure (see Adjusted Knaster in Raith, 2000)

Modifying the Step 4 of the previous procedure as:

Step 4' the surplus s is shared among all the companies proportionally to $P_i^*, i \in N$; STOP.

The output of the procedure is the equitable allocation (2):

$$\left(\frac{1}{n}\pi - \frac{1}{n}\sum_{j\in N} H_i(q_j^* R) + H_i(q_i^* R) + \frac{P_i^*}{\sum_{j\in N} P_j^*} s, q_i^* R\right)_{i\in N} = \left(H_i(q_i^* R) + \frac{P_i^*}{\sum_{j\in N} P_j^*} P^*, q_i^* R\right)_{i\in N}$$

Each company can be sure of the equitability of the solution only if it knows the evaluations of the other companies

The envy-free procedure (see Fragnelli and Marina, 2003)

(the ordering of the companies is such that for each quota of risk they have decreasing evaluations)

Step 1 let $b_1 = 0$ and for $i = 2, ..., n$ let $b_i = \sum_{k=1}^{i-1} H_k(q_k^* R) - H_{k+1}(q_k^* R)$;

Step 2 for $i = 1, ..., n$ let $r_i = \frac{P^* - \sum_{j \in N} b_j}{n}$;

Step 3 for $i = 1, ..., n$ let $c_i^* = H_i(q_i^* R) + b_i + r_i$ this is the final amount given to company i.

The output of the procedure is the envy-free allocation:

$$\left(H_i(q_i^* R) + b_i + \frac{1}{n} \left(P^* - \sum_{j \in N} b_j \right), q_i^* R \right)_{i \in N}$$

An Application

Environmental risks in Italy

The sole responsibility is given to a pool of 61 insurance companies whose evaluation of a random variable X is according to the variance principle:

$$H_i(X) = E(X) + a_i Var(X), \forall\ i \in N$$

where $a_i = \frac{a(N)}{\bar{q}_i}, i \in N$ with $a(N) = 0.1$

The distribution function of R is $F(x) = 1 - e^{-\mu x}$, with $\mu = \frac{1}{1.05}$, so $E(R) = 1.05$ and $Var(R) = 1.1025$

Let $\mathcal{B} = \left\{ (\bar{q}_{\pi(i)} R)_{i \in N}, \pi : N \to N \right\}$ then the unique optimal decomposition of the risk is $(q_i^* R)_{i \in N} = (\bar{q}_i R)_{i \in N}$

The maximum of $\sum_{j \in N} H_i(q_j^* R), i \in N$ and is 2.413, so according to Hypotheses 4 let $\pi = 2.5$

$\sum_{i \in N} H_i(q_i^* R)$ is 1.160

	COMPANY	$\bar{q}_i \times 100$
1	LE ASSICURAZIONI DI ROMA	0.286
2	BNC ASSICURAZIONI	0.286
3	GIULIANA ASSICURAZIONI	0.286
4	MAECI - SOC. MUTUA DI ASS.NI E RIASS.NI	0.286
5	RISPARMIO ASSICURAZIONI	0.286
6	S.E.A.R.	0.286
7	TICINO	0.306
8	ASSIMOCO	0.429
9	BERNESE ASS.NI-COMP. ITALO-SVIZZERA	0.429
10	LIGURIA	0.429
11	LLOYD ITALICO ASSICURAZIONI	0.429
12	MAECI ASSICURAZIONI E RIASSICURAZIONI	0.429
13	LA MANNHEIM	0.429
14	MEDIOLANUM ASSICURAZIONI	0.429
15	LA NATIONALE	0.429
16	NATIONALE SUISSE	0.429
17	NUOVA MAA ASSICURAZIONI	0.429
18	LA PIEMONTESE SOC. MUTUA DI ASS.NI	0.429
19	LA PIEMONTESE ASSICURAZIONI	0.429
20	SARA ASSICURAZIONI	0.429
21	SASA	0.429
22	SIAT-SOCIETA' ITALIANA ASS.NI E RIASS.NI	0.429
23	UNIVERSO ASSICURAZIONI	0.429
24	ITAS ASSICURAZIONI	0.529
25	ITAS SOC. DI MUTUA ASSICURAZIONE	0.529
26	IL DUOMO	0.574
27	UNIASS ASSICURAZIONI	0.686
28	AUGUSTA ASSICURAZIONI	0.717
29	GAN ITALIA	0.791
30	VITTORIA ASSICURAZIONI	0.840
31	ITALIANA ASSICURAZIONI	0.857

