

Procedural Approaches for Allocation Problems in Insurance

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Fairness in Co-Insurance

A risk may be too high for a single insurance company but not for a pool (if an adequate premium is paid)

The companies have different evaluations of the risks, depending on risk-aversion, information, size of the company etc.

Co-insurance problem

How to divide the risk and the premium among the companies, satisfying some fairness criteria

\mathcal{X} set of real valued random variables on a probability space

$R \in \mathcal{X}$ risk (non-negative random variable)

$\pi \in \mathbb{R}$ premium

$N = \{1, \dots, n\}$ set of companies

Quota share allocation

$(q_i \pi, q_i R)_{i \in N}$

where $(q_i)_{i \in N}$ is an n -dimensional real vector, s.t. $q_i > 0$ and $\sum_{i \in N} q_i = 1$

Is this solution fair?

- efficiency (weak Pareto optimality)
depends only on an optimal risk decomposition (Deprez and Gerber 1985)
- individual rationality
- envy-freeness
- proportionality
- equitability

(see also Brams and Taylor, 1996 and 1999)

The Aims

- analyze some possible fair divisions
- give some step-by-step procedures

Brams and Taylor (1999) deal *“with disputes ... in which everybody can win. For such disputes ... (they) describe and illustrate step-by-step procedures that help the disputants resolve their differences, capture the mutual gain, and reach a fair settlement”*

Haake, Raith and Su (2002) *“view a procedure as featuring the following characteristics: it is intuitive, meaning that each step must be easy to understand; it is plausible, meaning that each step must be simple to argue; and it is manageable, meaning that each step must be straightforward to compute ... see these subjective criteria as relevant for the practical implementation of a fair division outcome, in particular when parties in real life prefer to establish fairness by themselves, rather than trust the ‘magic’ of a computer algorithm”*

Procedural approach to fair division

Knaster (1946)

Steinhaus (1948)

Brams and Taylor (1996, 1999)

Raith (2000)

Haake, Raith and Su (2002)

The Problem

Decomposition of R $(X_i)_{i \in N}$ s.t. $X_i \in \mathcal{X}$, $\forall i \in N$ and $\sum_{i \in N} X_i = R$

Division of π $(c_i)_{i \in N}$ s.t. $c_i \in \mathbb{R}$, $\forall i \in N$ and $\sum_{i \in N} c_i = \pi$

The allocation $(c_i, X_i)_{i \in N}$ of (π, R) means that company $i \in N$ gets the stochastic amount $c_i - X_i$

Company $i \in N$ assigns to $X \in \mathcal{X}$ a value $m_i(X) \in [-\infty, +\infty]$ and let $\mathcal{L}_i = \{X \in \mathcal{X} \text{ s.t. } m_i(X) \in \mathbb{R}\}$

The preferences \succeq_i of company $i \in N$ over the set \mathcal{L}_i are such that:

$$X \succeq_i Y \iff m_i(X) \geq m_i(Y) \quad \forall X, Y \in \mathcal{L}_i$$

Hypothesis 1 For each company $i \in N$:

$$m_i(w) = w \quad \forall w \in \mathbb{R}$$

Hypothesis 2 For each company $i \in N$:

$$m_i(w + X) = w + m_i(X) \quad \forall w \in \mathbb{R}, \forall X \in \mathcal{L}_i$$

By Hypotheses 1 and 2 company $i \in N$ is indifferent between X and $m_i(X)$ and between $X - m_i(X)$ and a null amount, $\forall X \in \mathcal{L}_i$

Company $i \in N$ may insure a risk Y only if $-Y \in \mathcal{L}_i$

$H_i(Y) = -m_i(-Y)$ can be interpreted as the minimal premium amount that company i requires for covering the risk Y without suffering losses (not including commercial loading for commissions and expenses)

Hypotheses 1 and 2 hold for many of the premium calculation principles:

- *Net premium principle*

$$H(X) = E(X)$$

- *Variance principle*

$$H(X) = E(X) + \alpha \text{Var}(X), \quad \alpha > 0$$

- *Standard deviation principle*

$$H(X) = E(X) + \beta \sqrt{\text{Var}(X)}, \quad \beta > 0$$

(see Goovaerts, De Vylder and Haezendonck, 1984)

