A quantitative evaluation of veto power

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Motivation

power $\longrightarrow$ largely studied in literature, among others

- ordering $\rightarrow$ Shapley-Shubik [Shapley and Shubik, 1954]
- different majorities $\rightarrow$ Banzhaf-Coleman [Banzhaf, 1965] and [Coleman, 1971]
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M. Chessa

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- To evaluate the veto power we think about a quantitative approach;
- it is not necessary anymore that the power given to the single agents sum up to 1;
- we look for an index which gives power of veto 1 to a veto player;
- more than one can be a veto player;
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UNSC

[http://www.un.org/Docs/sc/index.html]

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Each Council member has one vote. Decisions on procedural matters are made by an affirmative vote of at least 9 of the 15 members. Decisions on substantive matters require nine votes, including the concurring votes of all five permanent members. This is the rule of "great Power unanimity", often referred to as the **veto power**.

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Some preliminaries definitions 1

We consider a weighted majority situation $[q; w_1, \ldots, w_n]$ and the associated **weighted majority game**

$$w(S) = \begin{cases} 1 & \text{if } \sum_{j \in S} w_j \geq q \\ 0 & \text{otherwise.} \end{cases}$$

if $v(S) = 1$ we say the coalition is **winning**, if $v(S) = 0$ is **losing**.

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Given a game (N, W), the set of coalitions 2^N splits into four classes [Carreras, 2005]:

- D (**decisive winning**): $S \in W$ s.t. $N \setminus S \notin W$;
- C (**conflictive winning**): $S \in W$ s.t. $N \setminus S \in W$;
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We define the **dual game** of (N, W) as (N, W^*), where $W = \{S \subseteq N : N \setminus S \notin W\}$, i.e. $v^*(S) = v(N) - v(N \setminus S)$.

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The decisiveness index

Carreras defines the **decisiveness degree** of the game (N, W) as

$$\delta(N, W) = \frac{|W|}{2^n}$$

where $n = |N|$.
It gives the probability that an abstract proposal passes in (N, W), where each agent $i \in N$ has only two options: voting for the proposal (Y) or voting against (N), with probability 1/2. The motion passes if and only if the set of agents that vote for Y is a winning coalition $S \in W$.
If a game is **proper** $(C = \emptyset)$ and **strong** $(Q = \emptyset)$ (i.e. **decisive**), then $\delta(N, W) = 1/2$.

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An obvious **protectionism index** for every simple game can be defined by taking

$$\delta^*(N, W) = \delta(N, W^*) = 1 - \delta(N, W)$$

It gives the probability that a proposal does not pass in (N, W).

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The decisiveness index for player i

We propose to define the **decisiveness degree** of player i as

$$\delta_i(N, W) = \frac{|W_i|}{2^{n-1}}$$

where $W_i = \{S \in W : \ i \in S\}$.
It gives the probability that a proposal passes in (N, W) when we already know that player i has voted for the proposal (Y) and each agent $j \in N$, $j \neq i$ has only two options: voting for the proposal (Y) or voting against (N), with probability 1/2. .

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Similarly, we define the probability that a proposal does not pass in (N, W) when we know that player i votes against the proposal (N) as

$$\delta_i^*(N, W) = \frac{2^{n-1} - |W| + |W_i|}{2^{n-1}}$$

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A quantitative evaluation of veto power

Definition

Given the game (N, W) we define the **veto power of player** i as $\delta_i^*(N, W)$.

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Remark

The following relations hold:

- $\delta_i(N,W)$ *and* $\delta_i^*(N,W)$ *are strictly related and this relation depends on the decisiveness index of the game:*

$$\delta_i^*(N,W) = 1 - 2\delta(N,W) + \delta_i(N,W)$$

- *when* i *is a veto player, i.e.* $\delta_i^*(N,W) = 1$, *we get* $\delta_i(N,W) = 2\delta(N,W)$;
- *if the game is decisive, then a veto player* ($\delta_i^*(N,W) = 1$) *is a dictator* ($\delta_i(N,W) = 1$).

