Procedural Approaches for Allocation Problems in Insurance

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**Fairness in Co-Insurance**

A risk may be too high for a single insurance company but not for a pool (if an adequate premium is paid)
The companies have different evaluations of the risks, depending on risk-aversion, information, size of the company etc.

**Co-insurance problem**

How to divide the risk and the premium among the companies, satisfying some fairness criteria

- \( \mathcal{X} \): set of real valued random variables on a probability space
- \( R \in \mathcal{X} \): risk (non-negative random variable)
- \( \pi \in \mathbb{R} \): premium
- \( N = \{1, ..., n\} \): set of companies

**Quota share allocation**

\( (q_i \pi, q_i R)_{i \in N} \)

where \( (q_i)_{i \in N} \) is an \( n \)-dimensional real vector, s.t. \( q_i > 0 \) and \( \sum_{i \in N} q_i = 1 \)
Is this solution fair?

- efficiency (weak Pareto optimality)
  depends only on an optimal risk decomposition (Deprez and Gerber 1985)
- individual rationality
- envy-freeness
- proportionality
- equitability

(see also Brams and Taylor 1996, 1999)
The Aims

- analyze some possible fair divisions
- give some step-by-step procedures

Brams and Taylor (1999) deal "with disputes ... in which everybody can win. For such disputes ... (they) describe and illustrate step-by-step procedures that help the disputants resolve their differences, capture the mutual gain, and reach a fair settlement"

Haake, Raith and Su (2002) "view a procedure as featuring the following characteristics: it is intuitive, meaning that each step must be easy to understand; it is plausible, meaning that each step must be simple to argue; and it is manageable, meaning that each step must be straightforward to compute ... see these subjective criteria as relevant for the practical implementation of a fair division outcome, in particular when parties in real life prefer to establish fairness by themselves, rather than trust the 'magic' of a computer algorithm"

Procedural approach to fair division

Knaster (1946)
Steinhaus (1948)
Brams and Taylor (1996, 1999)
Raith (2000)
Haake, Raith and Su (2002)
The Problem

Decomposition of $R \ (X_i)_{i \in N}$ s.t. $X_i \in \mathcal{X}, \ \forall \ i \in N$ and $\sum_{i \in N} X_i = R$

Division of $\pi \ (c_i)_{i \in N}$ s.t. $c_i \in \mathbb{R}, \ \forall \ i \in N$ and $\sum_{i \in N} c_i = \pi$

The allocation $(c_i, X_i)_{i \in N}$ of $(\pi, R)$ means that company $i \in N$ gets the stochastic amount $c_i - X_i$

Company $i \in N$ assigns to $X \in \mathcal{X}$ a value $m_i(X) \in [-\infty, +\infty]$ and let $\mathcal{L}_i = \{X \in \mathcal{X} \ s.t. \ m_i(X) \in \mathbb{R}\}$

The preferences $\succeq_i$ of company $i \in N$ over the set $\mathcal{L}_i$ are such that:

$$X \succeq_i Y \iff m_i(X) \geq m_i(Y) \quad \forall \ X, Y \in \mathcal{L}_i$$

**Hypothesis 1** For each company $i \in N$:

$$m_i(w) = w \quad \forall \ w \in \mathbb{R}$$

**Hypothesis 2** For each company $i \in N$:

$$m_i(w + X) = w + m_i(X) \quad \forall \ w \in \mathbb{R}, \forall \ X \in \mathcal{L}_i$$

By Hypotheses 1 and 2 company $i \in N$ is indifferent between $X$ and $m_i(X)$ and between $X - m_i(X)$ and a null amount, $\forall \ X \in \mathcal{L}_i$
Company $i \in N$ may insure a risk $Y$ only if $-Y \in \mathcal{L}_i$

$H_i(Y) = -m_i(-Y)$ can be interpreted as the minimal premium amount that company $i$ requires for covering the risk $Y$ without suffering losses (not including commercial loading for commissions and expenses)

Hypotheses 1 and 2 hold for many of the premium calculation principles:

- **Net premium principle**
  
  $H(X) = E(X)$

- **Variance principle**
  
  $H(X) = E(X) + \alpha V ar(X), \quad \alpha > 0$

- **Standard deviation principle**
  
  $H(X) = E(X) + \beta \sqrt{V ar(X)}, \quad \beta > 0$

(see Goovaerts, De Vylder and Haezendonck 1984)
Let
\[ \Delta = \{(q_i)_{i \in N} \in \mathbb{R}^N \text{ s.t. } q_i > 0, \sum_{i \in N} q_i = 1\} \]
\[ \mathcal{A} = \{(q_i R)_{i \in N} \in \mathcal{X}^n, \text{ s.t. } (q_i)_{i \in N} \in \Delta, -q_i R \in \mathcal{L}_i \ \forall \ i \in N\} \]

To reduce computational difficulties, consider a non-empty set of risk decompositions, \( \mathcal{B} \subseteq \mathcal{A} \), s.t. if \((q_i R)_{i \in N} \in \mathcal{B}\) then \((q_{\sigma(i)} R)_{i \in N} \in \mathcal{B}\) for each \(\sigma : N \to N\), permutation of \{1, ..., n\}

**Hypothesis 3** There exists a vector \((q^*_i)_{i \in N} \in \Delta\) such that \((q^*_i R)_{i \in N} \in \mathcal{B}\) and:
\[ \sum_{i \in N} H_i(q^*_i R) = \min_{(q_i R)_{i \in N} \in \mathcal{B}} \sum_{i \in N} H_i(q_i R) \]
\((q^*_i R)_{i \in N}\) is an optimal decomposition of the risk in \(\mathcal{B}\)
\[ Q^*_\mathcal{B}(R) = \sum_{i \in N} H_i(q^*_i R) \]

**Remark 1** In practice the companies consider a set \(\mathcal{B}\) containing a finite number of elements (Hypothesis 3 trivially holds). For an a priori fixed decomposition of the risk \((\bar{q}_i R)_{i \in N}\) then
\[ \mathcal{B} = \{(\bar{q}_{\sigma(i)} R)_{i \in N}, \sigma : N \to N, \text{ permutation of } \{1, ..., n\}\} \]

An optimal allocation of the risk may be obtained via the solution of a 0-1 linear assignment problem.
Let
\[ \mathcal{Z}(B) = C \times B = \text{set of the feasible allocations of } (\pi, R) \]
\[ \mathcal{O}(B) = \{(c_i, q_iR)_{i \in N} \in \mathcal{Z}(B) \mid \nexists (c'_i, q'_iR)_{i \in N} \in \mathcal{Z}(B) \text{ s.t. } (c'_i, q'_iR) \sqsupseteq_i (c_i, q_iR) \forall i \in N\} \]
\[ = \text{set of the feasible Pareto optimal allocations of } (\pi, R) \]

Proposition 1
\[ (c_i, q_iR)_{i \in N} \in \mathcal{O}(B) \iff (c_i, q_iR)_{i \in N} \in \mathcal{Z}(B) \text{ and } \sum_{i \in N} H_i(q_iR) = Q_B^*(R) \]
Let \((c_i, q_i^* R)_{i \in N}\) be a Pareto optimal allocation

Company \(i \in N\) assigns to the pair \((c_j, q_j^* R)\) received by company \(j \in N\) the value \(m_i(c_j, q_j^* R) = c_j - H_i(q_j^* R)\)

Let \(P_i = \pi - \sum_{j \in N} H_i(q_j^* R), i \in N\) be the value of the estate to divide (for \(i\)), after that each company \(j \in N\) received \(H_i(q_j^* R)\)

**Hypothesis 4** \(P_i \geq 0\), for each \(i \in N\)

Let \(P^* = \pi - Q^*_B(R)\) be the net profit to be shared after that each company \(i \in N\) received \(H_i(q_i^* R)\)

**Proposition 2** \(P^* \geq \frac{1}{n} \sum_{i \in N} P_i\)

If the companies reach an agreement on an optimal decomposition of the risk \((q_i^* R)_{i \in N}\) for which Hypothesis 4 holds, then by Proposition 2 \(P^* \geq 0\) and there exist allocations \((c_i, q_i^* R)_{i \in N} \in \mathcal{O} (\mathcal{B})\) that are:

- **individually rational**: \(m_i(c_i - q_i^* R) = c_i - H_i(q_i^* R) \geq 0, \forall i \in N\)
- **proportional**: \(c_i - H_i(q_i^* R) \geq \frac{1}{n} P_i, \forall i \in N\)
- **equitable**: \(\frac{c_i - H_i(q_i^* R)}{P_i} = \frac{c_j - H_j(q_j^* R)}{P_j}, \forall i, j \in N\)
- **envy-free**: \(m_i(c_i - q_i^* R) \geq m_i(c_j - q_j^* R), \forall i, j \in N\)
A simple proportional allocation

\[(c^p_i, q^*_i R)_{i \in N} = \left( H_i(q^*_i R) + \frac{1}{n} P_i + \frac{1}{n} \left( P^* - \sum_{j \in N} \frac{1}{n} P_j \right), q^*_i R \right)_{i \in N} \]  \hspace{1cm} (1)

The unique equitable allocation

\[(c^e_i, q^*_i R)_{i \in N} = \left( H_i(q^*_i R) + \frac{P_i}{\sum_{h \in N} P_h} P^*, q^*_i R \right)_{i \in N} \] \hspace{1cm} (2)

This allocation is also proportional:

\[c^e_i - H_i(q^*_i R) = \frac{P^*}{\sum_{h \in N} P_h} P_i \geq \frac{1}{n} P_i \]

where the inequality holds by Proposition 2
The Knaster’s Procedure

Input: \( m \) indivisible objects \( b_1, \ldots, b_m \) \((M = \{1, \ldots, m\})\) to divide among the players of the set \( N = \{1, \ldots, n\} \)

Output: assignment of each object to just one player and monetary compensations

**Step 1** each player \( i \in N \) evaluates each object \( b_k, k \in M \) as \( v_{ik} \); let \( E_i = \frac{1}{n} \sum_{k \in M} v_{ik} \) \((E_i \text{ is the initial proportional share according to the evaluations of } i)\);

**Step 2** each object \( b_k, k \in M \) is assigned to the player \( j(k) \) that gives the maximal evaluation \((j(k) = \arg \max (v_{ik}, i \in N))\); let \( v_k = v_{j(k),k} \) \((\text{if more players give the same evaluation of an object it is assigned randomly})\);

**Step 3** let \( G_i = \sum_{k:j(k)=i} v_k \) \((G_i \text{ is the value of the objects received by player } i)\);

**Step 4** let \( s = \sum_{i \in N} (G_i - E_i) \) \((s \text{ is the surplus})\);

**Step 5** let \( V_i = E_i + \frac{s}{n} \) \((V_i \text{ is the adjusted fair share})\);

**Step 6** for each player \( i \in N \), if the monetary amount \( V_i - G_i \) is positive the player \( i \) receives it in addition to his objects;

\[ \text{otherwise he has to pay } G_i - V_i; \text{ STOP.} \]
Remark 2

- The surplus $s$ is non-negative
- The division is proportional
- The division is equitable $\iff E_i = E_j, \forall i, j \in N$
- The sum of the compensations is zero (no money is required or produced)
Our Procedures

The proportional procedure
Each company \( i \in N \) submits, independently, its function \( H_i \) to a mediator, which selects an optimal decomposition of the risk \((q_j^* R)_{j \in N}\) for which the Hypothesis 4 holds.

**Step 1** each company \( i \in N \) receives \( \frac{1}{n} \pi \); 

**Step 2** each company \( i \in N \) pays \( \frac{1}{n} \sum_{j \in N} H_i(q_j^* R) \); 

**Step 3** each company \( i \in N \) receives the quota of risk \( q_i^* R \) and the amount \( H_i(q_i^* R) \); 

**Step 4** the surplus \( s = \frac{1}{n} \sum_{j \in N} \sum_{i \in N} H_i(q_j^* R) - \sum_{i \in N} H_i(q_i^* R) \) is equally shared among all the companies; STOP.

