POLITECNICO DI MILANO

Advanced Network Technologies Laboratory

Competitive Spectrum Sharing in Cognitive Radio Networks

Summer School on Game Theory and Telecommunications

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Credits

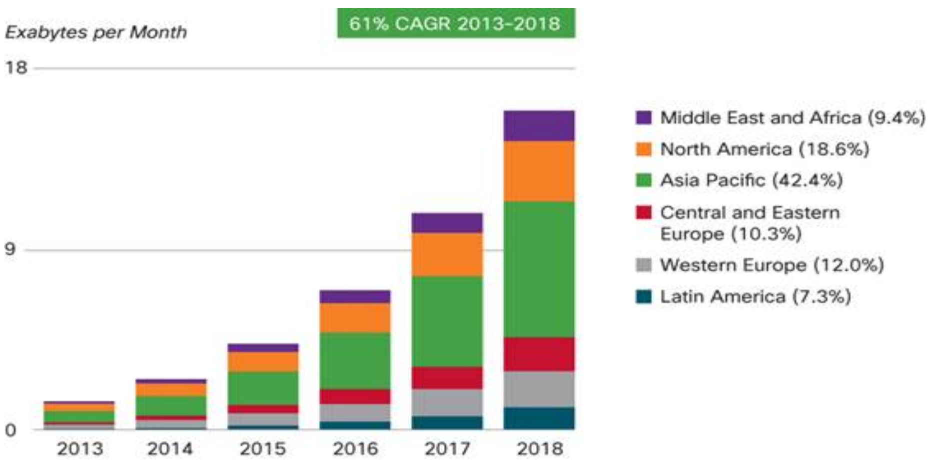
- Thanks to
  - Ilaria Malanchini *(Bell Labs, Stuttgart, Germany)*
  - Matteo Cesana *(Politecnico di Milano, Italy)*
  - Nicola Gatti *(Politecnico di Milano, Italy)*
  - Steven Weber *(Drexel University, Philadelphia, USA)*

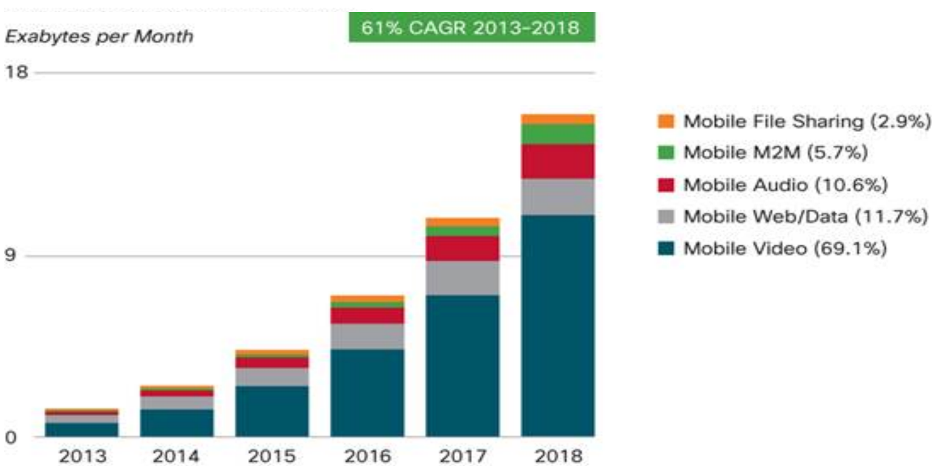
Outline

- Introduction (very brief) to Cognitive Radio Networks
- Spectrum Selection Game
  - Properties
  - Practical Aspects
- Queue Theory and Game Theory at work
- Power Game

Motivation for Cognitive Radio

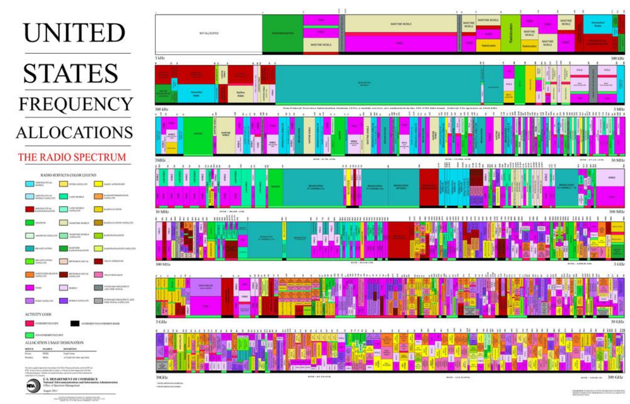
- Exponential mobile data traffic growth growth





Figures in parentheses refer to traffic share in 2018.
Source: Cisco VNI Mobile, 2014

- Fixed spectrum allocation by regulation authorities through auctions



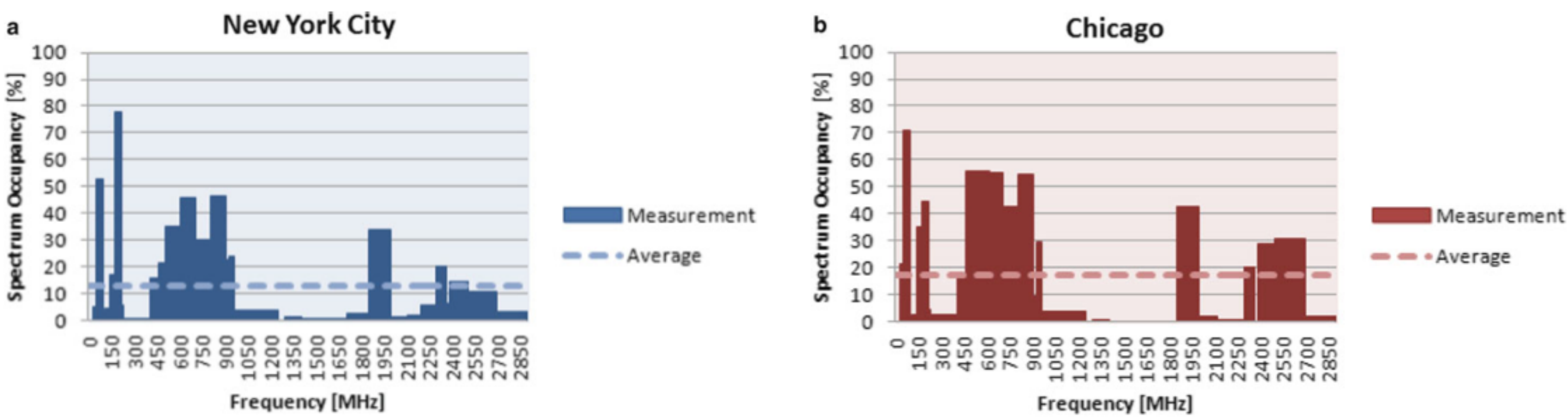
Graduatoria CHIUSA	800_G [Blocchi generici in banda 800 MHz]		
Vincente n. 1	Vodafone Omnitel N.V.	[tornata n. 283]	496 200 000.00
Vincente n. 2	Vodafone Omnitel N.V.	[tornata n. 283]	496 200 000.00
Vincente n. 3	Telecom Italia S.p.A.	[tornata n. 283]	496 100 000.00
Vincente n. 4	Telecom Italia S.p.A.	[tornata n. 284]	496 100 000.00
Vincente n. 5	Wind Telecomunicazioni S.p.A.	[tornata n. 282]	496 000 000.00
TOTALE			2 480 600 000.00

Graduatoria CHIUSA	1800_G [Blocchi generici in banda 1800 MHz]		
Vincente n. 1	Vodafone Omnitel N.V.	[tornata n. 435]	159 100 000.00
Vincente n. 2	Telecom Italia S.p.A.	[tornata n. 434]	159 000 000.00
Vincente n. 3	H3G S.p.A.	[tornata n. 415]	158 900 000.00
TOTALE			477 000 000.00

Graduatoria CHIUSA	2600_G [Blocchi FDD generici in banda 2600 MHz]		
Vincente n. 1	Telecom Italia S.p.A.	[tornata n. 456]	36 400 000.00
Vincente n. 2	H3G S.p.A.	[tornata n. 459]	36 400 000.00
Vincente n. 3	Telecom Italia S.p.A.	[tornata n. 460]	36 400 000.00
Vincente n. 4	Wind Telecomunicazioni S.p.A.	[tornata n. 456]	36 360 000.00
Vincente n. 5	Telecom Italia S.p.A.	[tornata n. 451]	36 320 000.00
Vincente n. 6	Vodafone Omnitel N.V.	[tornata n. 447]	36 060 000.00
Vincente n. 7	Vodafone Omnitel N.V.	[tornata n. 447]	36 060 000.00
Vincente n. 8	Vodafone Omnitel N.V.	[tornata n. 447]	36 060 000.00
Vincente n. 9	H3G S.p.A.	[tornata n. 459]	36 040 000.00
Vincente n. 10	Wind Telecomunicazioni S.p.A.	[tornata n. 445]	36 020 000.00
Vincente n. 11	Wind Telecomunicazioni S.p.A.	[tornata n. 445]	36 020 000.00
TOTALE			398 140 000.00

Spectrum underutilization

- 15%-85% of the spectrum is underutilized
- 3-day campaign in New York and Chicago in 2002 and 2005:

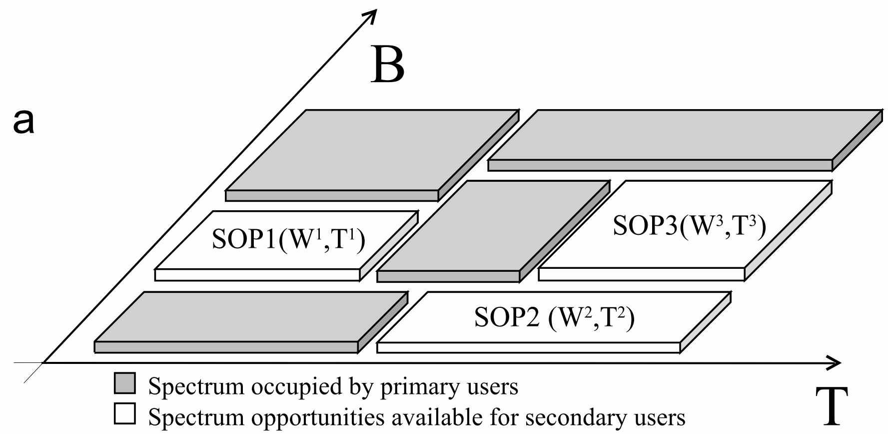


13.1% AVERAGE UTILIZATION **17.5%**

Cognitive Radio Networks: from Theory to Practice, Springer

Cognitive Radio Networks

- **Problem –** Licensed frequency assignment ➔ Underutilized spectrum portions both in time and in space.
- **Solution –** Access spectrum "holes" in a non-intrusive manner ➔ No interference to licensed users.
- **How to do that -** Cognitive cycle:
  - Detect unused spectrum portions, a.k.a. Spectrum Opportunities, SOPs *(Spectrum sensing)*
  - Characterize unused portions and assign a perceived quality *(Spectrum decision)*
  - Select best available SOP while coordinating with other secondary users *(Spectrum sharing)*
  - Handover towards other SOPs when current unavailable or better one shows up *(Spectrum mobility)*



Spectrum sensing

- External
  - Geo-location and spectrum databases
- Independent
  - Energy detector
  - Waveform-based (pattern matching)
  - Cyclostationarity-based (autocorrelation)
  - Radio identification
  - Matched-filtering
- Cooperative
  - Sharing of sensing information

Spectrum sharing scenarios

- Regulated scenario
  - Spectrum broker with full knowledge of the spectrum context
    - Occupation, load, bandwidth
  - Orchestrate spectrum assignment to maximize average quality perceived by SUs
- Unregulated scenario
  - Completely distributed process, competition among SUs
  - Optimizing their own experienced quality according to information on spectrum status

Spectrum sharing scenarios

- Regulated scenario
  - Spectrum broker with full knowledge of the spectrum cont
  - O ... e average qua

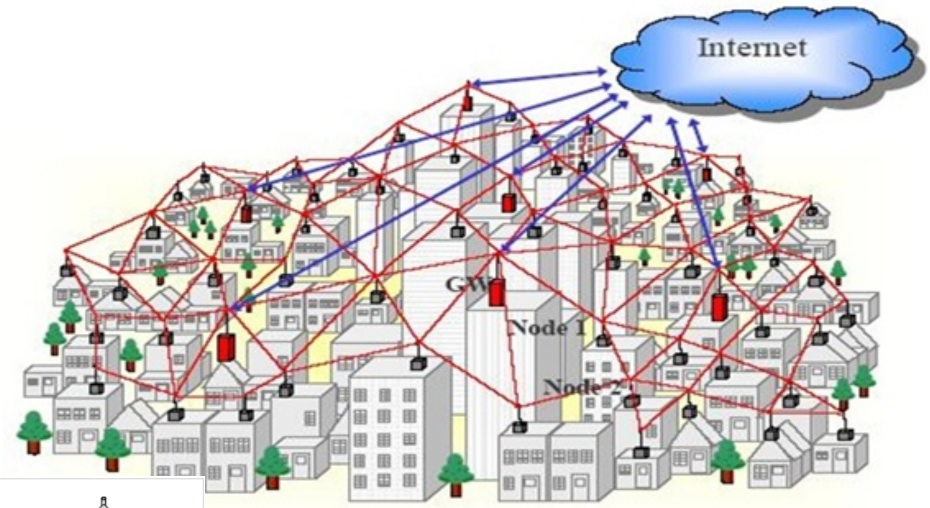
Pay attention when using Game Theory!