	COMPANY	$\bar{q}_i \times 100$
32	ROYAL & SUN ALLIANCE	0.857
33	WINTERTHUR ASSICURAZIONI	0.857
34	NAVALE ASSICURAZIONI	0.963
35	LEVANTE NORDITALIA ASSICURAZIONI	1.029
36	AURORA ASSICURAZIONI	1.071
37	METE ASSICURAZIONI	1.143
38	SOCIETA' CATTOLICA DI ASSICURAZIONE	1.186
39	ALLIANZ SUBALPINA	1.286
40	LLOYD ADRIATICO	1.340
41	F.A.T.A.	1.429
42	SOCIETA' REALE MUTUA DI ASSICURAZIONI	1.429
43	NUOVA TIRRENA	1.743
44	PADANA ASSICURAZIONI	2.143
45	COMPAGNIA ASSICURATRICE UNIPOL	2.231
46	AXA ASSICURAZIONI	2.460
47	NEW RE (*)	2.571
48	SCOR ITALIA RIASSICURAZIONI (*)	2.571
49	SOREMA (*)	2.571
50	GENERAL & COLOGNE RE (*)	2.714
51	BAYERISCHE RUCK (*)	2.857
52	TORO ASSICURAZIONI	2.857
53	MUNCHENER RUCK ITALIA (*)	3.286
54	ASSICURAZIONI GENERALI	5.263
55	ASSITALIA-LE ASSICURAZIONI D'ITALIA	5.263
56	COMPAGNIA DI ASSICURAZIONE DI MILANO	5.263
57	LA FONDIARIA ASSICURAZIONI	5.263
58	RIUNIONE ADRIATICA DI SICURTA'	5.263
59	SAI	5.263
60	ERC - FRANKONA AG (*)	5.714
61	SWISSE RE - ITALIA	7.714
TOTAL		100.000

(*) Reinsurance company

Comp	$\sum_{j\in N} H_i(q_j^* R)$	c_i^e	c_i^p	$q_i^*\pi$	c_i^{ef}
1	2.413	0.005	0.012	0.007	0.024
2	2.413	0.005	0.012	0.007	0.024
3	2.413	0.005	0.012	0.007	0.024
4	2.413	0.005	0.012	0.007	0.024
5	2.413	0.005	0.012	0.007	0.024
6	2.413	0.005	0.012	0.007	0.024
7	2.324	0.008	0.014	0.008	0.024
8	1.959	0.018	0.021	0.011	0.026
9	1.959	0.018	0.021	0.011	0.026
10	1.959	0.018	0.021	0.011	0.026
11	1.959	0.018	0.021	0.011	0.026
12	1.959	0.018	0.021	0.011	0.026
13	1.959	0.018	0.021	0.011	0.026
14	1.959	0.018	0.021	0.011	0.026
15	1.959	0.018	0.021	0.011	0.026
16	1.959	0.018	0.021	0.011	0.026
17	1.959	0.018	0.021	0.011	0.026
18	1.959	0.018	0.021	0.011	0.026
19	1.959	0.018	0.021	0.011	0.026
20	1.959	0.018	0.021	0.011	0.026
21	1.959	0.018	0.021	0.011	0.026
22	1.959	0.018	0.021	0.011	0.026
23	1.959	0.018	0.021	0.011	0.026
24	1.787	0.024	0.025	0.013	0.027
25	1.787	0.024	0.025	0.013	0.027
26	1.729	0.026	0.027	0.014	0.028
27	1.594	0.031	0.031	0.018	0.029
28	1.618	0.030	0.030	0.017	0.029
29	1.543	0.033	0.032	0.020	0.030
30	1.505	0.035	0.034	0.021	0.031
31	1.505	0.035	0.034	0.021	0.031
32	1.505	0.035	0.034	0.021	0.031
33	1.514	0.034	0.033	0.021	0.031
34	1.429	0.039	0.037	0.026	0.033
35	1.455	0.037	0.036	0.024	0.033
36	1.414	0.040	0.038	0.027	0.034
37	1.391	0.041	0.039	0.029	0.035
38	1.379	0.042	0.040	0.030	0.035
39	1.353	0.044	0.041	0.032	0.037
40	1.341	0.045	0.042	0.034	0.037
41	1.323	0.046	0.043	0.036	0.038
42	1.323	0.046	0.043	0.036	0.038
43	1.274	0.051	0.048	0.044	0.042
44	1.232	0.057	0.053	0.054	0.047
45	1.225	0.058	0.054	0.056	0.048
46	1.209	0.061	0.057	0.062	0.051
47	1.202	0.062	0.059	0.064	0.053
48	1.202	0.062	0.059	0.064	0.053
49	1.202	0.062	0.059	0.064	0.053
50	1.194	0.064	0.060	0.068	0.055
51	1.187	0.066	0.062	0.071	0.056
52	1.187	0.066	0.062	0.071	0.056
53	1.169	0.071	0.068	0.082	0.062
54	1.124	0.095	0.091	0.132	0.086
55	1.124	0.095	0.091	0.132	0.086
56	1.124	0.095	0.091	0.132	0.086
57	1.124	0.095	0.091	0.132	0.086
58	1.124	0.095	0.091	0.132	0.086
59	1.124	0.095	0.091	0.132	0.086
60	1.118	0.101	0.097	0.143	0.092
61	1.101	0.124	0.120	0.193	0.117

