# Game Theoretic Analysis of Transportation Problems

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## Summary

## The scenario

EEC directive 440/91 and EC directives 18/95 and 19/95
EU directives 12/01, 13/01 and 14/01
$\Downarrow$
reorganization of the railway sector in Europe
via separation between infrastructure management (IM) and transport operations
$\downarrow$
private railway undertakers (RUs) can access to the infrastructure

railway industry traditionally organized as vertically integrated firms
$\Downarrow$
inefficient allocation of scarce resources (track capacity)
arising from differential information, inappropriate incentives and existence of priority groups

EC/EU directives:

- efficient capacity allocation
- fair tariff system
- minimize the government subsidizations

EU directive 14/01:

- cooperation between IM and RUs
  - better exploitation of the track capacity
  - higher number of requests satisfied as best as possible
- fair tariff system
  - transparency
  - non-discriminatory access
  - equivalent tariffs for equivalent services
  - average costs
  - encouragement of optimal use of the network
  - reduction of the scarcity of the capacity of the network
  - coordination among the requests of RUs
  - enhancement of the available infrastructure capacity
  - incentivation of the investments by the IM
  - charging RUs for infrastructure maintenance

## Track capacity allocation

- Game theoretic approach
  - Brewer and Plott (1996)
    decentralized allocation process based on a binary conflict ascending price (BICAP) mechanisms
    the final allocation is efficient if the excluded agents fully reveal their willingness to pay
  - Bassanini and Nastasi (1997)
    three-stage model
    1. RUs ask for their preferred tracks, specifying a monetary evaluation
    2. IM assigns the available capacity of the network, maximizing the total assigned value (non cooperative market game)
    3. service prices for the users
  - Nilsson (1999)
    Vickrey-type mechanism (bidders appraise the value of a track in their hands)
- Combinatorial optimization approach
  - Caprara, Fischetti and Toth (2002)
    Graph-based algorithm:
    * nodes represent the different stations in different instants
    * arcs correspond to the movements of the trains between two stations or to the stops of the trains in the stations
    * collection of paths, satisfying some feasibility constraints and minimizing a utility function related to the ideal scheduling of the trains

## Access tariff for the railway transport operators

Hypotheses:

- A railway path is used by different types of trains belonging to several RUs
- The infrastructure costs have to be divided among these trains
  (joint cost allocation problem: Tijs and Driessen, 1986 - Young, 1994)
- The infrastructure consists of some "facilities": track, signalling system, stations, etc.
- Different groups of trains need these facilities at different levels
  fast trains $\leftrightarrow$ track and signalling system
  local trains $\leftrightarrow$ station services

The infrastructure is the "sum" of various facilities with different cost levels

Infrastructure costs are sum of:

- "building" costs (independent from number of users) $\rightarrow$ "airport game" (Littlechild and Thompson, 1977)
- "maintenance" costs (dependent on the number of users)

The "additive nature" suggests to use the Shapley value
Easily computable $\leftrightarrow$ very big amount of fees have to be computed by IM every new season

## Related problems

- bridges used by small and big cars
- queue management (Garcia and Garcia-Jurado, 2000)
- operating costs for a consortium for urban solid wastes collection and disposal (Fragnelli and Iandolino, 2004)
- sharing the costs related to the operating-theatre in a hospital (González and Herrero, 2004)

## The game theoretical model

**Definition 1** *Suppose we are given $k$ groups of players $g_1, \ldots, g_k$ with $n_1, \ldots, n_k$ players respectively and $k$ non-negative numbers $b_1, \ldots, b_k$. The* building cost game *corresponding to $g_1, \ldots, g_k$ and $b_1, \ldots, b_k$ is the cooperative (cost) game $(N, c_b)$ with $N = \cup_{i=1}^{k} g_i$ and cost function $c_b$ defined by*