Let

$$\Delta = \{(q_i)_{i \in N} \in \mathbb{R}^N \text{ s.t. } q_i > 0, \sum_{i \in N} q_i = 1\}$$

$$\mathcal{A} = \{(q_i R)_{i \in N} \in \mathcal{X}^n, \text{ s.t. } (q_i)_{i \in N} \in \Delta, -q_i R \in \mathcal{L}_i \forall i \in N\}$$

To reduce computational difficulties, consider a non-empty set of risk decompositions, $\mathcal{B} \subseteq \mathcal{A}$, s.t. if $(q_i R)_{i \in N} \in \mathcal{B}$ then $(q_{\sigma(i)} R)_{i \in N} \in \mathcal{B}$ for each $\sigma : N \rightarrow N$, permutation of $\{1, \dots, n\}$

Hypothesis 3 *There exists a vector $(q_i^*)_{i \in N} \in \Delta$ such that $(q_i^* R)_{i \in N} \in \mathcal{B}$ and:*

$$\sum_{i \in N} H_i(q_i^* R) = \min_{(q_i R)_{i \in N} \in \mathcal{B}} \sum_{i \in N} H_i(q_i R)$$

$(q_i^* R)_{i \in N}$ is an optimal decomposition of the risk in \mathcal{B}

$$Q_{\mathcal{B}}^*(R) = \sum_{i \in N} H_i(q_i^* R)$$

Remark 1 *In practice the companies consider a set \mathcal{B} containing a finite number of elements (Hypothesis 3 trivially holds). For an a priori fixed decomposition of the risk $(\bar{q}_i R)_{i \in N}$ then*

$$\mathcal{B} = \{(\bar{q}_{\sigma(i)} R)_{i \in N}, \sigma : N \rightarrow N, \text{ permutation of } \{1, \dots, n\}\}$$

An optimal allocation of the risk may be obtained via the solution of a 0-1 linear assignment problem

Let

$\mathcal{Z}(\mathcal{B}) = C \times \mathcal{B} =$ set of the feasible allocations of (π, R)

$\mathcal{O}(\mathcal{B}) = \{(c_i, q_i R)_{i \in N} \in \mathcal{Z}(\mathcal{B}) \mid \nexists (c'_i, q'_i R)_{i \in N} \in \mathcal{Z}(\mathcal{B}) \text{ s.t. } (c'_i, q'_i R) \supseteq_i (c_i, q_i R) \forall i \in N\}$
 = set of the feasible Pareto optimal allocations of (π, R)

Proposition 1

$$(c_i, q_i R)_{i \in N} \in \mathcal{O}(\mathcal{B}) \iff (c_i, q_i R)_{i \in N} \in \mathcal{Z}(\mathcal{B}) \text{ and } \sum_{i \in N} H_i(q_i R) = Q_{\mathcal{B}}^*(R)$$

Let $(c_i, q_i^* R)_{i \in N}$ be a Pareto optimal allocation

Company $i \in N$ assigns to the pair $(c_j, q_j^* R)$ received by company $j \in N$ the value $m_i(c_j, q_j^* R) = c_j - H_i(q_j^* R)$

Let $P_i^* = \pi - \sum_{j \in N} H_i(q_j^* R)$, $i \in N$ be the value of the estate to divide (for i), after that each company $j \in N$ received $H_i(q_j^* R)$

Hypothesis 4 $P_i^* \geq 0$, for each $i \in N$

Let $P^* = \pi - Q_{\mathcal{B}}^*(R)$ be the net profit to be shared after that each company $i \in N$ received $H_i(q_i^* R)$

Proposition 2 $P^* \geq \frac{1}{n} \sum_{i \in N} P_i^*$

If the companies reach an agreement on an optimal decomposition of the risk $(q_i^* R)_{i \in N}$ for which Hypothesis 4 holds, then by Proposition 2 $P^* \geq 0$ and there exist allocations $(c_i, q_i^* R)_{i \in N} \in \mathcal{O}(\mathcal{B})$ that are:

- *individually rational*: $m_i(c_i - q_i^* R) = c_i - H_i(q_i^* R) \geq 0, \forall i \in N$
- *proportional*: $c_i - H_i(q_i^* R) \geq \frac{1}{n} P_i^*, \forall i \in N$
- *equitable*: $\frac{c_i - H_i(q_i^* R)}{P_i^*} = \frac{c_j - H_j(q_j^* R)}{P_j^*}, \forall i, j \in N$
 $(P_i^* > 0, \forall i \in N)$
- *envy-free*: $m_i(c_i - q_i^* R) \geq m_i(c_j - q_j^* R), \forall i, j \in N$

A simple proportional allocation

$$(c_i^p, q_i^* R)_{i \in N} = \left(H_i(q_i^* R) + \frac{1}{n} P_i^* + \frac{1}{n} \left(P^* - \sum_{j \in N} \frac{1}{n} P_j^* \right), q_i^* R \right)_{i \in N} \quad (1)$$

The unique equitable allocation

$$(c_i^e, q_i^* R)_{i \in N} = \left(H_i(q_i^* R) + \frac{P_i^*}{\sum_{h \in N} P_h^*} P^*, q_i^* R \right)_{i \in N} \quad (2)$$

This allocation is also proportional:

$$c_i^e - H_i(q_i^* R) = \frac{P^*}{\sum_{h \in N} P_h^*} P_i^* \geq \frac{1}{n} P_i^*$$

where the inequality holds by Proposition 2

The Knaster's Procedure

Input: m indivisible objects b_1, \dots, b_m ($M = \{1, \dots, m\}$) to divide among the players of the set $N = \{1, \dots, n\}$

Output: assignment of each object to just one player and monetary compensations

Step 1 each player $i \in N$ evaluates each object $b_k, k \in M$ as v_{ik} ; let $E_i = \frac{1}{n} \sum_{k \in M} v_{ik}$ (E_i is the initial proportional share according to the evaluations of i);

Step 2 each object $b_k, k \in M$ is assigned to the player $j(k)$ that gives the maximal evaluation ($j(k) = \operatorname{argmax} (v_{ik}, i \in N)$); let $v_k = v_{j(k),k}$ (if more players give the same evaluation of an object it is assigned randomly);

Step 3 let $G_i = \sum_{k:j(k)=i} v_k$ (G_i is the value of the objects received by player i);

Step 4 let $s = \sum_{i \in N} (G_i - E_i)$ (s is the surplus);

Step 5 let $V_i = E_i + \frac{s}{n}$ (V_i is the adjusted fair share);

Step 6 for each player $i \in N$, if the monetary amount $V_i - G_i$ is positive the player i receives it in addition to his objects;

otherwise he has to pay $G_i - V_i$; STOP.

Remark 2

- *The surplus s is non-negative*
- *The division is proportional*
- *The division is equitable $\Leftrightarrow E_i = E_j, \forall i, j \in N$*
- *The sum of the compensations is zero (no money is required or produced)*

Our Procedures

The proportional procedure

Each company $i \in N$ submits, independently, its function H_i to a mediator, which selects an optimal decomposition of the risk $(q_j^* R)_{j \in N}$ for which the Hypothesis 4 holds.

Step 1 each company $i \in N$ receives $\frac{1}{n}\pi$;

Step 2 each company $i \in N$ pays $\frac{1}{n} \sum_{j \in N} H_i(q_j^* R)$;

Step 3 each company $i \in N$ receives the quota of risk $q_i^* R$ and the amount $H_i(q_i^* R)$;

Step 4 the surplus $s = \frac{1}{n} \sum_{j \in N} \sum_{i \in N} H_i(q_j^* R) - \sum_{i \in N} H_i(q_i^* R)$ is equally shared among all the companies;
STOP.