Comp	$m_i(c_i^e - q_i^* R)$	$m_i(c_i^p - q_i^* R)$	$m_i(q_i^* \pi - q_i^* R)$	$m_i(c_i^{ef} - q_i^* R)$
1	0.002	0.009	0.004	0.021
2	0.002	0.009	0.004	0.021
3	0.002	0.009	0.004	0.021
4	0.002	0.009	0.004	0.021
5	0.002	0.009	0.004	0.021
6	0.002	0.009	0.004	0.021
7	0.004	0.010	0.004	0.021
8	0.014	0.016	0.006	0.021
9	0.014	0.016	0.006	0.021
10	0.014	0.016	0.006	0.021
11	0.014	0.016	0.006	0.021
12	0.014	0.016	0.006	0.021
13	0.014	0.016	0.006	0.021
14	0.014	0.016	0.006	0.021
15	0.014	0.016	0.006	0.021
16	0.014	0.016	0.006	0.021
17	0.014	0.016	0.006	0.021
18	0.014	0.016	0.006	0.021
19	0.014	0.016	0.006	0.021
20	0.014	0.016	0.006	0.021
21	0.014	0.016	0.006	0.021
22	0.014	0.016	0.006	0.021
23	0.014	0.016	0.006	0.021
24	0.018	0.019	0.007	0.021
25	0.018	0.019	0.007	0.021
26	0.019	0.020	0.008	0.021
27	0.022	0.022	0.009	0.021
28	0.023	0.022	0.010	0.021
29	0.024	0.023	0.011	0.021
30	0.025	0.024	0.011	0.021
31	0.025	0.024	0.011	0.021
32	0.025	0.024	0.011	0.021
33	0.025	0.024	0.011	0.021
34	0.026	0.025	0.013	0.021
35	0.027	0.025	0.014	0.021
36	0.027	0.025	0.014	0.021
37	0.028	0.026	0.015	0.022
38	0.028	0.026	0.016	0.022
39	0.029	0.026	0.017	0.022
40	0.029	0.027	0.019	0.022
41	0.029	0.027	0.018	0.022
42	0.029	0.027	0.019	0.022
43	0.031	0.028	0.023	0.022
44	0.032	0.029	0.033	0.023
45	0.032	0.028	0.030	0.023
46	0.032	0.029	0.034	0.023
47	0.032	0.028	0.029	0.023
48	0.032	0.029	0.034	0.023
49	0.032	0.029	0.034	0.023
50	0.033	0.029	0.038	0.023
51	0.033	0.029	0.036	0.023
52	0.033	0.029	0.044	0.024
53	0.033	0.029	0.038	0.023
54	0.034	0.030	0.071	0.025
55	0.034	0.030	0.071	0.025
56	0.034	0.030	0.071	0.025
57	0.034	0.030	0.071	0.025
58	0.034	0.030	0.071	0.025
59	0.034	0.030	0.071	0.025
60	0.035	0.030	0.077	0.025
61	0.035	0.030	0.103	0.027

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Thanks!