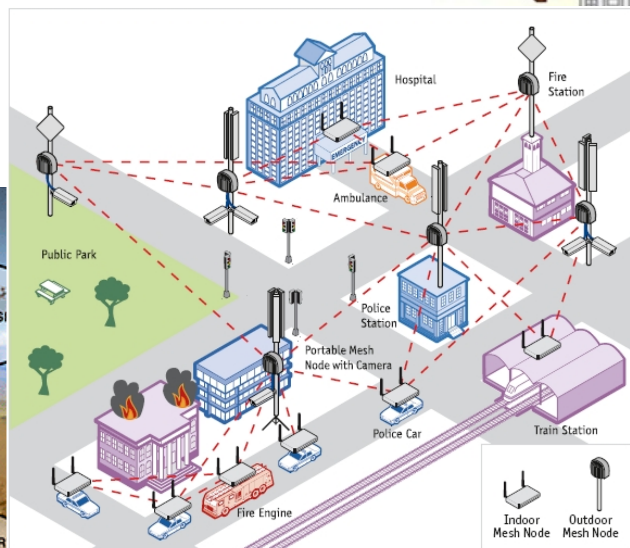
Don't introduce competition in scenarios where single-minded approaches are the norm.

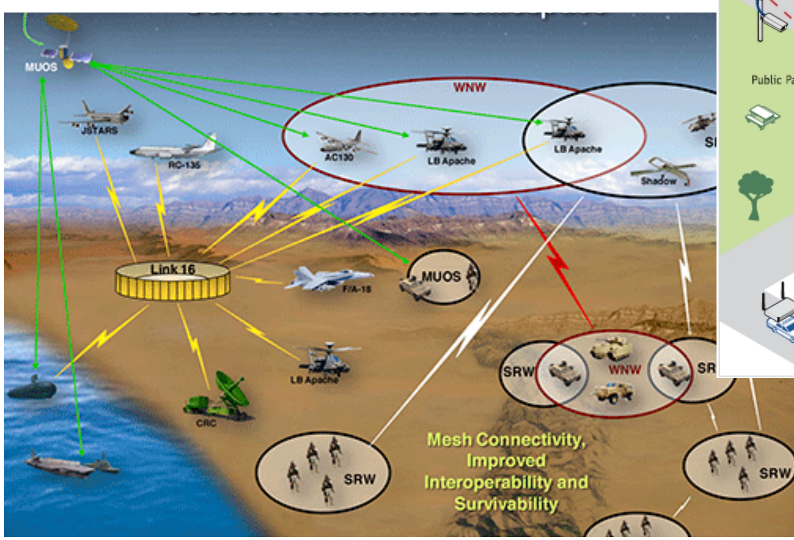
- Unregulated scenario
  - Completely distributed process, competiti on SUs
  - Optimizing their own experienced quality ac information on spectrum status

CRN applications

- Cognitive mesh networks for last-mile Internet
- Public safety networks
- Disaster relief and emergency networks
- Battlefield military networks
- Leased networks



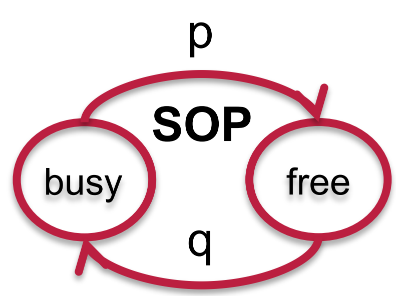




Spectrum Selection Game

Spectrum Selection Game

- Spectrum is divided in sub-bands: Spectrum OPportunities (SOPs)
- Secondary users (SUs) can occupy SOPs only if they are vacant, i.e., no primary user (PU) is using the SOP
- SUs tuned on the same SOP interfere each other if closer than interference range
- We define:
  - SU set ***N***: set of secondary users
  - SOP set ***B***: set of available spectrum opportunities



Spectrum Selection Game (cont'd)

- *SSG*:
  - Player set N : set of (secondary) users
  - Strategy sets B_i : set of available SOPs for user i
  - Cost functions c_i : $c_i(s,n_{s,i})$
    - s in B_i
    - $n_{s,i}$: users that interfere with i using SOP s
    - c_i is monotonically increasing in $n_{s,i}$

$$SSG = \left\langle N, \{B_i\}_{i \in N}, \{c_i(s, n_{s,i})\}_{i \in N, s \in B_i} \right\rangle$$

  - Snapshot of spectrum status
- User i plays:

$$s^* = \operatorname*{argmin}_{s \in B_i} c_i(s, n_{s,i})$$

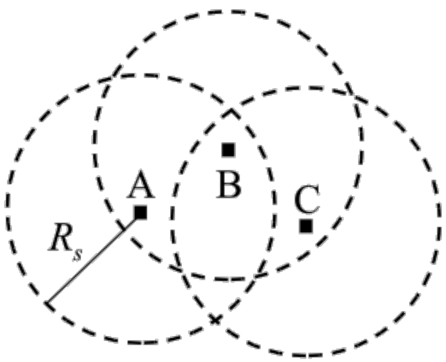
SSG properties

- SSG is a *congestion game*, specifically a *crowding game*
  - single-choice: only one SOP per SU
  - player-specific cost function: each SU can have different cost function
  - non-weighted: SUs congest resources with the same weight
- Theoretical result[1]:
  - It admits at least one pure-strategy Nash Equilibrium for any cost function that is increasing in the level of congestion

[1] I. Milchtaich, "Congestion games with player-specific payoff functions," *Games and Economic Behavior*, vol. 13, no. 1, pp. 111–124, 1996.

Crowding Game Equivalence

- *SSG* is equivalent to a non-weighted single-choice Crowding Game (*CG*)
- Subtle point
  - *CG*: $c_i(s,n_s)$, n_s number of players that choose resource *s*
  - *SSG*: $c_i(s,n_{s,i})$, different players can perceive different congestion levels $n_{s,i}$ due to interference range



everybody selects the same SOP s

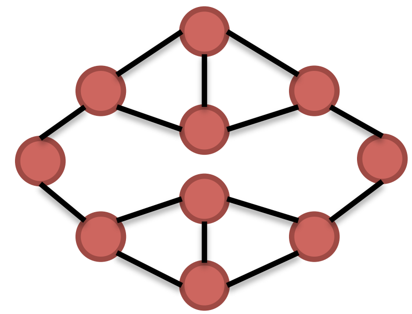
$n_{s,A} = 2$

$n_{s,B} = 3$

$n_{s,C} = 2$

Network Games

- Players select path from a source to a destination
- Edges are resources and players' costs are the sum of the costs of the chosen resources
  - Multiple-choice congestion game

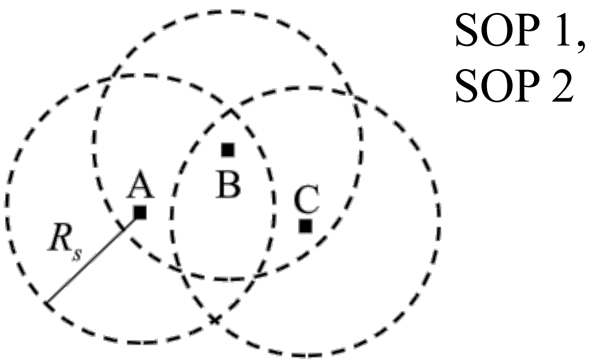


- We use linear player-specific cost function
  - $c_i(s,n_s) = a_{i,s}n_s$
- However, by opportunistically setting $a_{i,s}$
  - Each player makes *essentially* one choice
  - *Essentially* → there is a dominant choice independently of the other players in all but one node

Crowding Game Equivalence (cont'd)

- Edge weights are player specific parameters ($a_{A,s}$, $a_{B,s}$, $a_{C,s}$)
- Only at source we have a non-trivial choice for every player
- Aim is to construct an equivalent game that produces the same costs of the original game.

$c_i(s, n_{s,i}) = n_{s,i}$

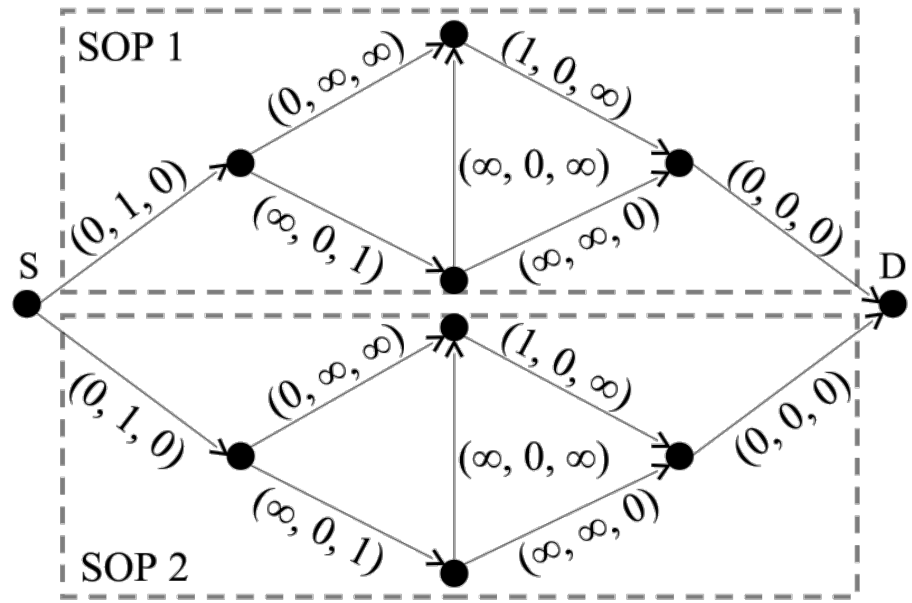


$SOP\ 2 \quad c_A = 2$

$SOP\ 2 \quad c_B = 3$

$SOP\ 2 \quad c_C = 2$

$\equiv$



Crowding Game Equivalence (cont'd)

- Edge weights are player specific parameters ($a_{A,s}$, $a_{B,s}$, $a_{C,s}$)
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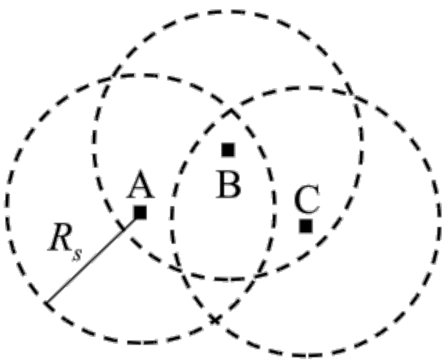
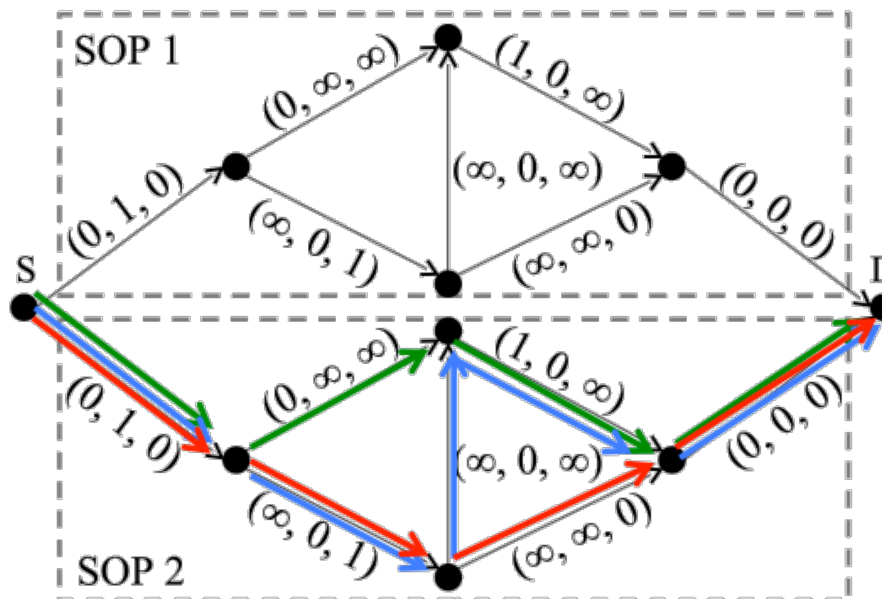
$c_i(s, n_{s,i}) = n_{s,i}$



SOP 1,
SOP 2

$\equiv$



SOP 2 $c_A = 2$
SOP 2 $c_B = 3$
SOP 2 $c_C = 2$

Analysis of SSG

- Cost function
  - How to translate SOP quality in costs?
    - Engineering

- Characterization of Equilibria
  - Find Equilibria
  - Investigate about Price of Stability and Price of Anarchy
    - Mathematics

Assessing Quality of SOP

- Parameters
  - SOP Bandwidth: Total bit/s
  - SOP Holding Time: the longer the less SU has to switch
  - SOP Congestion: number of interfering users
- We define
  - $W_{s,i}$ proportional to inverse of the Bandwidth
  - $T_{s,i}$ proportional to inverse of the Holding Time
- Three cost functions
  1) Simple: $c_i\left(s,n_{s,i}\right)=n_{s,i}$
  2) Additive: $c_i\left(s,n_{s,i}\right)=\lambda_i n_{s,i}W_{s,i}+\left(1-\lambda_i\right)T_{s,i}$
  3) Multiplicative: $c_i\left(s,n_{s,i}\right)=n_{s,i}W_{s,i}T_{s,i}$

Assessing Quality of SOP

- Parameters
  - SOP Bandwidth: Total bit/s
  - SOP ... to

Pay attention to the objective of your cost function!!!