The output of the procedure is the proportional allocation (1):

$$\left(\frac{1}{n}\pi - \frac{1}{n} \sum_{j \in N} H_i(q_j^* R) + H_i(q_i^* R) + \frac{1}{n}s, q_i^* R \right)_{i \in N} = \left(H_i(q_i^* R) + \frac{1}{n}P_i^* + \frac{1}{n} \left(P^* - \frac{1}{n} \sum_{j \in N} P_j^* \right), q_i^* R \right)_{i \in N}$$

Remark 3

- *The surplus s is non negative by Proposition 2*
- *In Step 2 each company pays one n -th of its evaluation of each quota of risk $q_j^* R, j \in N$*
- *After Step 2 each company has received the amount $\frac{1}{n} P_i^*$ that is non negative by Hypothesis 4*
- *In Step 3 each company receives its quota of risk $q_i^* R$ and its evaluation of it. Note that the sum of the amounts received by the companies in Step 3 is not greater than the sum of the amounts paid in Step 2 by Proposition 2*

The equitable procedure (see Adjusted Knaster in Raith, 2000)

Modifying the Step 4 of the previous procedure as:

Step 4' the surplus s is shared among all the companies proportionally to $P_i^*, i \in N$; STOP.

The output of the procedure is the equitable allocation (2):

$$\left(\frac{1}{n}\pi - \frac{1}{n} \sum_{j \in N} H_i(q_j^* R) + H_i(q_i^* R) + \frac{P_i^*}{\sum_{j \in N} P_j^*} s, q_i^* R \right)_{i \in N} = \left(H_i(q_i^* R) + \frac{P_i^*}{\sum_{j \in N} P_j^*} P^*, q_i^* R \right)_{i \in N}$$

Each company can be sure of the equitability of the solution only if it knows the evaluations of the other companies

The envy-free procedure (see Fragnelli and Marina, 2003)

(the ordering of the companies is such that for each quota of risk they have decreasing evaluations)

Step 1 let $b_1 = 0$ and for $i = 2, \dots, n$ let $b_i = \sum_{k=1}^{i-1} H_k(q_k^* R) - H_{k+1}(q_k^* R)$;

Step 2 for $i = 1, \dots, n$ let $r_i = \frac{P^* - \sum_{j \in N} b_j}{n}$;

Step 3 for $i = 1, \dots, n$ let $c_i^* = H_i(q_i^* R) + b_i + r_i$ this is the final amount given to company i .

The output of the procedure is the envy-free allocation:

$$\left(H_i(q_i^* R) + b_i + \frac{1}{n} \left(P^* - \sum_{j \in N} b_j \right), q_i^* R \right)_{i \in N}$$

An Application

Environmental risks in Italy

The sole responsibility is given to a pool of 61 insurance companies whose evaluation of a random variable X is according to the variance principle:

$$H_i(X) = E(X) + a_i \text{Var}(X), \forall i \in N$$

where $a_i = \frac{a(N)}{\bar{q}_i}$, $i \in N$ with $a(N) = 0.1$

The distribution function of R is $F(x) = 1 - e^{-\mu x}$, with $\mu = \frac{1}{1.05}$, so $E(R) = 1.05$ and $\text{Var}(R) = 1.1025$

Let $\mathcal{B} = \{(\bar{q}_{\pi(i)} R)_{i \in N}, \pi : N \rightarrow N\}$ then the unique optimal decomposition of the risk is $(q_i^* R)_{i \in N} = (\bar{q}_i R)_{i \in N}$

The maximum of $\sum_{j \in N} H_i(q_j^* R)$, $i \in N$ and is 2.413, so according to Hypotheses 4 let $\pi = 2.5$