$$c_b(S) = b_1 + \cdots + b_{j(S)} \qquad S \subseteq N$$

*where $j(S) = \max \{j \, : \, S \cap g_j \neq \emptyset\}$*

**Definition 2** *Suppose we are given $k$ groups of players $g_1, \ldots, g_k$ with $n_1, \ldots, n_k$ players respectively and $k(k+1)/2$ non-negative numbers $\{\alpha_{i,j}\}_{i,j \in \{1,\ldots,k\}, j \geq i}$. The* maintenance cost game *corresponding to $g_1, \ldots, g_k$ and $\{\alpha_{i,j}\}_{i,j \in \{1,\ldots,k\}, j \geq i}$ is the cooperative (cost) game $(N, c_m)$ with $N = \cup_{i=1}^{k} g_i$ and cost function $c_m$ defined by*

$$c_m(S) = \sum_{i=1}^{j(S)} |S \cap g_i| A_{i,j(S)} \qquad S \subseteq N$$

*where $A_{i,j} = \alpha_{i,i} + ... + \alpha_{i,j}$ for all $i, j \in \{1, ..., k\}$ with $j \geq i$*

Interpretation for a player in $g_i$:
$\alpha_{i,i}$ is the maintenance cost up to level $i$
$\alpha_{i,i+1}$ is the extra maintenance cost up to level $i+1$
$A_{i,j}$ is the maintenance cost up to level $j$

## The solution

Shapley value

$$\phi_i(v) = \frac{1}{n!} \sum_{\pi} [v(P(\pi; i) \cup \{i\}) - v(P(\pi; i))]$$

where $P(\pi; i)$ are the predecessors of $i$ in permutation $\pi$

Properties:

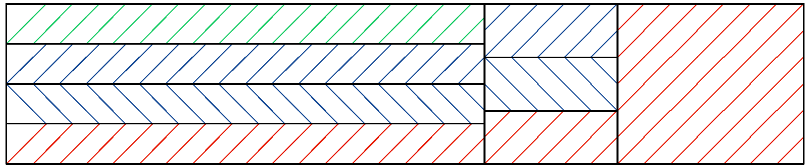
- additive solution
- easy formula for a player $i \in g_s$, $s = 1, ..., k$:

$$\begin{aligned} \phi_i(c_b) &= \sum_{l=1,\dots,s} \frac{b_l}{G_{lk}} \\ \phi_i(c_m) &= \alpha_{s,s} + \sum_{l=s+1,\dots,k} \alpha_{s,l} \frac{G_{lk}}{G_{lk}+1} + \sum_{l=2,\dots,s} \sum_{j=1,\dots,l-1} \alpha_{j,l} \frac{|g_j|}{(G_{lk})(G_{lk}+1)} \end{aligned}$$

where $G_{lk} = \left| \bigcup_{h=l,\dots,k} g_h \right|$

$N = g_1 \cup g_2 \cup g_3;\, g_1 = \{\mathbf{1}\};\, g_2 = \{\mathbf{2}, \mathbf{3}\};\, g_3 = \{\mathbf{4}\}$