Have clear in mind the behavior of a rationale player!

- We define
  - $W_{s,i}$ proportional to inverse of the Bandwidth
  - $T_{s,i}$ proportional to inverse of the Holding Tim
- Three cost functions
  1) Simple: $c_i\left(s,n_{s,i}\right)=n_{s,i}$
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  3) Multiplicative: $c_i\left(s,n_{s,i}\right)=n_{s,i}W_{s,i}T_{s,i}$

Finding Nash Equilibrium

- Several alternative ways
  - Representing the game with a table
  - Drawing best response curves
  - Play the game
    - f.i., best response dynamics, if the game admits Finite Improvement Property with best response
  - Solving a set of equations
    - <u>Using a Mathematical Programming Model</u>

Mathematical Programming Formulation

- Three main ingredients
  - Decision variables
    - SOP selected by each SU
  - Constraints
    - Each SU can choose a single SOP
    - Solution must be a Nash Equilibrium
  - Objective function
    - Define the quality of equilibrium
- This linear Integer Programming (IP) model can be solved with standard tools
  - AMPL/OPL modeling language
  - CPLEX/GUROBI solver engine

$$y_{i,k} \begin{cases} 1 & \text{if SU } i \text{ selects SOP } k \\ 0 & \text{otherwise} \end{cases}$$

$$\min/\max \sum_{k \in B_i} y_{ik} c_i\left(k, n_{k,i}\right)$$

such that

$$\sum_{k \in B_i} y_{ik} = 1 \qquad \forall i \in N$$

$$y_{im} c_i\left(m, n_{m,i}\right) \leq c_i\left(k, n_{k,i}\right) \qquad \forall i \in N, m, k \neq m \in B_i$$

$$y_{i,m} \in \{0,1\} \qquad \forall i \in N, m \in B_i$$

MIN gives you the best NE
MAX gives you the worst NE

Quality of reached equilibria

- Solve the centralized problem optimally using previous IP model
  - MIN objective function
  - remove NE constraint
- Compare
  - Best NE against OPT: Price of Stability
  - Worst NE against OPT: Price of Anarchy

SPECTRUM OPPORTUNITIES FEATURES

Spectrum Class	Low Activity						Medium Activity						High Activity					
	Low Opportunity			High Opportunity			Low Opportunity			High Opportunity			Low Opportunity			High Opportunity		
Spectrum band k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
p	0.2	0.2	0.2	0.2	0.2	0.2	0.5	0.5	0.5	0.5	0.5	0.5	0.8	0.8	0.8	0.8	0.8	0.8
q	0.1	0.1	0.1	0.5	0.5	0.5	0.3	0.3	0.3	0.8	0.8	0.8	0.3	0.3	0.3	0.9	0.9	0.9
Bandwidth [KHz]	250	100	70	250	100	70	250	100	70	250	100	70	250	100	70	250	100	70
W^k	1	2.5	3.5	1	2.5	3.5	1	2.5	3.5	1	2.5	3.5	1	2.5	3.5	1	2.5	3.5
Holding Time [sec]	5	5	5	5	5	5	2	2	2	2	2	2	1.25	1.25	1.25	1.25	1.25	1.25
T^k	1	1	1	1	1	1	2.5	2.5	2.5	2.5	2.5	2.5	4	4	4	4	4	4

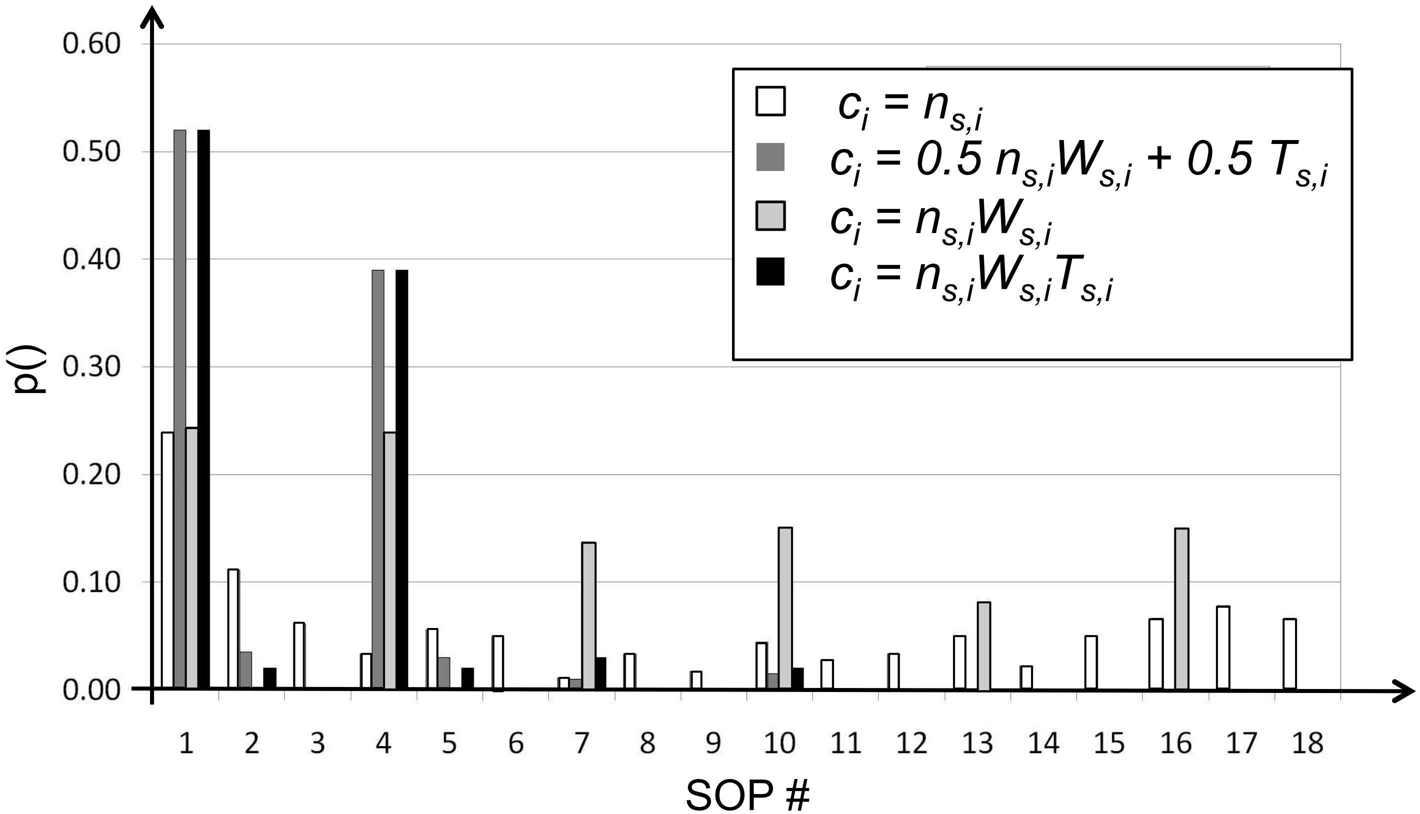
Anarchy is rather efficient !

RESULTS OBTAINED ON A UNIFORM TOPOLOGY WITH $n = 20$ SECONDARY USERS, $L = 500$ AND $r = 100$ METERS

ost Function	(1)	(2)											(3)
		$\lambda = 0$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
$\overline{x}_i^k$	1.000	3.250	1.220	1.220	1.220	1.220	1.220	1.030	1.030	1.000	1.000	1.000	1.220
$\overline{W}^k$	2.186	1.008	1.135	1.135	1.135	1.135	1.098	1.000	1.000	1.000	1.000	1.000	1.060
Bandwidth [KHz]	150.83	249.25	236.50	236.50	236.50	236.50	240.25	250.00	250.00	250.00	250.00	250.00	244.00
$\overline{T}^k$	2.250	1.000	1.000	1.000	1.000	1.000	1.038	1.278	1.278	1.323	1.323	2.125	1.075
Holding Time [sec]	3.250	5.000	5.000	5.000	5.000	5.000	4.925	4.445	4.445	4.389	4.389	3.270	4.850
PoS	1.000	1.000	1.004	1.008	1.012	1.015	1.025	1.004	1.006	1.000	1.000	1.000	1.043
PoA	1.000	1.000	1.030	1.059	1.086	1.111	1.116	1.206	1.091	1.042	1.022	1.000	1.092

Experimental results

- Probability of a generic user to occupy a SOP



Spectrum Class	Low Activity						Medium Activity						High Activity					
	Low Opportunity			High Opportunity			Low Opportunity			High Opportunity			Low Opportunity			High Opportunity		
Spectrum band k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
p	0.2	0.2	0.2	0.2	0.2	0.2	0.5	0.5	0.5	0.5	0.5	0.5	0.8	0.8	0.8	0.8	0.8	0.8
q	0.1	0.1	0.1	0.5	0.5	0.5	0.3	0.3	0.3	0.8	0.8	0.8	0.3	0.3	0.3	0.9	0.9	0.9
Bandwidth $[KHz]$	250	100	70	250	100	70	250	100	70	250	100	70	250	100	70	250	100	70
W^k	1	2.5	3.5	1	2.5	3.5	1	2.5	3.5	1	2.5	3.5	1	2.5	3.5	1	2.5	3.5
Holding Time $[sec]$	5	5	5	5	5	5	2	2	2	2	2	2	1.25	1.25	1.25	1.25	1.25	1.25
T^k	1	1	1	1	1	1	2.5	2.5	2.5	2.5	2.5	2.5	4	4	4	4	4	4

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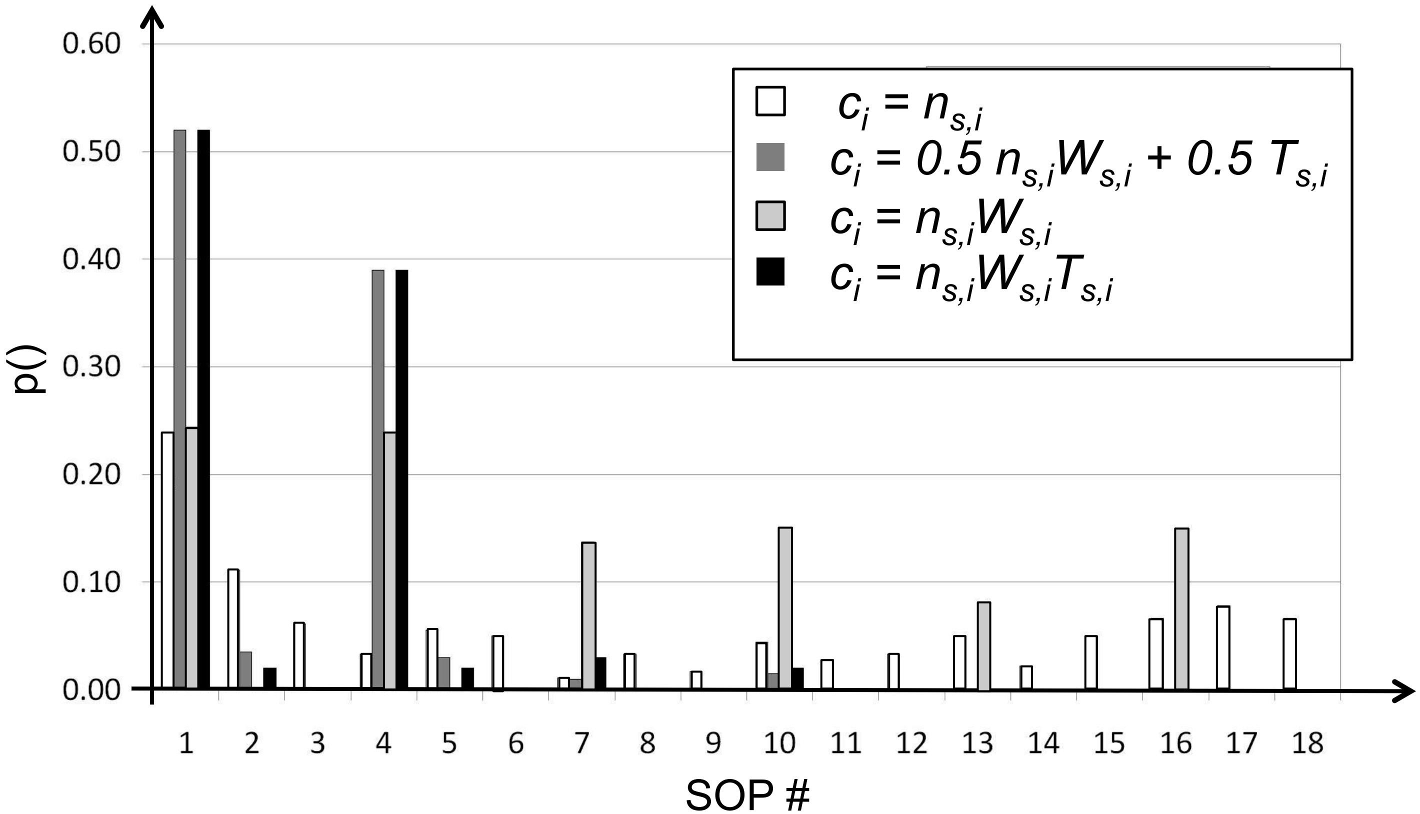