The output of the procedure is the proportional allocation (1):

\[
\left( \frac{1}{n} \pi \right)_{i \in N} - \left( \frac{1}{n} \sum_{j \in N} H_i(q_j^* R) + H_i(q_i^* R) + \frac{1}{n} s, q_i^* R \right)_{i \in N} = \left( H_i(q_i^* R) + \frac{1}{n} P_i + \frac{1}{n} \left( P^* - \frac{1}{n} \sum_{j \in N} P_j^* \right), q_i^* R \right)_{i \in N}
\]
Remark 3

- The surplus $s$ is non negative by Proposition 2
- In Step 2 each company pays one $n$-th of its evaluation of each quota of risk $q_j^* R, j \in N$
- After Step 2 each company has received the amount $\frac{1}{n} P_i^*$ that is non negative by Hypothesis 4
- In Step 3 each company receives its quota of risk $q_i^* R$ and its evaluation of it. Note that the sum of the amounts received by the companies in Step 3 is not greater than the sum of the amounts paid in Step 2 by Proposition 2
The equitable procedure (see Adjusted Knaster by Raith 2000)

Modifying the Step 4 of the previous procedure as:

**Step 4’** the surplus $s$ is shared among all the companies proportionally to $P_i^*, i \in N$; STOP.

The output of the procedure is the equitable allocation (2):

$$\frac{1}{n} \frac{\pi}{n} - \frac{1}{n} \sum_{j \in N} H_i(q_j^* R) + H_i(q_i^* R) + \frac{P_i^*}{\sum_{j \in N} P_j^*} s, q_i^* R$$

$$= \left( \frac{H_i(q_i^* R) + \frac{P_i^*}{\sum_{j \in N} P_j^*} P_i^*, q_i^* R}{i \in N} \right)$$

Each company can be sure of the equitability of the solution only if it knows the evaluations of the other companies.
The envy-free procedure (see Fragnelli and Marina 2003) (the ordering of the companies is such that for each quota of risk they have decreasing evaluations)

**Step 1** let $b_1 = 0$ and for $i = 2, \ldots, n$ let $b_i = \sum_{k=1}^{i-1} H_k(q_k^* R) - H_{k+1}(q_k^* R);$  

**Step 2** for $i = 1, \ldots, n$ let $r_i = \frac{P^* - \sum_{j \in N} b_j}{n};$

**Step 3** for $i = 1, \ldots, n$ let $c_i^* = H_i(q_i^* R) + b_i + r_i$ this is the final amount given to company $i.$

The output of the procedure is the envy-free allocation:

$$\left( H_i(q_i^* R) + b_i + \frac{1}{n} \left( P^* - \sum_{j \in N} b_j \right), q_i^* R \right)_{i \in N}$$
**An Application**

Environmental risks in Italy  
The sole responsibility is given to a pool of 61 insurance companies whose evaluation of a random variable \( X \) is according to the variance principle:

\[
H_i(X) = E(X) + a_i \text{Var}(X), \quad \forall \ i \in N
\]

where \( a_i = \frac{a(N)}{\bar{q}_i}, \ i \in N \) with \( a(N) = 0.1 \)

The distribution function of \( R \) is \( F(x) = 1 - e^{-\mu x} \), with \( \mu = \frac{1}{1.05} \), so \( E(R) = 1.05 \) and \( \text{Var}(R) = 1.1025 \)

Let \( \mathcal{B} = \{(\bar{q}_{\pi(i)}R)_{i \in N}, \pi : N \to N\} \) then the unique optimal decomposition of the risk is \( (q_i^* R)_{i \in N} = (\bar{q}_i R)_{i \in N} \)