## Building cost game

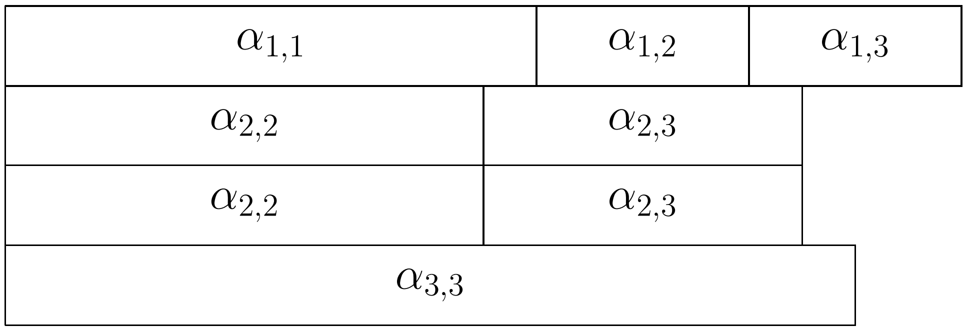



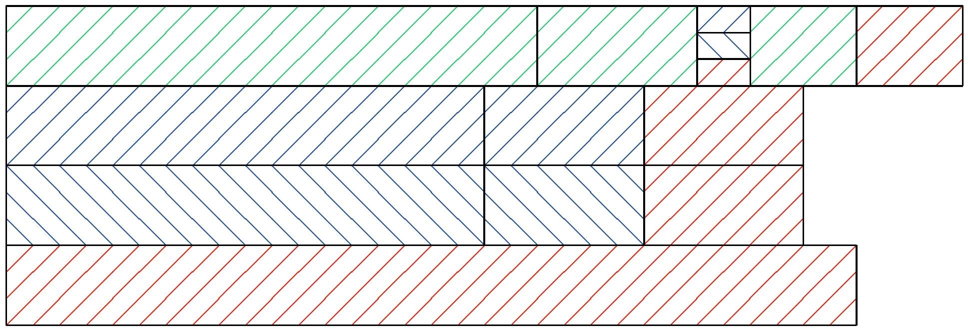
$\phi_1(c_b) = \frac{1}{4}b_1$

$\phi_2(c_b) = \phi_3(c_b) = \frac{1}{4}b_1 + \frac{1}{3}b_2$

$\phi_4(c_b) = \frac{1}{4}b_1 + \frac{1}{3}b_2 + b_3$

## Maintenance cost game



$\phi_1(c_m) = \alpha_{1,1} + \frac{3}{4}\alpha_{1,2} + \frac{1}{2}\alpha_{1,3}$

$\phi_2(c_m) = \phi_3(c_m) = \alpha_{2,2} + \frac{1}{2}\alpha_{2,3} + \frac{1}{3} \times \frac{1}{4}\alpha_{1,2}$

$\phi_4(c_m) = \alpha_{3,3} + \frac{1}{3} \times \frac{1}{4}\alpha_{1,2} + \frac{1}{2}\alpha_{1,3} + 2 \times \frac{1}{2}\alpha_{2,3}$

**Remark 1** *Applying the infrastructure cost games to a consortium for collection and disposal of urban solid wastes (Fragnelli and Iandolino, 2004), an undesired behavior of the Shapley value showed up*
*If the number of players in each group increases proportionally the Shapley value of the maintenance cost game smooths the differences among the amount charged to the different groups*

*This may be avoided using the Owen value (Owen, 1977)*

**Theorem 1** *Let* $(N, c^m)$ *be the maintenance cost game corresponding to the groups* $g_1,...,g_k$*, with* $n_1,...,n_k$ *players respectively and to the non-negative numbers* $\{\alpha_{l,m}\}_{l,m\in\{1,...,k\},m\geq l}$*. If the groups* $g_1,...,g_k$ *correspond to the* a priori unions, *then the Owen value for a player in the group* $g_i, i = 1, ..., k$ *is:*

$$\Omega_i(c^m) = \sum_{H\subset G_{i-1}} \frac{h!(k-h-1)!}{k!} \left(\frac{1}{|g_i|}\beta(H) + \alpha_{i,i}\right) + \sum_{H\not\subset G_{i-1}} \frac{h!(k-h-1)!}{k!} A_{i,j(H)}$$

*where* $h = |H|$*,* $G_{i-1} = \{g_1, ..., g_{i-1}\}$*,* $\beta(H) = A_{j(H),i} \sum_{j|g_j\in H} |g_j|$ *is the upgrading cost for the players in* $H$ *and* $j(H) = max\ \{j|g_j \in H\}$