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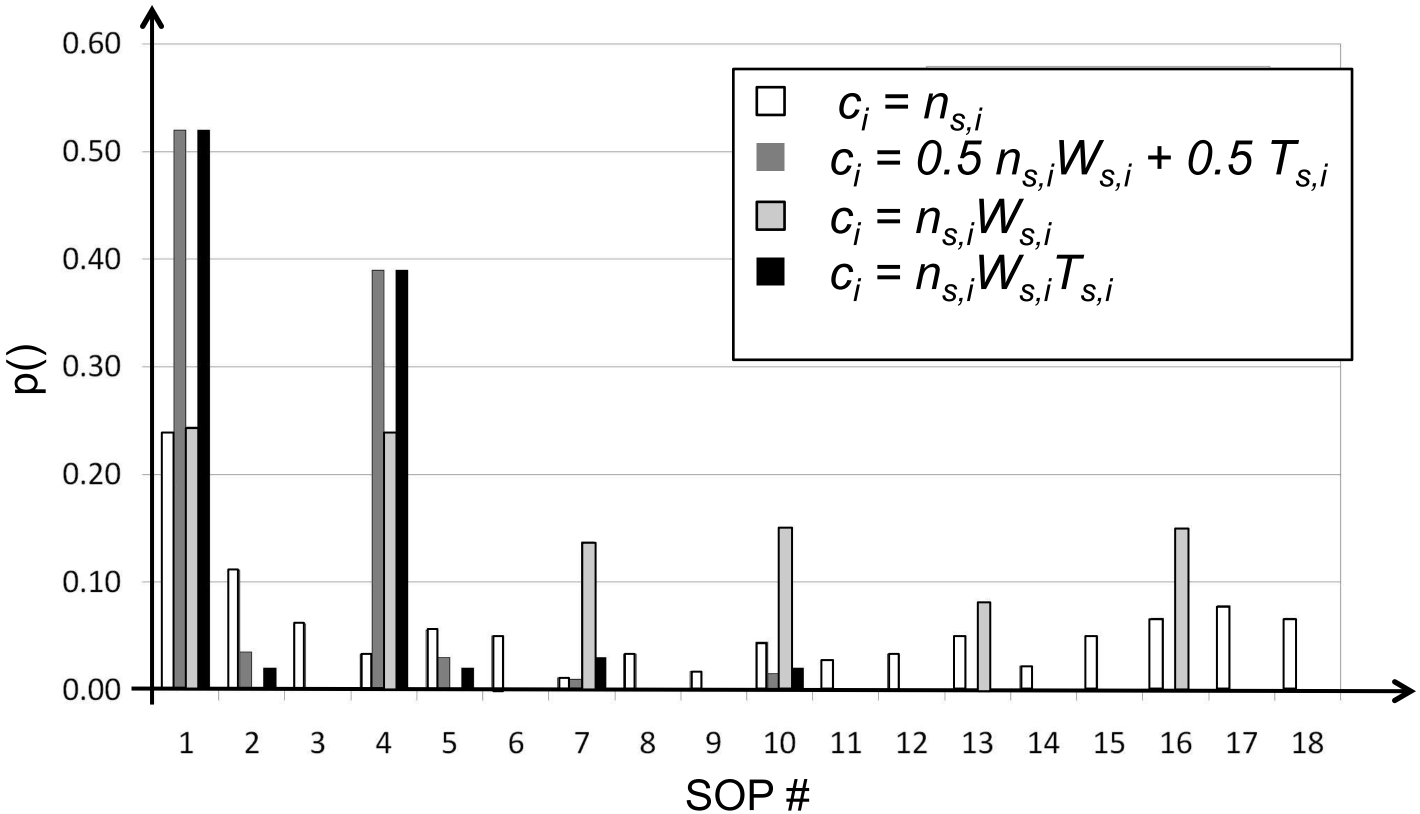
Holding Time $[sec]$	3	5	5	3	5	5	2	2	2	2	2	2	1.25	1.25	1.25	1.25	1.25	1.25
T^k	1	1	1	1	1	1	2.5	2.5	2.5	2.5	2.5	2.5	4	4	4	4	4	4

Experimental results

- Probability of a generic user to occupy a SOP



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q	0.1	0.1	0.1	0.5	0.5	0.5	0.3	0.3	0.3	0.8	0.8	0.8	0.3	0.3	0.3	0.9	0.9	0.9
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Holding Time $[sec]$	[illegible]	[illegible]	5	[illegible]	[illegible]	5	[illegible]	2	2	[illegible]	2	2	1.25	1.25	1.25	1.25	1.25	1.25
T^k	1	1	1	1	1	1	2.5	2.5	2.5	2.5	2.5	2.5	4	4	4	4	4	4

Practical Aspects

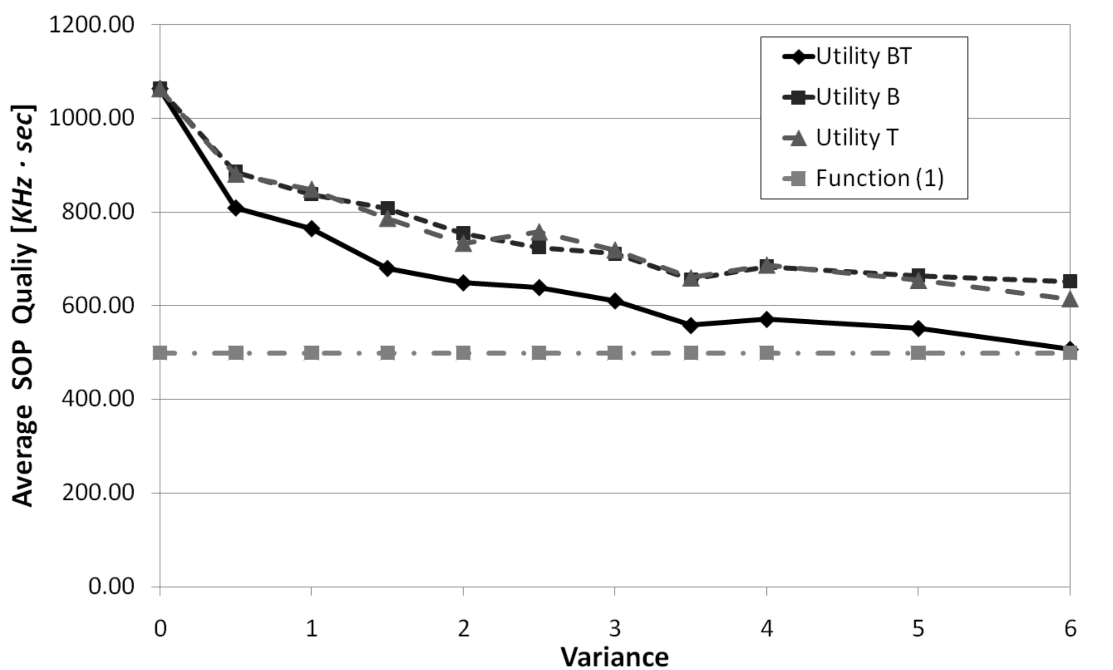
Practical Aspects

I like it !!!

I’m a nerdy engineer.

Parameters affected by Gaussian error

- Users get information by spectrum sensing, monitoring radio transmissions and exchanging data with neighbors
- Parameters are in general obtained from the average on multiple values
  ➔ **Imperfect Knowledge**
- Performance degradation in terms of perceived SOP quality ([Bandwidth · Holding Time/Interfering Users])



Parameters affected by Gaussian error

- Users get information by spectrum sensing, monitoring radio transmissions and exchanging data with neighbors
- Parameters are in general obtained from the average on multiple values
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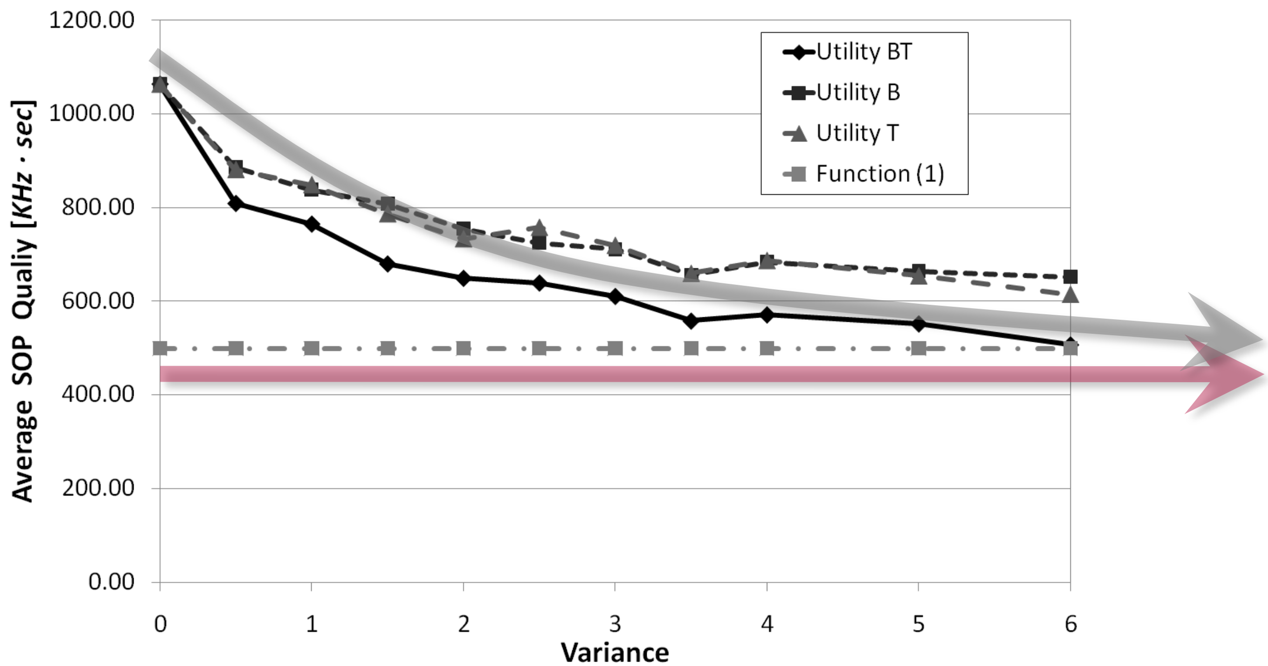

Parameters affected by Gaussian error

- Users get information by spectrum sensing, monitoring radio transmissions and exchanging data with neighbors
- Parameters are in general obtained from the average on multiple values ➔ **Imperfect Knowledge**
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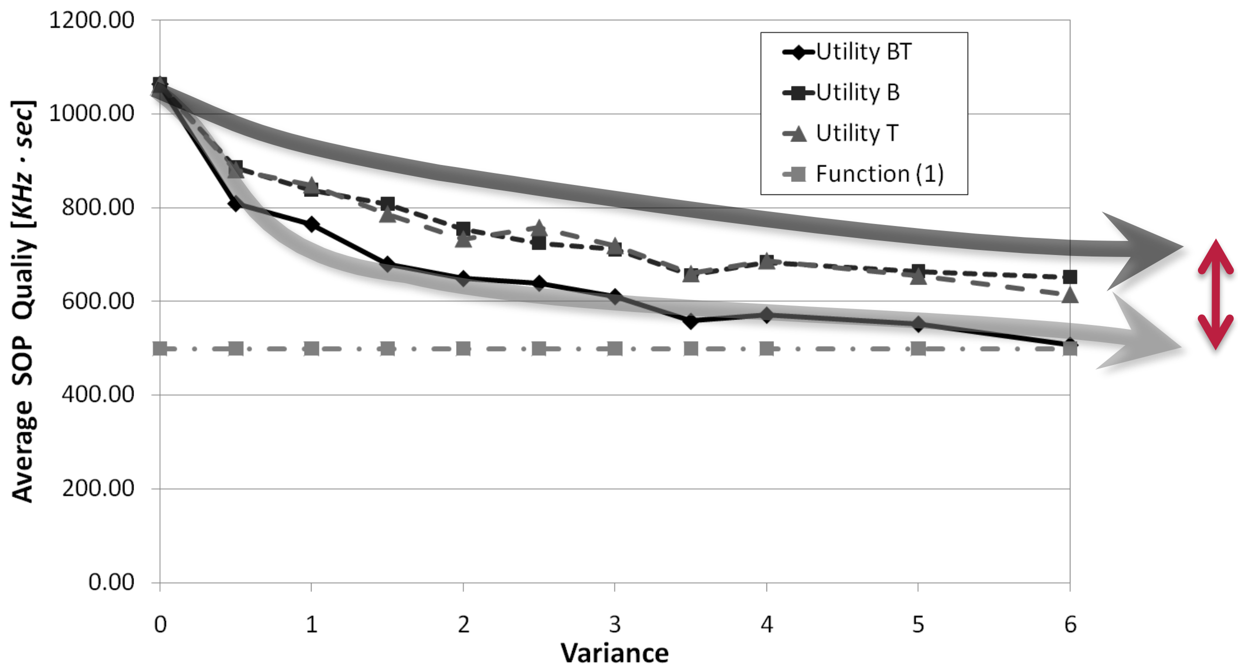
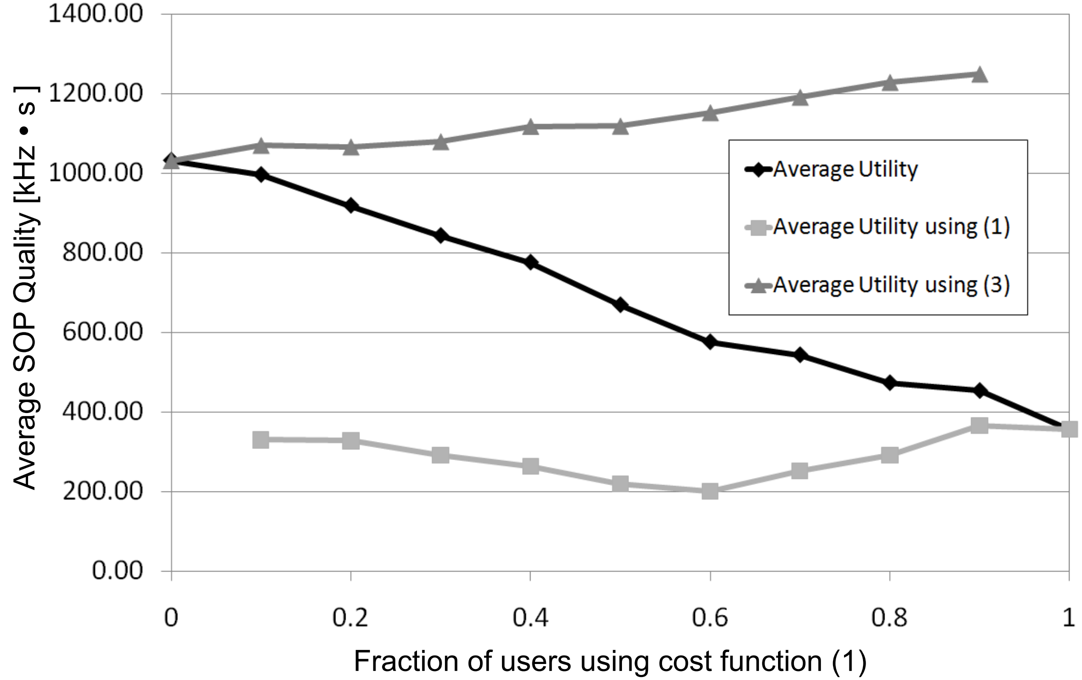