$\sum_{i \in N} H_i(q_i^* R)$ is 1.160

COMPANY	$\bar{q}_i \times 100$	COMPANY	$\bar{q}_i \times 100$
1 LE ASSICURAZIONI DI ROMA	0.286	32 ROYAL & SUN ALLIANCE	0.857
2 BNC ASSICURAZIONI	0.286	33 WINTERTHUR ASSICURAZIONI	0.857
3 GIULIANA ASSICURAZIONI	0.286	34 NAVALE ASSICURAZIONI	0.963
4 MAECI - SOC. MUTUA DI ASS.NI E RIASS.NI	0.286	35 LEVANTE NORDITALIA ASSICURAZIONI	1.029
5 RISPARMIO ASSICURAZIONI	0.286	36 AURORA ASSICURAZIONI	1.071
6 S.E.A.R.	0.286	37 METE ASSICURAZIONI	1.143
7 TICINO	0.306	38 SOCIETA' CATTOLICA DI ASSICURAZIONE	1.186
8 ASSIMOCO	0.429	39 ALLIANZ SUBALPINA	1.286
9 BERNESE ASS.NI-COMP. ITALO-SVIZZERA	0.429	40 LLOYD ADRIATICO	1.340
10 LIGURIA	0.429	41 F.A.T.A.	1.429
11 LLOYD ITALICO ASSICURAZIONI	0.429	42 SOCIETA' REALE MUTUA DI ASSICURAZIONI	1.429
12 MAECI ASSICURAZIONI E RIASSICURAZIONI	0.429	43 NUOVA TIRRENA	1.743
13 LA MANNHEIM	0.429	44 PADANA ASSICURAZIONI	2.143
14 MEDIOLANUM ASSICURAZIONI	0.429	45 COMPAGNIA ASSICURATRICE UNIPOL	2.231
15 LA NATIONALE	0.429	46 AXA ASSICURAZIONI	2.460
16 NATIONALE SUISSE	0.429	47 NEW RE (*)	2.571
17 NUOVA MAA ASSICURAZIONI	0.429	48 SCOR ITALIA RIASSICURAZIONI (*)	2.571
18 LA PIEMONTESE SOC. MUTUA DI ASS.NI	0.429	49 SOREMA (*)	2.571
19 LA PIEMONTESE ASSICURAZIONI	0.429	50 GENERAL & COLOGNE RE (*)	2.714
20 SARA ASSICURAZIONI	0.429	51 BAYERISCHE RUCK (*)	2.857
21 SASA	0.429	52 TORO ASSICURAZIONI	2.857
22 SIAT-SOCIETA' ITALIANA ASS.NI E RIASS.NI	0.429	53 MUNCHENER RUCK ITALIA (*)	3.286
23 UNIVERSO ASSICURAZIONI	0.429	54 ASSICURAZIONI GENERALI	5.263
24 ITAS ASSICURAZIONI	0.529	55 ASSITALIA-LE ASSICURAZIONI D'ITALIA	5.263
25 ITAS SOC. DI MUTUA ASSICURAZIONE	0.529	56 COMPAGNIA DI ASSICURAZIONE DI MILANO	5.263
26 IL DUOMO	0.574	57 LA FONDIARIA ASSICURAZIONI	5.263
27 UNIASS ASSICURAZIONI	0.686	58 RIUNIONE ADRIATICA DI SICURTA'	5.263
28 AUGUSTA ASSICURAZIONI	0.717	59 SAI	5.263
29 GAN ITALIA	0.791	60 ERC - FRANKONA AG (*)	5.714
30 VITTORIA ASSICURAZIONI	0.840	61 SWISSE RE - ITALIA	7.714
31 ITALIANA ASSICURAZIONI	0.857	TOTAL	100.000