*For a building cost game* $(N, c^b)$ *corresponding to the groups* $g_1,...,g_k$*, with* $n_1,...,n_k$ *players respectively and to the non-negative numbers* $b_1,...,b_k$*, with* a priori unions $g_1,...,g_k$*, then the Owen value for a player in the group* $g_i, i = 1, ..., k$ *is:*

$$\Omega_i(c^b) = \sum_{H\subset G_{i-1}} \frac{h!(k-h-1)!}{k!} \left(\frac{b_i - b_{j(H)}}{|g_i|}\right)$$

## Balancedness of one facility infrastructure cost games

**Proposition 1** *Let $(N, c)$ be a one facility infrastructure cost game with groups $g_1$,...,$g_k$, with $n_1$,...,$n_k$ players respectively and non-negative numbers $b_1$,...,$b_k$ and $\{\alpha_{i,j}\}_{i,j \in \{1,...,k\}, j \geq i}$. Then $(N, c)$ is balanced iff:*

$$\sum_{i=1,...,j} n_i \left( A_{i,k} - A_{i,j} \right) \leq \sum_{i=1,...,j} b_i$$

*for every $j = 1, ..., k-1$.*

Interpretation:
the extra maintenance costs up to the level $k$ for the players in $g_1 \cup \cdots \cup g_j$ have to be not greater than their building cost

## The nucleolus

For a one facility infrastructure game with groups $g_1$,...,$g_k$, with $n_1$,...,$n_k$ players respectively and non-negative numbers $b_1$,...,$b_k$ and $\{\alpha_{i,j}\}_{i,j\in\{1,...,k\},j\geq i}$ the nucleolus corresponds to the allocation:

$$\Phi_i(c) = A_{i,k} + z_i,\ \ i = 1, ..., k$$

where:

- $z_1$, ..., $z_k$ are defined recursively by:

$$z_1 = \min_{1\leq j\leq k}\left\{\frac{\sum_{l=1,...,j}\hat{b}_l}{W_j}\right\}$$
$$z_i = \min_{i\leq j\leq k}\left\{\frac{\sum_{l=1,...,j}\hat{b}_l - (n_1 z_1 + ... + n_{i-1}z_{i-1})}{W_j - \sum_{l=1,...,i-1} n_l}\right\},\ \ i = 2, ..., k$$

- $\hat{b}_1$,...,$\hat{b}_k$ are defined by the linear system:

$$\begin{cases} \hat{b}_1 & = b_1 - n_1\left(A_{1,k} - A_{1,1}\right) \\ \hat{b}_1 + \hat{b}_2 & = \sum_{i=1,2} b_i - \sum_{i=1,2} n_i\left(A_{i,k} - A_{i,2}\right) \\ ... & ... \\ \hat{b}_1 + \hat{b}_2 + ... + \hat{b}_{k-1} & = \sum_{i=1,...,k-1} b_i - \sum_{i=1,...,k-1} n_i\left(A_{i,k} - A_{i,k-1}\right) \\ \hat{b}_1 + \hat{b}_2 + ... + \hat{b}_{k-1} + \hat{b}_k & = \sum_{i=1,...,k} b_i \end{cases}$$

- $W_j = \sum_{l=1,...,j} n_l + 1$ for $j = 1, ..., k-1$ and $W_k = \sum_{l=1,...,k} n_l$

## Infrastructure cost games

Let $m$ be an arbitrary number of facilities, with no special requirements upon the ordering of the wishes of the coalitions, but the groups are the same for each facility

An infrastructure cost game $(N, c)$ with groups of players $g_1$,...,$g_k$, with $n_1$,...,$n_k$ players respectively is defined by:

$$c = c^1 + ... + c^m$$

where $(N, c^l), l = 1, ..., m$ is a one facility infrastructure cost game with groups of players $g_{\pi^l(1)}$,...,$g_{\pi^l(k)}$ ($\pi^l$ is a permutation of $\{1, ..., k\}$)

If an infrastructure cost game is the sum of balanced one facility infrastructure cost games then it is balanced
The converse is not true