Different knowledge about SOP parameters

- Different knowledge on SOP ➔ users play using different cost functions.
- Example:
  - Users using (1) $c_i = n_{s,i}$ only know congestion levels
  - Users using (3) $c_i = n_{s,i} W_{s,i} T_{s,i}$ have complete information

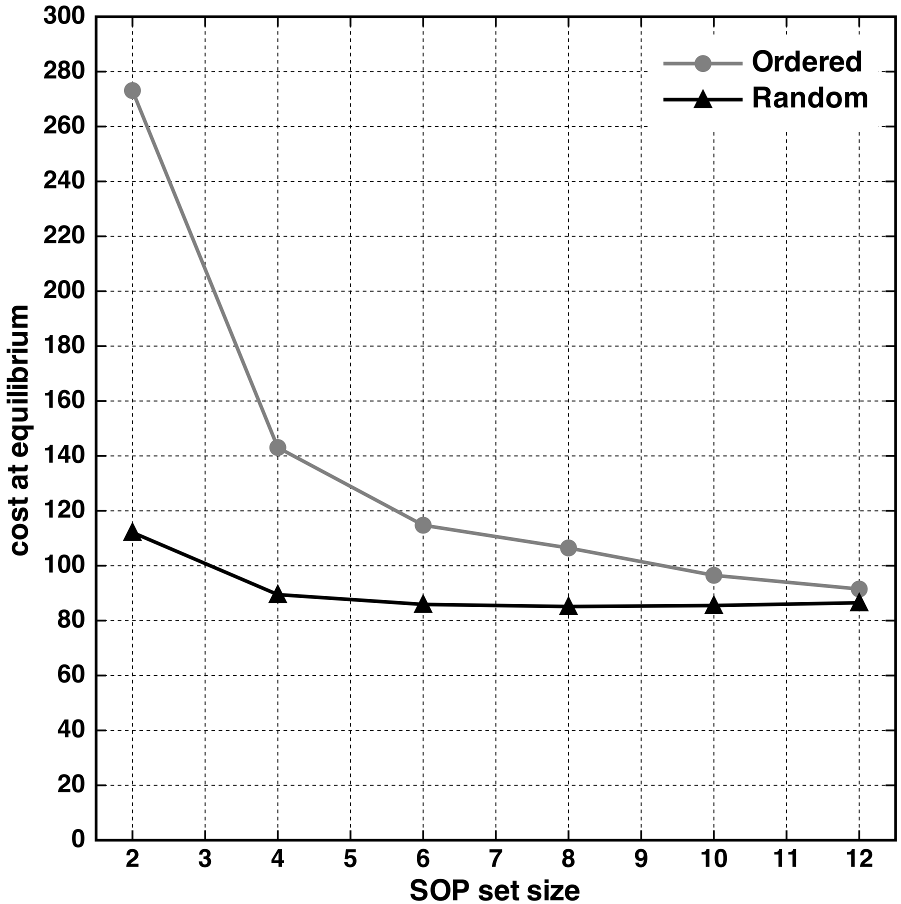


Size of SOP sets

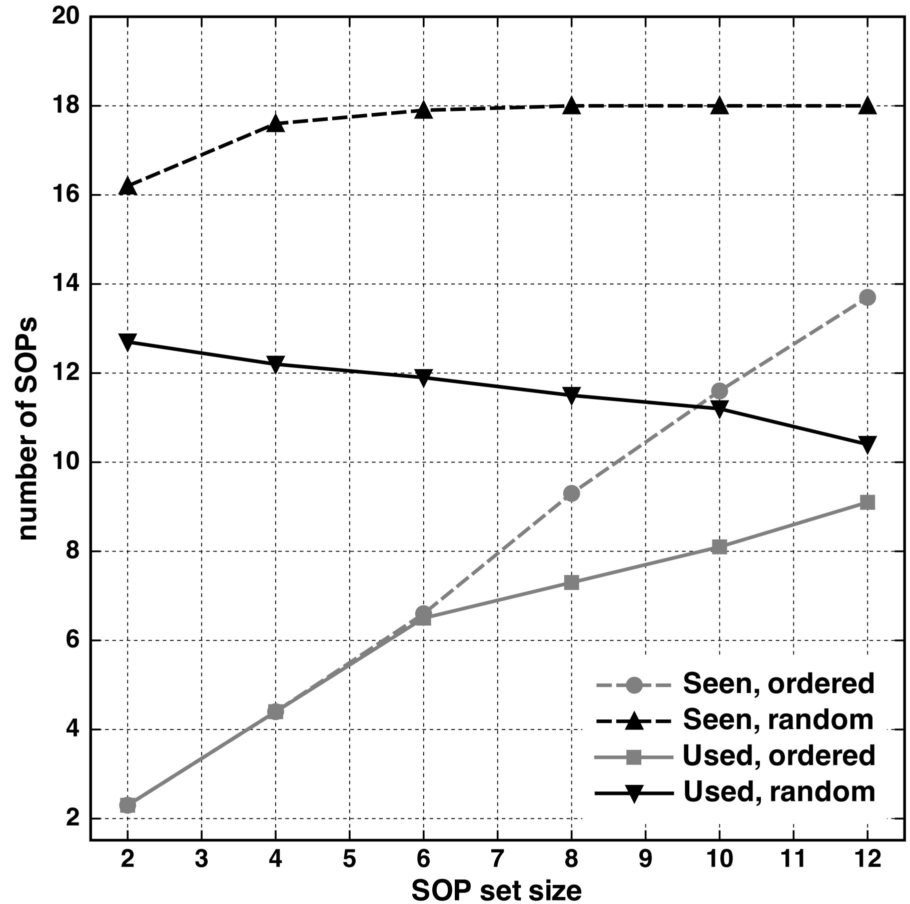
- Sometimes the whole spectrum cannot be entirely scanned before transmitting due to time constraints.
- Only up to *B* of all the available SOPs can be used in each user's set.
- Selection schemes:
  - **Ordered:** every user uses (almost) the same SOP set, first best *B* SOPs (lowest cost).
  - **Random:** users randomly and independently select which SOPs to include, up to *B.*
- Users play choosing SOPs only within the *B* SOPs in their sets.

Experimental Results

Cost at equilibrium



Number of different seen/used SOPs in the entire set of users



Paradox

- Increasing size of SOP set can sometimes lead to worse equilibria in the random approach.
- Example with 6 users and initial 2-SOP sets:

User	1st #, [W T]	2nd #, [W T]
A	#4, 1.00	#13, 4.00
B	#4, 1.00	#8, 6.25
C	#4, 1.00	#13, 4.00
D	#4, 1.00	#8, 6.25
E	#4, 1.00	#13, 4.00
F	#12, 8.75	#18,14.00

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Best NE
social cost = 28.75

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F	**#12, 8.75**	#18,14.00

User	1st #, [W T]	2nd #, [W T]	3rd #, [W T]
A	#4, 1.00	#13, 4.00	#18,14.00
B	#4, 1.00	#8, 6.25	#18,14.00
C	#4, 1.00	#13, 4.00	#18,14.00
D	#4, 1.00	#8, 6.25	#18,14.00
E	#4, 1.00	#13, 4.00	#18,14.00
F	#12, 8.75	#18,14.00	#4, 1.00

One more SOP…

Paradox

- Increasing size of SOP set can sometimes lead to worse equilibria in the random approach.
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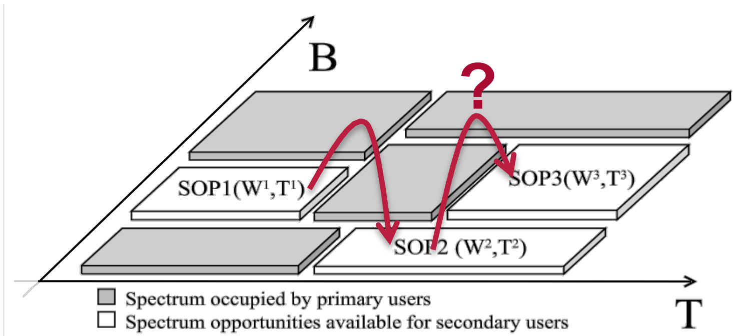
E	#4, 1.00	**#13, 4.00**	#18,14.00
F	#12, 8.75	#18,14.00	**#4, 1.00**

Best NE
social cost = 29 > 28.75 !!!

Time-varying Scenario

Spectrum mobility with Multi-stage Games

- Time varying scenario, multiple epochs:
  - Move to a new SOP when primary user shows up in the current one
  - To jump or not to jump when better SOPs appear ?



- At each epoch, users:
  - are currently using a SOP (from the previous epoch)
  - must choose if staying or moving and where moving
- Different cost function:
  - $c_i(s,n_{s,i}) = n_{s,i}W_{s,i}T_{s,i} + K_{ms}$
- K_{ms} : switching cost in terms of switching delay or energy or simply will to not move.

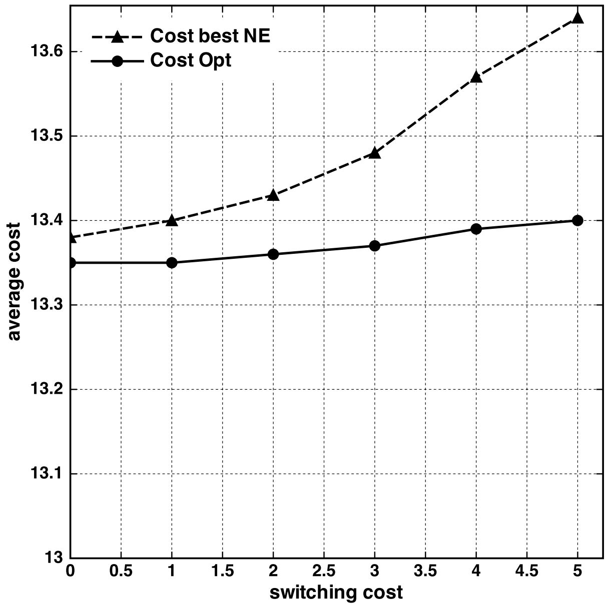
What about complexity ???

- Multi-stage game ➔ Extensive-form Game
- We need a **sub-game perfect equilibrium**
- Strategy that is a NE in each sub-game
  - 1 sub-game for each choice of each user of each epoch:
    - $[\,[\text{SOPS}]^{\text{USERS}}\,]^{\text{EPOCHS}}$ sub-games !!!
- Two approaches:
  - Playing on-line, stage-by-stage equilibrium
  - Playing with look-ahead: users know SOP availability status of the next epoch.
    - Users considers both current SOP and one in the next epoch. Next epoch, again, users compute optimal strategy taking into account current and next epoch. Sliding two-epoch window over the epoch sequence
- Smaller instances !!!