The maximum of \( \sum_{j \in N} H_i(q_j^* R), \ i \in N \) and is 2.413, so according to Hypotheses 4 let \( \pi = 2.5 \)

\[
\sum_{i \in N} H_i(q_i^* R) \text{ is } 1.160
\]
| COMPANY                                      | $\bar{q}_i \times 100$ | COMPANY                                      | $\bar{q}_i \times 100$
|----------------------------------------------|------------------------|----------------------------------------------|------------------------
| LE ASSICURAZIONI DI ROMA                    | 0.286                  | ROYAL & SUN ALLIANCE                         | 0.857                  
| BNC ASSICURAZIONI                            | 0.286                  | WINTERTHUR ASSICURAZIONI                     | 0.857                  
| GIULIANA ASSICURAZIONI                       | 0.286                  | NAVELE ASSICURAZIONI                         | 0.963                  
| MAECI - SOC. MUTUA DI ASS.NI E RIASS.NI      | 0.286                  | LEVANTE NORDITALIA ASSICURAZIONI             | 1.029                  
| RISPARMIO ASSICURAZIONI                      | 0.286                  | AURORA ASSICURAZIONI                         | 1.071                  
| S.E.A.R.                                     | 0.286                  | METE ASSICURAZIONI                           | 1.143                  
| TICINO                                       | 0.306                  | SOCIETA' CATTOLICA DI ASSICURAZIONE          | 1.186                  
| ASSIMOCO                                     | 0.429                  | ALLIANZ SUBALPINA                            | 1.286                  
| BERNESE ASS.NI-COMP. ITALO-SVIZZERA          | 0.429                  | LLOYD ADRIATICO                              | 1.340                  
| LIGURIA                                      | 0.429                  | F.A.T.A.                                     | 1.429                  
| LLOYD ITALICO ASSICURAZIONI                  | 0.429                  | SOCIETA' REALE MUTUA DI ASSICURAZIONI        | 1.429                  
| MAECI ASSICURAZIONI E RIASSICURAZIONI        | 0.429                  | NUOVA TIRRENA                                | 1.743                  
| LA MANNHEIM                                  | 0.429                  | PADANA ASSICURAZIONI                         | 2.143                  
| MEDIOLANUM ASSICURAZIONI                     | 0.429                  | COMPAGNIA ASSICURATRICE UNIPOL               | 2.231                  
| LA NATIONALE                                 | 0.429                  | AXA ASSICURAZIONI                            | 2.460                  
| NATIONALE SUISSE                             | 0.429                  | NEW RE (*)                                   | 2.571                  
| NUOVA MAA ASSICURAZIONI                      | 0.429                  | SCOR ITALIA RIASSICURAZIONI (*)              | 2.571                  
| LA PIEMONTESE SOC. MUTUA DI ASS.NI           | 0.429                  | SOREMA (*)                                   | 2.571                  
| LA PIEMONTESE ASSICURAZIONI                  | 0.429                  | GENERAL & COLOGNE RE (*)                    | 2.714                  
| SARA ASSICURAZIONI                           | 0.429                  | BAYERISCHE RUCK (*)                          | 2.857                  
| SASA                                         | 0.429                  | TORO ASSICURAZIONI                           | 2.857                  
| SIAT-SOCIETA' ITALIANA ASS.NI E RIASS.NI     | 0.429                  | MUNCHENER RUCK ITALIA (*)                    | 3.286                  
| UNIVERSO ASSICURAZIONI                       | 0.429                  | ASSICURAZIONI GENERALI                      | 5.263                  
| ITAS ASSICURAZIONI                           | 0.529                  | ASSITALIA-LE ASSICURAZIONI D’ITALIA          | 5.263                  
| ITAS SOC. DI MUTUA ASSICURAZIONE             | 0.529                  | COMPAGNIA DI ASSICURAZIONE DI MILANO         | 5.263                  
| IL DUOMO                                     | 0.574                  | LA FONDIARIA ASSICURAZIONI                   | 5.263                  
| UNIASS ASSICURAZIONI                         | 0.686                  | RIUNIONE ADRIATICA DI SICURTA’              | 5.263                  
| AUGUSTA ASSICURAZIONI                        | 0.717                  | SAI                                          | 5.263                  
| GAN ITALIA                                   | 0.791                  | ERC - FRANKONA AG (*)                        | 5.714                  
| VITTORIA ASSICURAZIONI                       | 0.840                  | SWISSE RE - ITALIA                          | 7.714                  
| ITALIANA ASSICURAZIONI                       | 0.857                  | TOTAL                                        | 100.000                

(*) Reinsurance company
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