Proposition

$\delta_i^*(N,W) = \delta_i(N,W^*)$, *for every* $i \in N$

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- *when* i *is a veto player, i.e.* $\delta_i^*(N, W) = 1$, *we get* $\delta_i(N, W) = 2\delta(N, W)$;
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We consider the simple weighted majority game $[6; 2, 3, 5]$.
The winning coalitions are $\{1, 3\}$, $\{2, 3\}$ *e* $\{1, 2, 3\}$.
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Player 3 is a veto player, voting against a proposal **he is sure not to make it approved**, but voting in favor **he is not sure to make it approved!**
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A quantitative evaluation of veto power

M. Chessa

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We can modelize the situation as a **game with incomplete information played by bayesian players** [Harsanyi, 1967]. In such a game the players are uncertain about some important parameters of the game. In our model we assume that each player can be of two types: player in favor of the proposal or player against the proposal. Obviously one player knows his own type, but he gives a probability on the type of the other players.
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Formally a *game with incomplete information played by bayesian players* , or simply a *bayesian game*, is a 5-tuple $(N, \{C_i\}_{i\in N}, \{T_i\}_{i\in N}, \{p_{ik}\}_{i\in N,\ k\in T_i}, \{u_i\}_{i\in N})$ where

- N is the set of players;
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- $u_i : \prod_{j\in N} C_j \times \prod_{j\in N} T_j \to \mathbb{R}$ is the utility function of player i.

A pure strategy for player i is a function $s_i : T_i \to C_i$ and Σ_i is the set of all the pure strategies of i. A mixed strategy for player i is a function $\sigma_i : C_i \times T_i \to [0, 1]$ with $\sum_{c\in C_i} \sigma_i(c, t) = 1$ for each $t \in T_i$.

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Formally a *game with incomplete information played by bayesian players* , or simply a *bayesian game*, is a 5-tuple $(N, \{C_i\}_{i \in N}, \{T_i\}_{i \in N}, \{p_{ik}\}_{i \in N,\ k \in T_i}, \{u_i\}_{i \in N})$ where

- N is the set of players;
- C_i is the set of the actions of player i;
- T_i is the set of types of player i;
- p_{ik} is the probability of player i of being of type k, with $k \in T_i$, $\sum_{k \in T_i} p_{ik} = 1$;
- $u_i : \prod_{j \in N} C_j \times \prod_{j \in N} T_j \to \mathbb{R}$ is the utility function of player i.

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Example *(1)*

- $N = \{1, 2, 3\}$;
- $C_i = \{Y, N\}$ are the actions, for each $i \in N$;
- $T_i = \{P, Q\}$ are the types, for each $i \in N$;
- $p_{ik} = 1/2$ are the probabilities on the types given a priori, for each $i \in N$, $k \in T_i$;
- the outcome of the game is given by "the law is approved", if the parties which voted Y have total number of seats greater than of equal to the majority quota, "the law is not approved" otherwise;
- the payoff of each party is 1 if it is of type P and the law is approved or if it is of type Q and the law is not approved, 0 otherwise. Formally

$$u_i(s_1, \dots, s_n) = \begin{cases} 1 & \text{if } T_i = P \text{ and } \sum_{j \in N : s_j(T_j) = Y} w_j \geq q \\ 1 & \text{if } T_i = Q \text{ and } \sum_{j \in N : s_j(T_j) = Y} w_j < q \\ 0 & \text{otherwise} \end{cases}$$

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Strategic form of the game

$(1_P, 2_P, 3_P)$	$\begin{pmatrix} (1,1,1) & (1,1,1) \\ (1,1,1) & (0,0,0) \end{pmatrix}$	$\begin{pmatrix} (0,0,0) & (0,0,0) \\ (0,0,0) & (0,0,0) \end{pmatrix}$
$(1_P, 2_P, 3_Q)$	$\begin{pmatrix} (1,1,0) & (1,1,0) \\ (1,1,0) & (0,0,1) \end{pmatrix}$	$\begin{pmatrix} (0,0,1) & (0,0,1) \\ (0,0,1) & (0,0,1) \end{pmatrix}$
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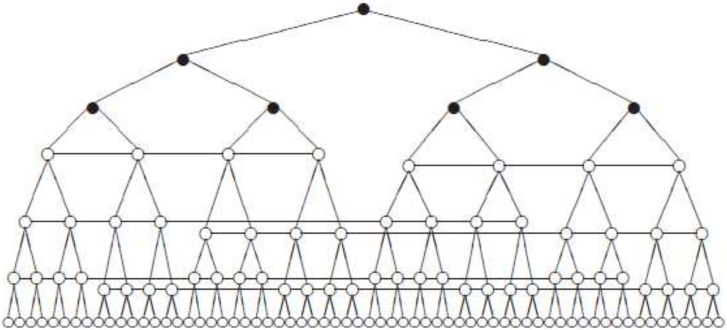
Extensive form of the game



Remark

We can observe:

- *for each player of type in favor of the proposal it is better to vote Y;*
- *for each player against the proposal it is better to vote N;*
- $((Y, N), (Y, N), (Y, N))$ *is a Nash Equilibrium for the game;*
- δ_i *represents what player i of type in favor of the proposal can obtain at the equilibrium;*
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Thank you for your attention