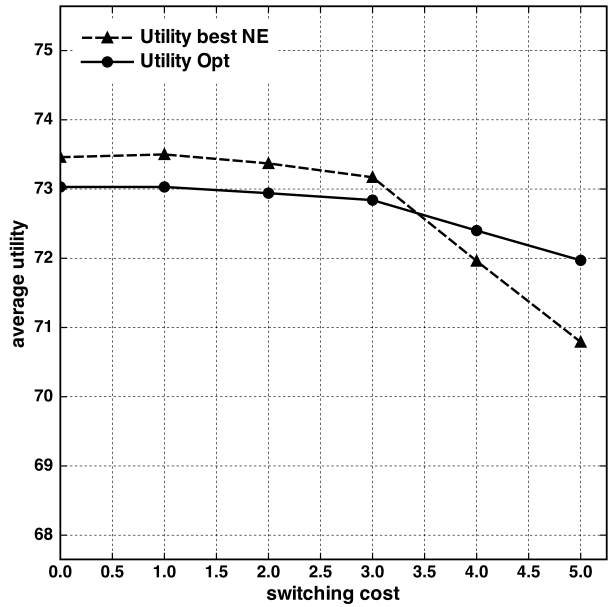
Experimental Results

- Stage-by-stage

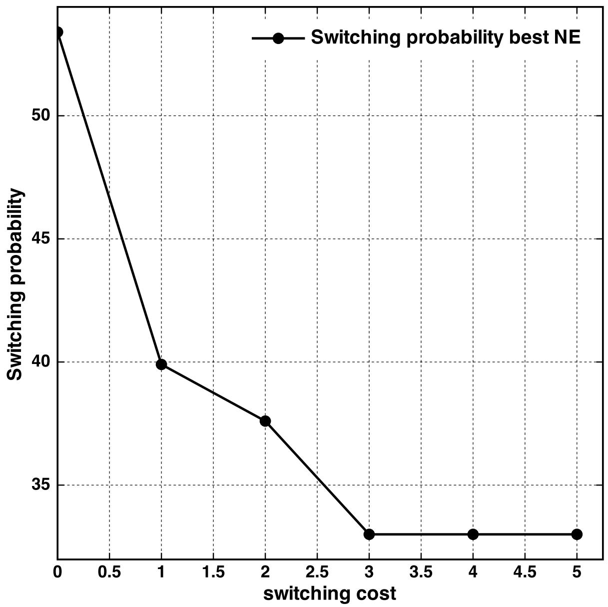
Cost



Utility



Switching Prob.



Experimental Results

- Stage-by-stage

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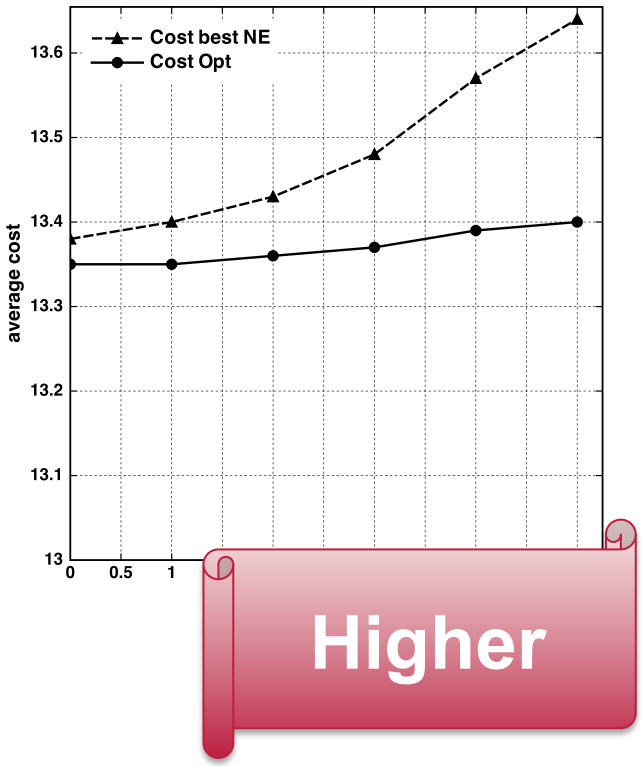
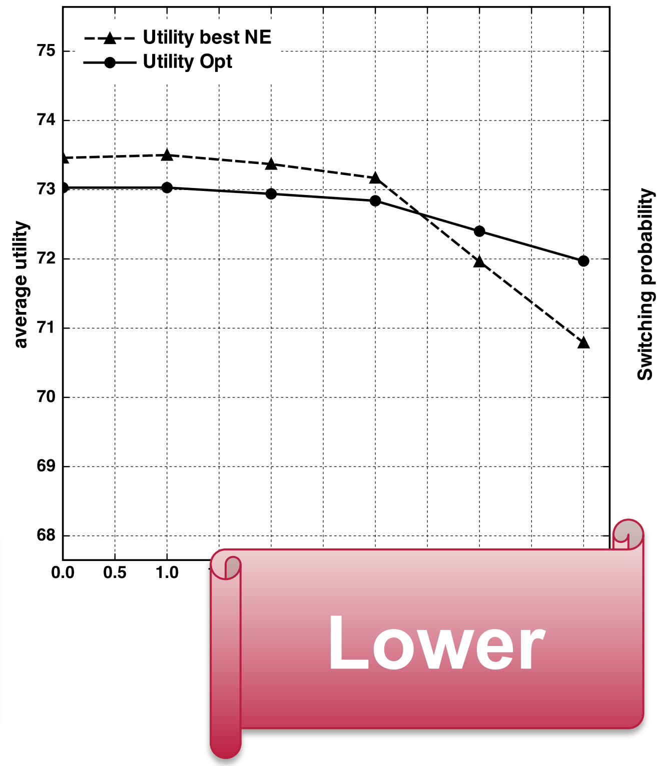


Utility



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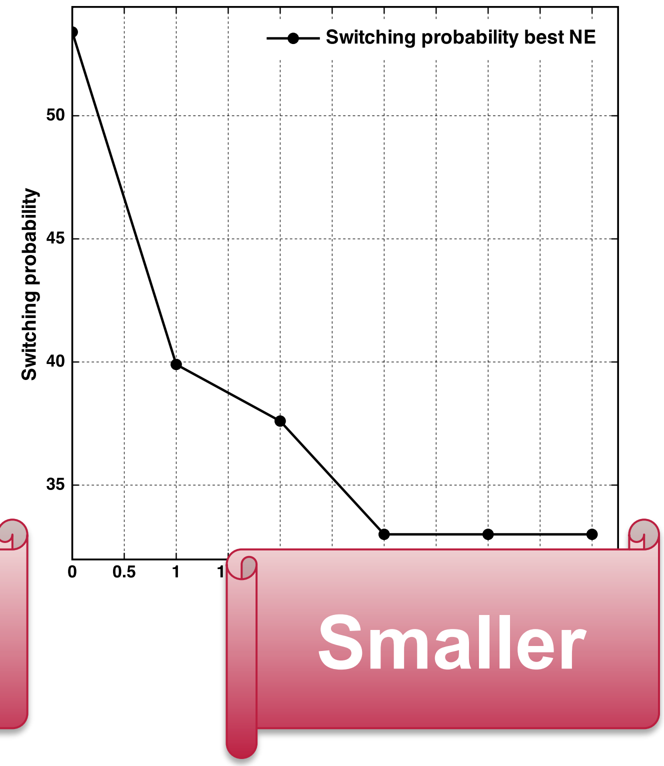
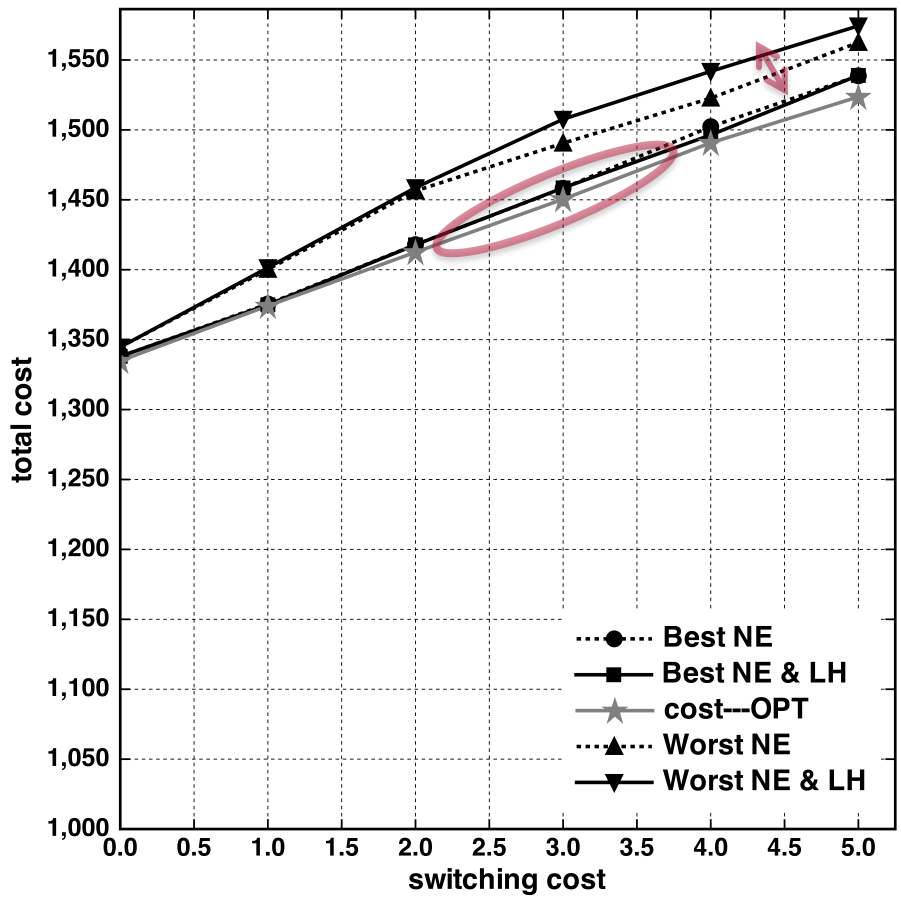


- Stage-by-stage and Look-ahead

Total Cost



- SOP costs include holding time
- Users prefer stable SOPs, information on next epoch is not so important

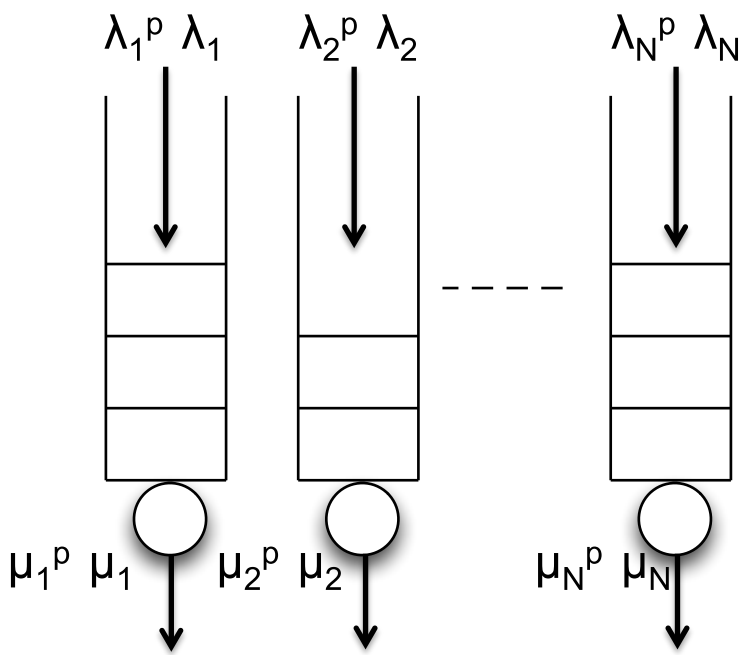
Game + Queue Theory

Reference scenario

- Set of available channels $i=1..N$
- PU transmissions
  - PU arrivals: $Poi(\lambda^p_i)$
  - Average channel occupation time: $1/\mu^p_i$
- SU transmissions
  - Average time length over channel i: $1/\mu_i$
  - Arrivals split over available channels

$$\lambda_{tot} = \sum \lambda_i$$

- Ideal collision management
- Preemption-repeat strategy
  - SUs back-off at PU arrival
  - Re-tx of the entire packet as the channel frees up



Spectrum quality measure

- Transmission delay: time required by SU transmission to go through the channel
  - Channel quality: bandwidth and retransmissions
  - Congestion level: queueing
- Computed using Pollaczek-Khintchine result:

$$d_i(\lambda_i) = \frac{\frac{\lambda_i}{\mu_i} E[Z_i^s]}{1 - \frac{\lambda_i}{\mu_i}} + E[C_i^s]$$

$E[C^s{}_i]$ = extended service time considering PU interruptions

$E[Z^s{}_i]$ = residual extended service time seen by a SU packet entering at channel i

- Closed form expressions in F. Borgonovo, M. Cesana, L. Fratta, "*Throughput and delay bounds for cognitive transmissions*", *Advances in Ad Hoc Networking,* Springer, 2008, vol. 265, pp. 179-190

Optimal regulated scenario

- Spectrum broker optimally subdivides SUs among available channels
- Optimization problem:

$$\text{minimize} \quad S(\boldsymbol{\lambda}) = \sum_{i=1}^{N} \lambda_i d_i(\lambda_i)$$

$$\text{s. t.} \quad \sum_{i=1}^{N} \lambda_i = \lambda_{tot},$$

$$\lambda_i \geq 0 \; i \in \mathcal{I},$$

- Solution $\boldsymbol{\lambda}_{opt} = [\lambda_1, \lambda_2, \ldots, \lambda_N]$
- Social welfare: $S(\boldsymbol{\lambda})$ average delay