(*) Reinsurance company

$Comp$	$\sum_{j \in N} H_i(q_j^* R)$	c_i^e	c_i^p	$q_i^* \pi$	c_i^{ef}	$Comp$	$\sum_{j \in N} H_i(q_j^* R)$	c_i^e	c_i^p	$q_i^* \pi$	c_i^{ef}
1	2.413	0.005	0.012	0.007	0.024	32	1.505	0.035	0.034	0.021	0.031
2	2.413	0.005	0.012	0.007	0.024	33	1.514	0.034	0.033	0.021	0.031
3	2.413	0.005	0.012	0.007	0.024	34	1.429	0.039	0.037	0.026	0.033
4	2.413	0.005	0.012	0.007	0.024	35	1.455	0.037	0.036	0.024	0.033
5	2.413	0.005	0.012	0.007	0.024	36	1.414	0.040	0.038	0.027	0.034
6	2.413	0.005	0.012	0.007	0.024	37	1.391	0.041	0.039	0.029	0.035
7	2.324	0.008	0.014	0.008	0.024	38	1.379	0.042	0.040	0.030	0.035
8	1.959	0.018	0.021	0.011	0.026	39	1.353	0.044	0.041	0.032	0.037
9	1.959	0.018	0.021	0.011	0.026	40	1.341	0.045	0.042	0.034	0.037
10	1.959	0.018	0.021	0.011	0.026	41	1.323	0.046	0.043	0.036	0.038
11	1.959	0.018	0.021	0.011	0.026	42	1.323	0.046	0.043	0.036	0.038
12	1.959	0.018	0.021	0.011	0.026	43	1.274	0.051	0.048	0.044	0.042
13	1.959	0.018	0.021	0.011	0.026	44	1.232	0.057	0.053	0.054	0.047
14	1.959	0.018	0.021	0.011	0.026	45	1.225	0.058	0.054	0.056	0.048
15	1.959	0.018	0.021	0.011	0.026	46	1.209	0.061	0.057	0.062	0.051
16	1.959	0.018	0.021	0.011	0.026	47	1.202	0.062	0.059	0.064	0.053
17	1.959	0.018	0.021	0.011	0.026	48	1.202	0.062	0.059	0.064	0.053
18	1.959	0.018	0.021	0.011	0.026	49	1.202	0.062	0.059	0.064	0.053
19	1.959	0.018	0.021	0.011	0.026	50	1.194	0.064	0.060	0.068	0.055
20	1.959	0.018	0.021	0.011	0.026	51	1.187	0.066	0.062	0.071	0.056
21	1.959	0.018	0.021	0.011	0.026	52	1.187	0.066	0.062	0.071	0.056
22	1.959	0.018	0.021	0.011	0.026	53	1.169	0.071	0.068	0.082	0.062
23	1.959	0.018	0.021	0.011	0.026	54	1.124	0.095	0.091	0.132	0.086
24	1.787	0.024	0.025	0.013	0.027	55	1.124	0.095	0.091	0.132	0.086
25	1.787	0.024	0.025	0.013	0.027	56	1.124	0.095	0.091	0.132	0.086
26	1.729	0.026	0.027	0.014	0.028	57	1.124	0.095	0.091	0.132	0.086
27	1.594	0.031	0.031	0.018	0.029	58	1.124	0.095	0.091	0.132	0.086
28	1.618	0.030	0.030	0.017	0.029	59	1.124	0.095	0.091	0.132	0.086
29	1.543	0.033	0.032	0.020	0.030	60	1.118	0.101	0.097	0.143	0.092
30	1.505	0.035	0.034	0.021	0.031	61	1.101	0.124	0.120	0.193	0.117
31	1.505	0.035	0.034	0.021	0.031						