**Example 1** *Consider the two facilities infrastructure cost game* $(N, c)$*, where the ordering of the three groups involved are:*

| | | | |
|---|---|---|---|
| *facility 1* | $g_1$ | $g_2$ | $g_3$ |
| *facility 2* | $g_2$ | $g_1$ | $g_3$ |

*Let* $g_1 = \{1\}$*,* $g_2 = \{2\}$*, and* $g_3 = \{3\}$
$(N, c^1)$ *and* $(N, c^2)$ *are defined by the numbers:*

$$\begin{array}{lll} b_1^1 = b_1^2 = 1 & b_2^1 = b_2^2 = 9 & b_3^1 = b_3^2 = 1 \\ \alpha_{1,1}^1 = \alpha_{1,1}^2 = 1 & \alpha_{1,2}^1 = \alpha_{1,2}^2 = 1 & \alpha_{1,3}^1 = \alpha_{1,3}^2 = 1 \\ & \alpha_{2,2}^1 = \alpha_{2,2}^2 = 2 & \alpha_{2,3}^1 = \alpha_{2,3}^2 = 1 \\ & & \alpha_{3,3}^1 = \alpha_{3,3}^2 = 3 \end{array}$$

$(N, c^1)$ *is not balanced:*

$$c^1(123) = 20 > 2 + 17 = c^1(1) + c^1(23)$$

$(N, c^2)$ *is not balanced:*

$$c^2(123) = 20 > 2 + 17 = c^2(2) + c^2(13)$$

$(N, c)$ *with* $c(1) = 14$*,* $c(2) = 14$*,* $c(3) = c(12) = 28$*,* $c(13) = c(23) = 34$*,* $c(123) = 40$ *is balanced as* $(6, 6, 28)$ *is a core element*

## A case study

Data from Baumgartner (1997) - Fees in CHF

For each facility the costs can be decomposed into:

- a fixed part (building cost game)
- a variable part (maintenance cost game)

For each kilometer of track, there are two kind of costs: *renewal costs (RWC)* and *repairing costs (RPC)*
Two facilities: *track renewal* and *track repairing*

Renewal costs per kilometer and per year

$$RWC = 0.001125X + 11,250$$

Repairing costs per kilometer and per year

$$RPC_s = 0.001X + 10,000 \qquad \text{for slow trains}$$
$$RPC_f = 0.00125X + 12,500 \qquad \text{for fast trains}$$

where $X$ measures the "number" of trains, expressed in yearly TGCK (Tons Gross and Complete per Kilometer)

One kilometer of line used by a total weight of $10^7$ TGCK (about 20,000 trains)
15,000 slow trains and 5,000 fast trains
$(N, c)$ is given by:

- $N = g_s \cup g_f$, where $n_s =$ 15,000 and $n_f =$ 5,000
- $c = c^1 + c^2$, where $c^1$ (track renewal) and $c^2$ (track repairing) are one facility infrastructure cost games with groups of players ordered as $g_s$, $g_f$, characterized by the parameters:

| | | | | | |
|---|---|---|---|---|---|
| $c^1$ | $b_s^1 = 11,250$ | $b_f^1 = 0$ | $\alpha_{s,s}^1 = 0.5625$ | $\alpha_{s,f}^1 = 0$ | $\alpha_{f,f}^1 = 0.5625$ |
| $c^2$ | $b_s^2 = 10,000$ | $b_f^2 = 2,500$ | $\alpha_{s,s}^2 = 0.5$ | $\alpha_{s,f}^2 = 0.125$ | $\alpha_{f,f}^2 = 0.625$ |

For a slow and a fast train respectively, we have:

- $\phi_s(c) = \dfrac{b_s^1}{n_s + n_f} + \alpha_{s,s}^1 + \dfrac{b_s^2}{n_s + n_f} + \alpha_{s,s}^2 + \alpha_{s,f}^2 \dfrac{n_f}{n_f + 1} = 2.25$
- $\phi_f(c) = \dfrac{b_s^1}{n_s + n_f} + \alpha_{f,f}^1 + \dfrac{b_s^2}{n_s + n_f} + \dfrac{b_f^2}{n_f} + \alpha_{f,f}^2 + \alpha_{s,f}^2 \dfrac{n_s}{n_f(n_f + 1)} = 2.75$