Competitive scenario and equilibrium

- SUs selfishly select the best channel to use
  - Non-cooperative Game
- Number SUs is large, single demand is infinitesimal contribution with respect to the overall demand
- Stable repartition defined by Wardrop Equilibrium
  - All the used channels feature a transmission delay which is equal or less than the transmission delay of any other used channel
- *Wardrop Equilibrium:* $\boldsymbol{\lambda}_w = [\lambda_1, \lambda_2, \ldots, \lambda_N]$

$$\lambda_k > 0 \quad \text{iff} \quad d_k(\lambda_k) \le d_i(\lambda_i), \quad \forall i,k \in I,\ i \ne k$$

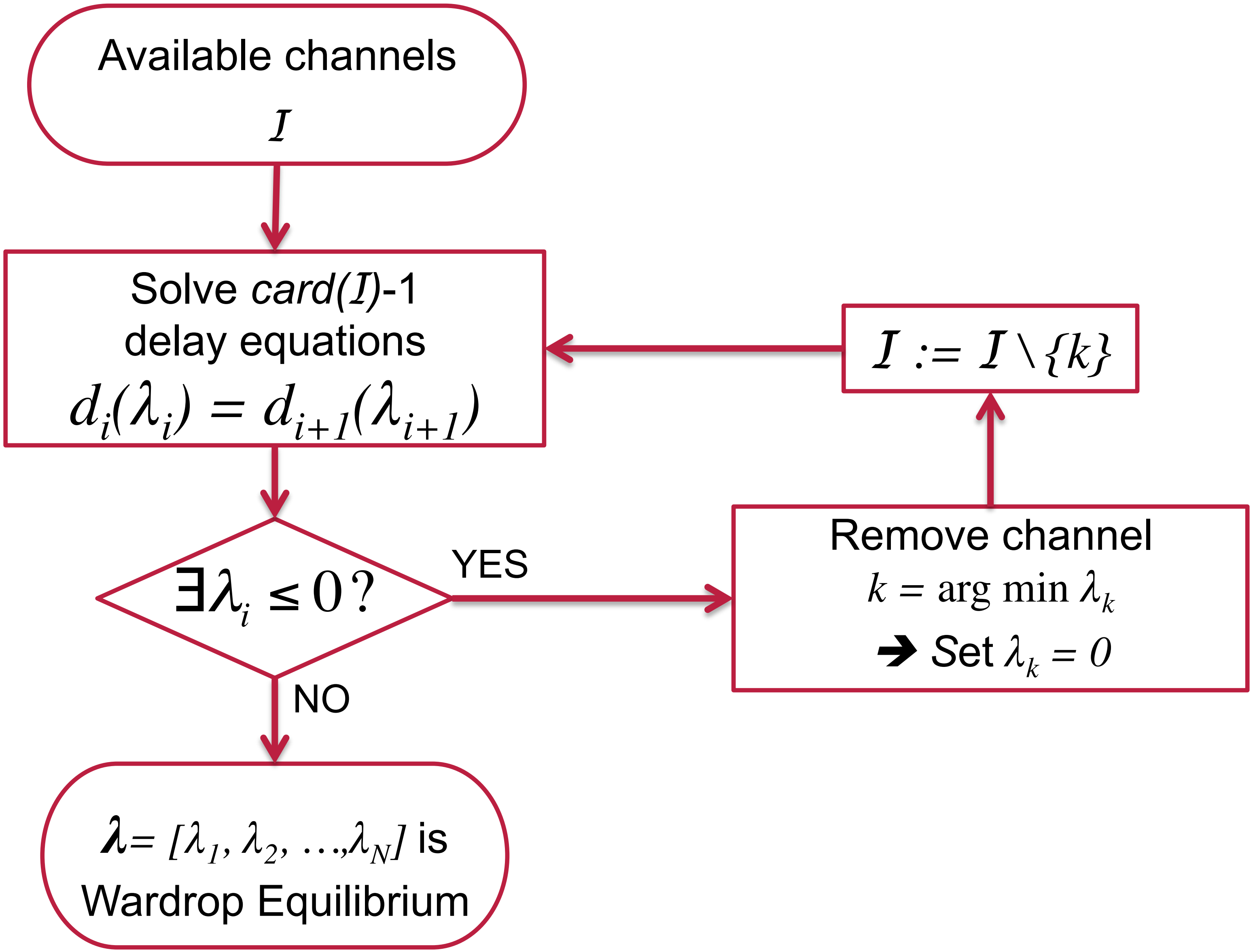
Finding the Wardrop Equilibrium

- Delay function is continuous and non-decreasing in λ
  ➔ Unique Equilibrium

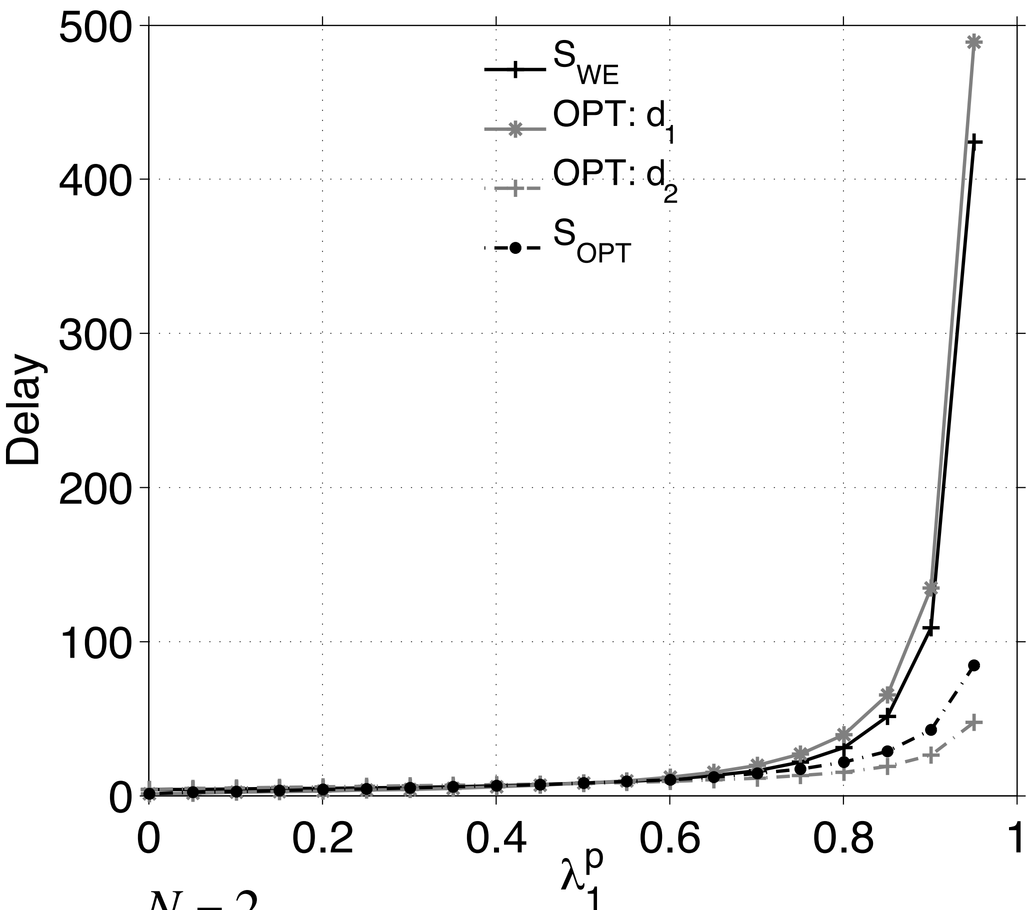
- Practically:
  – Find a non-negative flow repartition where the delay at each used channel is equal

$$\begin{cases} d_i(\lambda_i) = d_k(\lambda_k) \quad \forall i, k \in \mathcal{I} : \lambda_i > 0, \lambda_k > 0 \\ \sum_{i \in \mathcal{I}} \lambda_i = \lambda_{tot} \end{cases}$$

Finding the Wardrop Equilibrium



Delay: Optimal vs Wardrop



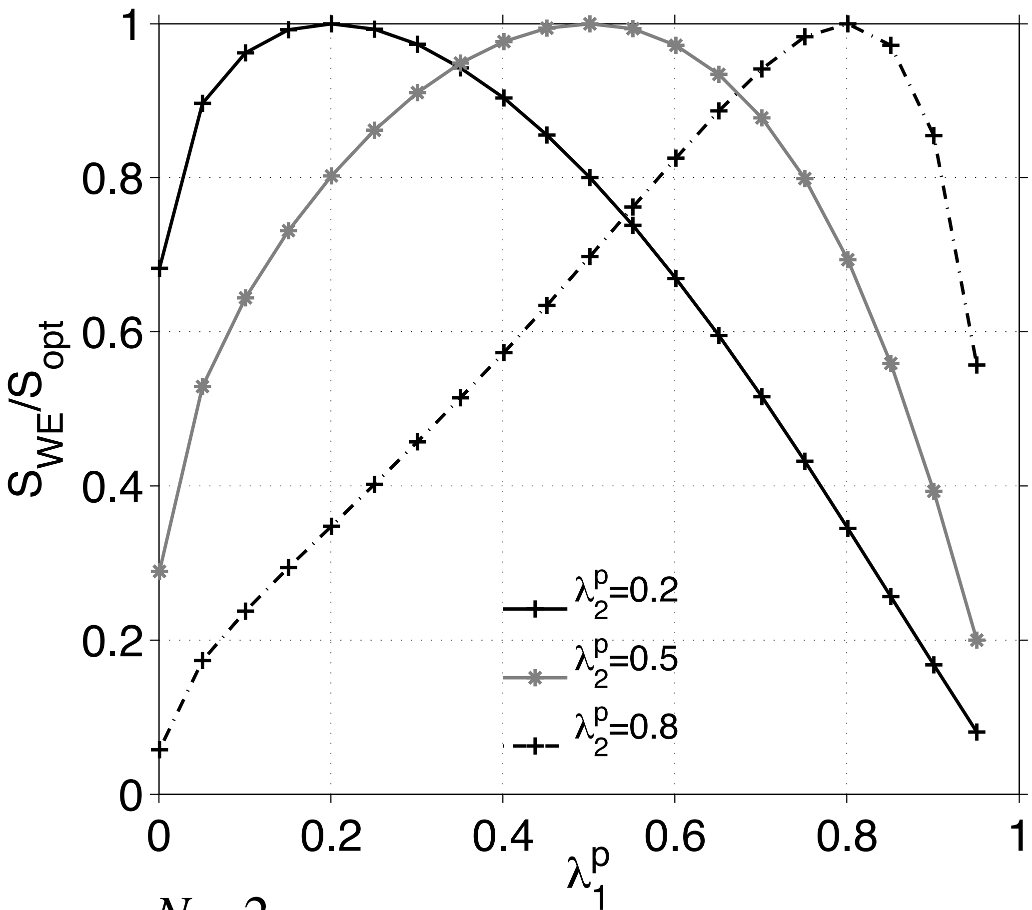
$N = 2$

$\lambda_2^p = 0.5$

$\mu = \mu^p = 1$

- Optimal Social Welfare is better than at Wardrop Equilibrium
- Optimization:
  - delay channel 1 ≠ delay channel 2
- Wardrop:
  - delay channel 1 =
  - delay channel 2 =
  - Social welfare S_{WE}

Quality of Equilibria

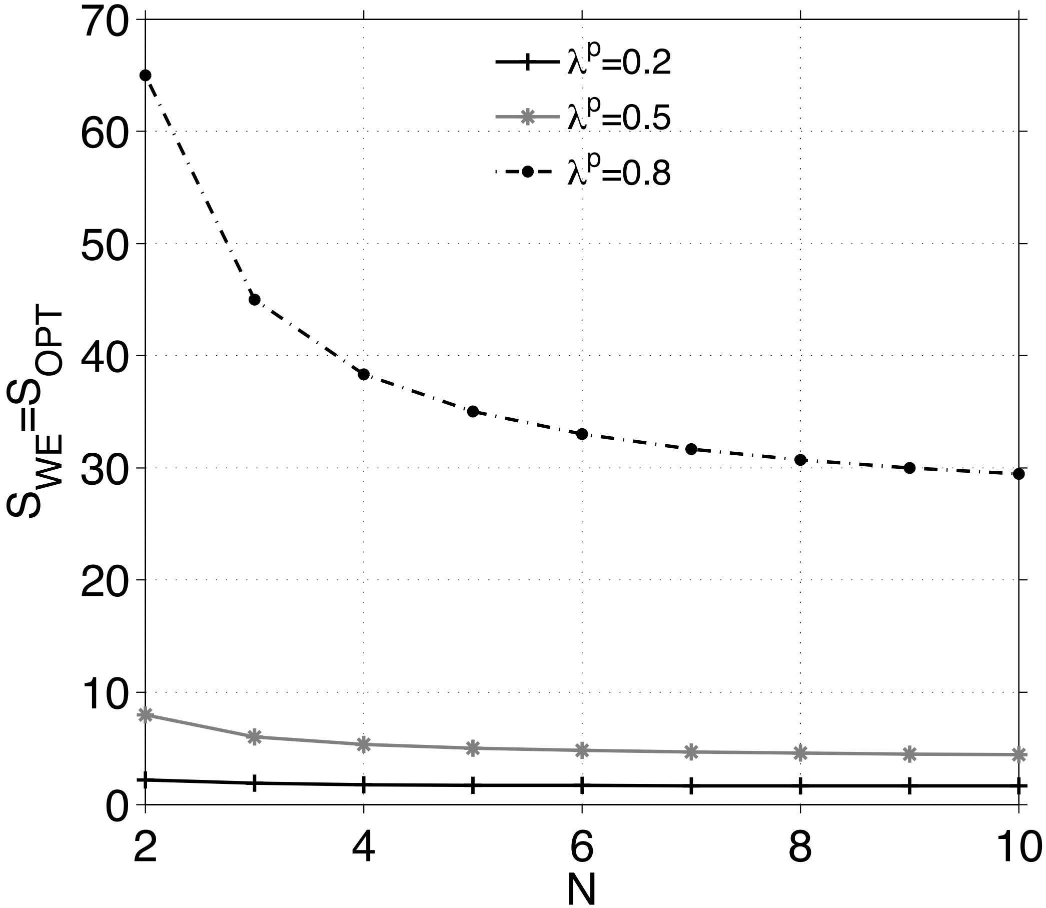


$N = 2$

$\mu = \mu^p = 1$

- Ratio between Social Welfare at Wardrop equilibrium and at the optimum
- Wardrop repartition is optimal when PU traffic is homogeneous
- Heterogeneity can severely harm efficiency of the unregulated scenario