<i>Comp</i>	$m_i(c_i^e - q_i^*R)$	$m_i(c_i^p - q_i^*R)$	$m_i(q_i^*\pi - q_i^*R)$	$m_i(c_i^{ef} - q_i^*R)$	<i>Comp</i>	$m_i(c_i^e - q_i^*R)$	$m_i(c_i^p - q_i^*R)$	$m_i(q_i^*\pi - q_i^*R)$	$m_i(c_i^{ef} - q_i^*R)$
1	0.002	0.009	0.004	0.021	32	0.025	0.024	0.011	0.021
2	0.002	0.009	0.004	0.021	33	0.025	0.024	0.011	0.021
3	0.002	0.009	0.004	0.021	34	0.026	0.025	0.013	0.021
4	0.002	0.009	0.004	0.021	35	0.027	0.025	0.014	0.021
5	0.002	0.009	0.004	0.021	36	0.027	0.025	0.014	0.021
6	0.002	0.009	0.004	0.021	37	0.028	0.026	0.015	0.022
7	0.004	0.010	0.004	0.021	38	0.028	0.026	0.016	0.022
8	0.014	0.016	0.006	0.021	39	0.029	0.026	0.017	0.022
9	0.014	0.016	0.006	0.021	40	0.029	0.027	0.019	0.022
10	0.014	0.016	0.006	0.021	41	0.029	0.027	0.018	0.022
11	0.014	0.016	0.006	0.021	42	0.029	0.027	0.019	0.022
12	0.014	0.016	0.006	0.021	43	0.031	0.028	0.023	0.022
13	0.014	0.016	0.006	0.021	44	0.032	0.029	0.033	0.023
14	0.014	0.016	0.006	0.021	45	0.032	0.028	0.030	0.023
15	0.014	0.016	0.006	0.021	46	0.032	0.029	0.034	0.023
16	0.014	0.016	0.006	0.021	47	0.032	0.028	0.029	0.023
17	0.014	0.016	0.006	0.021	48	0.032	0.029	0.034	0.023
18	0.014	0.016	0.006	0.021	49	0.032	0.029	0.034	0.023
19	0.014	0.016	0.006	0.021	50	0.033	0.029	0.038	0.023
20	0.014	0.016	0.006	0.021	51	0.033	0.029	0.036	0.023
21	0.014	0.016	0.006	0.021	52	0.033	0.029	0.044	0.024
22	0.014	0.016	0.006	0.021	53	0.033	0.029	0.038	0.023
23	0.014	0.016	0.006	0.021	54	0.034	0.030	0.071	0.025
24	0.018	0.019	0.007	0.021	55	0.034	0.030	0.071	0.025
25	0.018	0.019	0.007	0.021	56	0.034	0.030	0.071	0.025
26	0.019	0.020	0.008	0.021	57	0.034	0.030	0.071	0.025
27	0.022	0.022	0.009	0.021	58	0.034	0.030	0.071	0.025
28	0.023	0.022	0.010	0.021	59	0.034	0.030	0.071	0.025
29	0.024	0.023	0.011	0.021	60	0.035	0.030	0.077	0.025
30	0.025	0.024	0.011	0.021	61	0.035	0.030	0.103	0.027
31	0.025	0.024	0.011	0.021					

References

- [1] Brams SJ, Taylor AD (1996) *Fair-Division*. Cambridge University Press, New York, USA
- [2] Brams SJ, Taylor AD (1999) *The Win-Win Solution*. W.W.Norton & Company, London, UK
- [3] Deprez O, Gerber HU (1985) On Convex Principle of Premium Calculation. *Insurance: Mathematics and Economics* 4: 179-189
- [4] Fragnelli V, Marina ME (2003) A Fair Procedure in Insurance. *Insurance: Mathematics and Economics* 33: 75-85
- [5] Fragnelli V, Marina ME (2004) Co-Insurance Games and Environmental Pollution Risk. in "Game Practice and the Environment" (Eds. Carraro C, Fragnelli V) Edward Elgar Publishing, Cheltenham, UK : 145-163
- [6] Goovaerts MJ, De Vylder F, Haezendonck J (1984) *Insurance Premiums*. North-Holland, Amsterdam, The Netherlands
- [7] Haake CJ, Raith MG, Su FE (2002) Bidding for Envy-Freeness: A Procedural Approach to n-Player Fair-Division Problems. *Social Choice and Welfare* 19: 723-749
- [8] Knaster B (1946) Sur le Problème du Partage Pragmatique de H Steinhaus. *Annales de la Société Polonaise de Mathématique* 19: 228-230
- [9] Myerson RB (1991) *Game Theory: Analysis of Conflict*. Harvard University Press, Cambridge, USA
- [10] Nash JF (1950) The Bargaining Problem. *Econometrica* 21: 155-162
- [11] Raith MG (2000) Fair Negotiation Procedures. *Mathematical Social Science* 39: 303-322
- [12] Steinhaus H (1948) The Problem of Fair Division. *Econometrica* 16: 101-104
- [13] Suijs J, Borm P (1999) Stochastic Cooperative Games: Superadditivity, Convexity, and Certainty Equivalents. *Games and Economic Behavior* 27: 331-345
- [14] Thomson W (1994) Cooperative Models of Bargaining. in "Handbook of Game Theory" vol. 2 (Aumann RJ, Hart S eds). Elsevier, Amsterdam, The Netherlands: 1237-1284

Thanks!

