A new family of power indices for voting games

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Motivation

Power indices → measure of the influence of each member of a coalition on decisions
In the last decade → increasing attention in political science:
- study the voting power among EU states
- analyze the effects of institutional reforms

*Felsenthal and Machover (1997)*
*Nurmi (1997 and 2000)*
*Nurmi and Meskanen (1999)*
*Dowding (2000)*
*Aleskerov et al. (2002)*
Aims

For policy makers:
Provide a power index with predictive value, designing a family of power indices that may be tailored on different situations with a suitable setting of some parameters

For game theorists:
Take into account the role of a party in favoring the formation of a majority, even if its influence on the majority in terms of seats (or percentage of votes) is null

*Moretti and Patrone, 2008 and related comments*
The Archetype

A suitable analysis of the ideologies allows to represent the parties on a left-right axis → ordering of the parties

↓

contiguous winning coalition $S$: for all $i, j \in S$ if there exists $k \in N$ with $i < k < j$ then $k \in S$
• Let $W^c = \{S_1, S_2, \ldots, S_m\}$ be the set of contiguous winning coalitions

• Equal sharing of the unitary power among coalitions $S_j \in W^c$

• Equal sharing of the quota assigned to coalition $S_j$ among its members

• Each player $i \in N$ sums up all the amounts received in the coalitions he belongs to

$$FP_i = \sum_{S_j \ni i, S_j \in W^c} \frac{1}{m} \frac{1}{S_j}, \quad i \in N$$
The Plausibility Criterion

- Set of winning coalitions
- Probability of a coalition to form
- Sharing rule inside each winning coalition
Set of winning coalitions

- **office-seeking**: size of the coalition
  - minimal winning coalition (MWC)  
    *von Neumann and Morgenstern, 1953*
  - minimal number coalition (MNC): *minimum of the parties*
    *Leiserson, 1966*
  - minimal size coalition (MSC): *minimum of the seats*
    *Riker, 1962*

- **policy-seeking**: ideological connection
  *Axelrod (1970)*
  *De Swaan (1973)*
  *Aleskerov (2008)*

- other
Probability of a coalition to form

Equiprobability of coalitions may be not appealing

- ideological distance between the extreme parties
  *Bilal et al. (2001)*
- distance between the preferred point of each party and the position of the coalition
  *Bilal et al. (2001)*
- number of parties (proxy of ideological distance)
- number of parties or number of seats (cost of coordination and agreement): the larger the number, the lower the probability
- estimation on historical data
  *Heard and Swartz (1998)*
Sharing rule inside each winning coalition

Equal sharing rule may be not suitable
• number of seats of each party
• critical role of parties (distance from potential opposition)
• median parties within the coalition

*Bäck, 2001*
Examples and Comparisons - 1

Let $[9; 6, 2, 6]$ be a weighted majority situation.
A unique contiguous winning coalition: $\{1,2,3\}$

Equally sharing the power among the players:

$$FP = (1/3, 1/3, 1/3)$$

$$\varphi = \delta = h = (1/2, 0, 1/2)$$
Examples and Comparisons - 2

Let \([6; 1, 5, 2, 3]\) be a weighted majority situation. Four contiguous winning coalitions: \(\{2,3\}, \{1,2,3\}, \{2,3,4\}, \{1,2,3,4\}\)

\[FP = (0.146, 0.354, 0.354, 0.146)\]

Proportional sharing w.r.t. the number of seats

\[FP = (0.054, 0.573, 0.229, 0.143)\]

Sharing w.r.t. number of parties (probability = \((0.235, 0.176, 0.353, 0.235)\)) and proportional sharing w.r.t. the number of seats

\[FP = (0.045, 0.597, 0.239, 0.119)\]

\[\varphi = (0, 0.666, 0.167, 0.167), \delta = h = (0, 0.50, 0.25, 0.25)\]

A unique minimal contiguous winning coalition: \(\{2,3\}\)

\[FP = (0, 0.50, 0.50, 0)\]
A player \( i \in S \) is contiguous-critical for a contiguous winning coalition \( S \) if \( S \setminus \{i\} \) is losing or is no longer contiguous.

Let \([24; 13, 8, 2, 10, 11]\) be a weighted majority situation. Three contiguous winning coalitions: \(\{1, 2, 3, 4\}, \{2, 3, 4, 5\}, \{1, 2, 3, 4, 5\}\). Sharing vectors w.r.t. contiguous-critical parties:

\[(1/4, 1/4, 1/4, 1/4), (1/4, 1/4, 1/4, 1/4), (0, 1/3, 1/3, 1/3, 0)\]

\[FP = (3/36, 10/36, 10/36, 10/36, 3/36)\]
Further Researches

- Coherence in the choice of the parameters
  - minimal size coalition ↔ number of seats
  - minimal number coalition ↔ number of parties

- Analysis of the selection rules for the parameters in order to obtain classical power indices

- Multidimensional approach would allow taking account of the multifaceted nature of voters’ decisions

- Endogeneity due to strategic behavior of both voters and parties
Main References