## Further researches

1. Tariff system for freight trains

EU directive 14/01:

- favor sustainable mobility
- better balance of transport between modes
- efficient use of international freight corridors
- discounts for efficient use of the underutilized lines
- direct charge of direct costs
- appropriate charges for paths allocated but not used
- reducing scarcity
- limiting environmental impact, mainly noise pollution

Development of a unified formula for all European IMs $\Rightarrow$ simpler procedures for RUs (Branzei et al., 2005)

Cost categories

$\alpha$ Basic costs

$\beta$ Facility and service costs

$\gamma$ Energy costs

$\delta$ Scarcity costs

$\varepsilon$ Other costs

Subscripts $0$ and $1$ refer to fixed costs and per Km costs, respectively

$$T = \alpha_0 + \sum_{s \in S} h(tt)\alpha_1^{lt(s)} km^s + \sum_{f \in F} \beta_0^{ft(f)} + \gamma_1^p \sum_{s \in S} km^s + \sum_{s \in S} \delta_1^{sc(s,t)} km^s + \sum_{f \in F} \varepsilon_0^f + \sum_{s \in S} [\varepsilon_0^s + \varepsilon_1^s km^s]$$

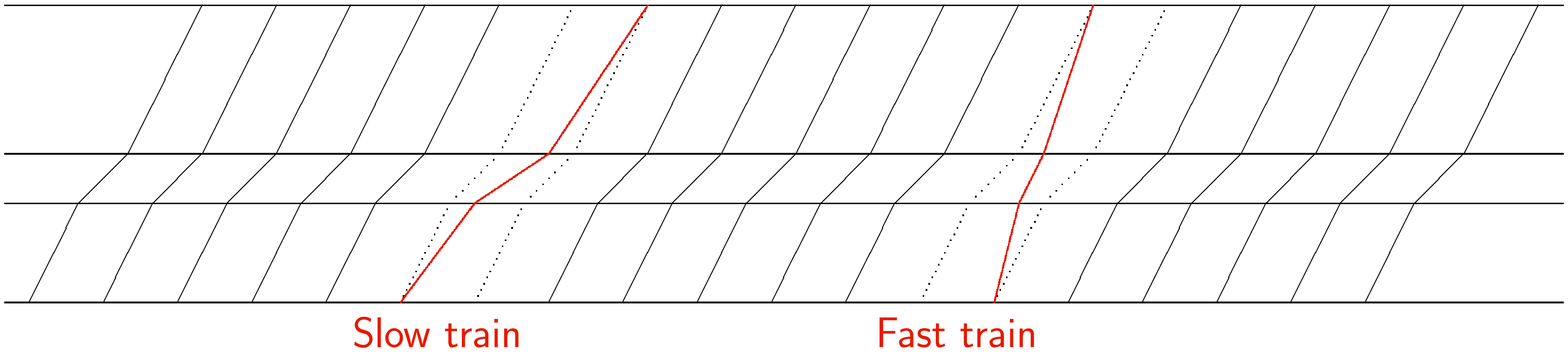
| | | |
|---|---|---|
| where | $tt$ | type of train |
| | $S$ | set of segments of the path |
| | $F$ | set of facilities used along the path |
| | $p$ | power type of train |
| | $km^s$ | length of segment $s \in S$ in km |
| | $lt(s)$ | line type of segment $s \in S$ |
| | $ft(f)$ | type of facility $f \in F$ |
| | $sc(s,t)$ | scarcity index of segment $s \in S$, during time period $t$ |

## 2. Railway timetable

homotachicity (Bagnasco, 2005) ↔ higher regularity

⇓

improve the exploitation of the capacity of a line



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# Thanks!