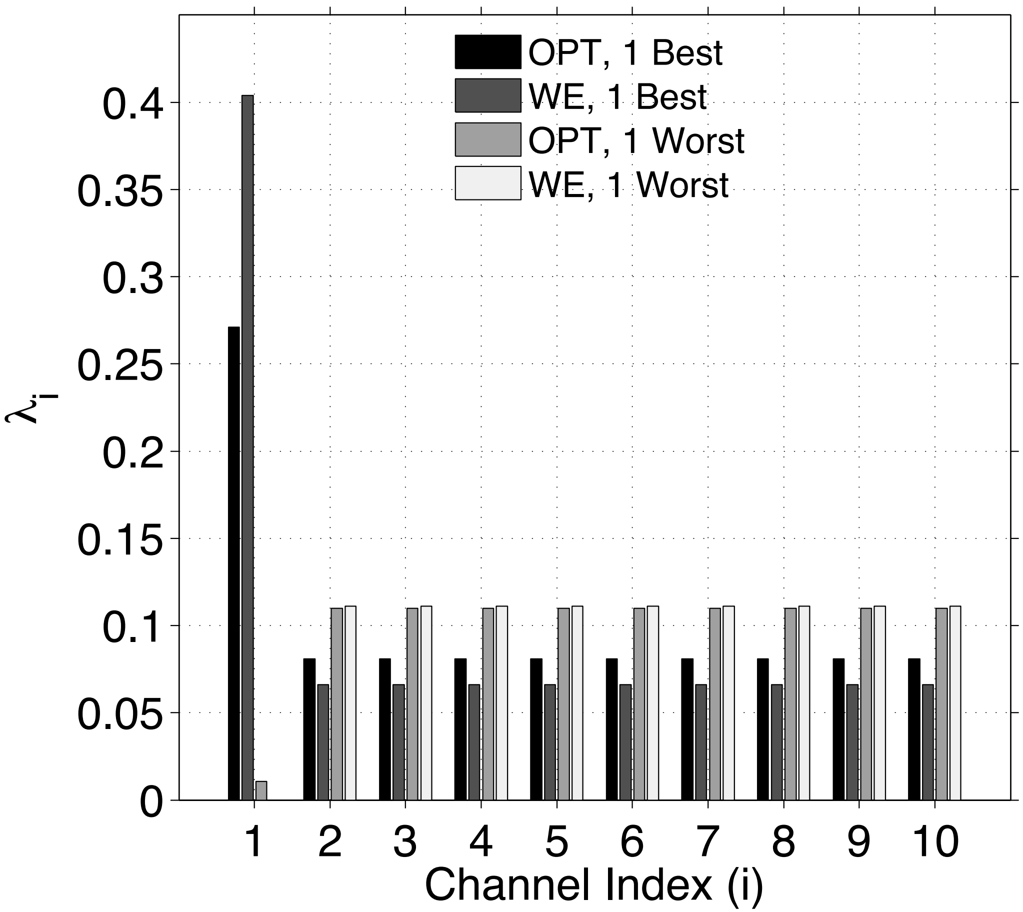
Increasing available channels



$\mu = \mu^p = 1$

- Homogeneous PU behavior
- Wardrop Equilibrium is always optimal
- Adding channels decreases SU delay, in particular when PU are aggressive

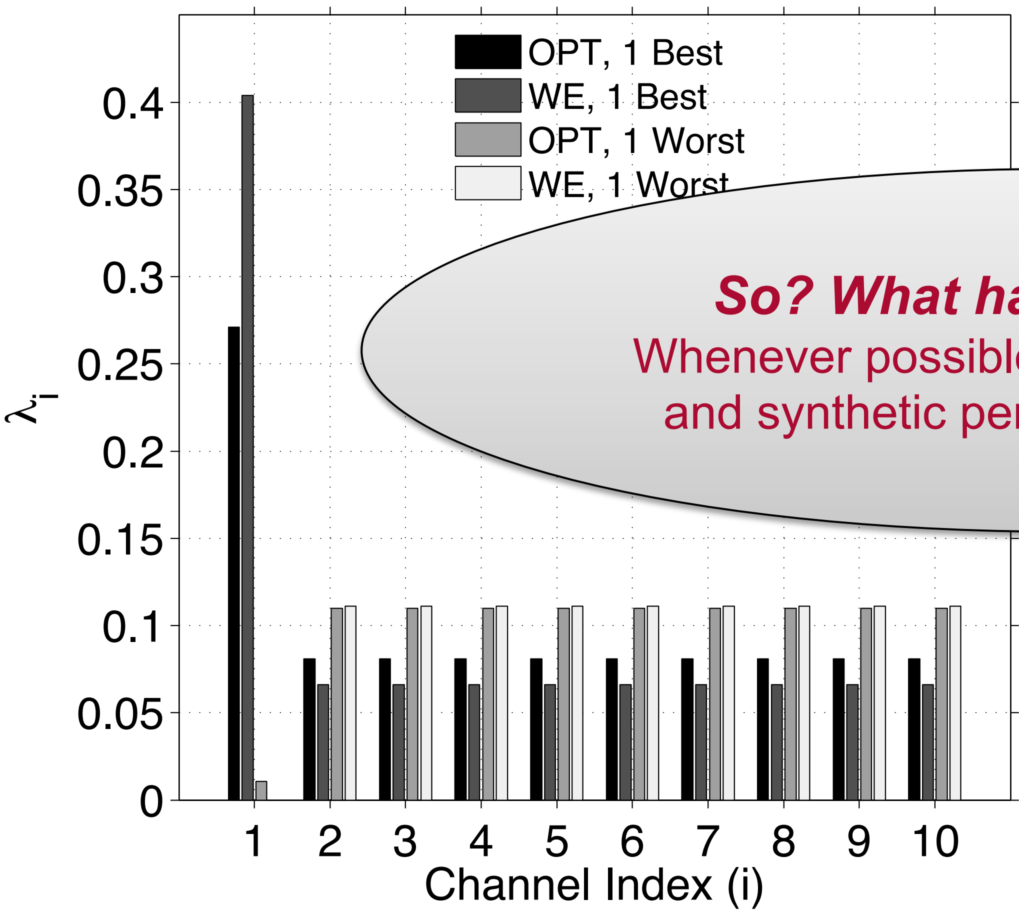
Spectrum Heterogeneity



$\mu = \mu^p = 1$

- Changing quality of 1st channel
  - 1 Best: λ^1_p = 0.4, λ^i_p = 0.5 others
  - 1 Worst: : λ^1_p = 0.6, λ^i_p = 0.5 others
- 1 Best:
  - Most of the SUs choose channel 1
- 1 Worst:
  - Channel 1 never used
- Quality: 0.96-0.98
  - Not too heterogeneous

Spectrum Heterogeneity



$\mu = \mu^p = 1$

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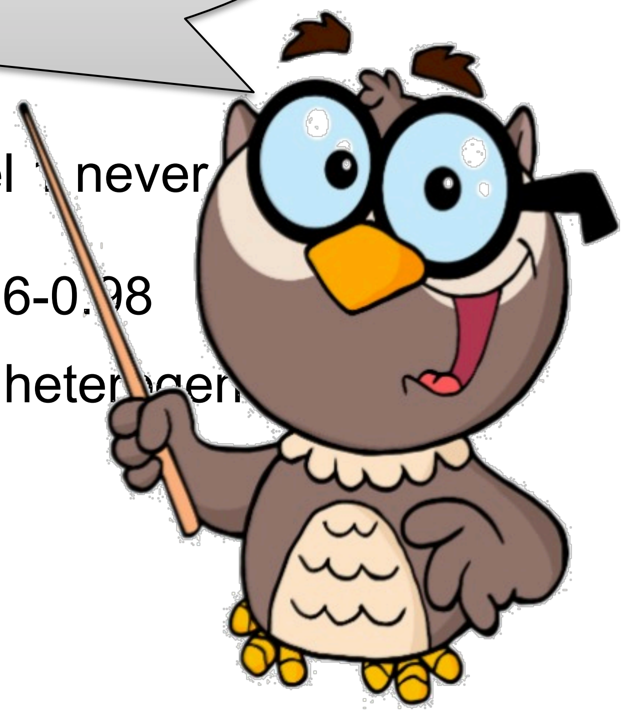
  - se channel 1
- 1 Worst:
  - Channel 1 never
- Quality: 0.96-0.98
  - Not too heterogen

So? What have we learned?

Whenever possible, try to get an intuitive and synthetic perspective of the work.



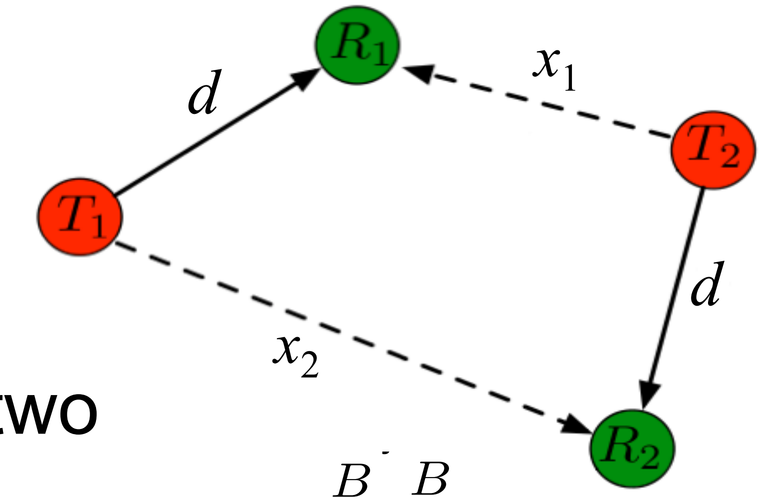
Lesson learned

- Homogeneous spectrum status
  - Anarchy leads to optimality
- Heterogeneity needs a controller
  - Unless we accept higher social costs
- Further investigation
  - Penalty/incentives to improve the quality of unregulated scenario

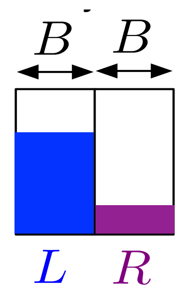
Playing with Power

Spectrum Sharing Game

- Players: Two transmitter and receiver pairs

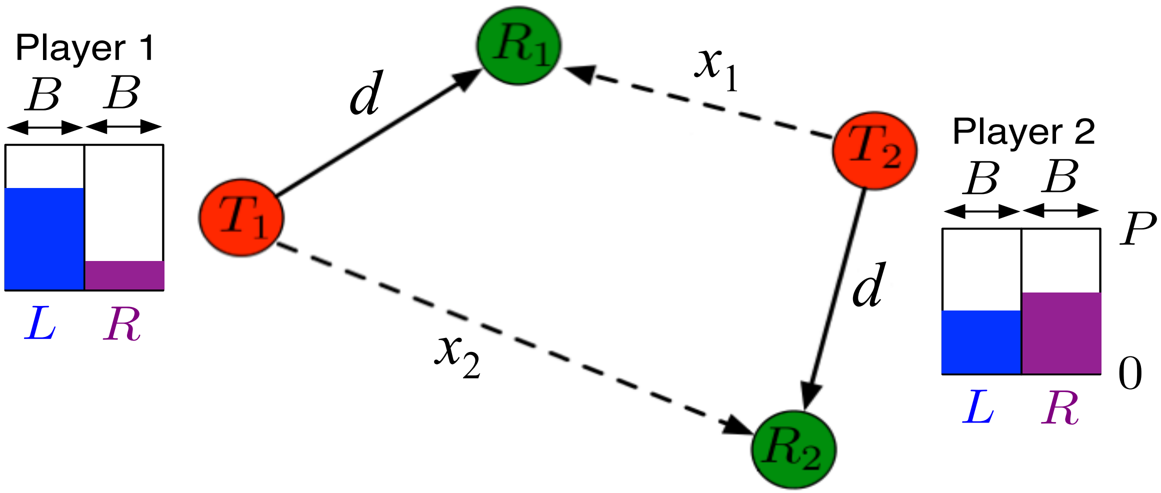


- Actions: power splits over the two bands:



- Payoffs: sum of the achievable Shannon rates $P_i \in [0, 1]$

Spectrum Sharing Problem



Optimum?

low interference → split power

high interference → different bands

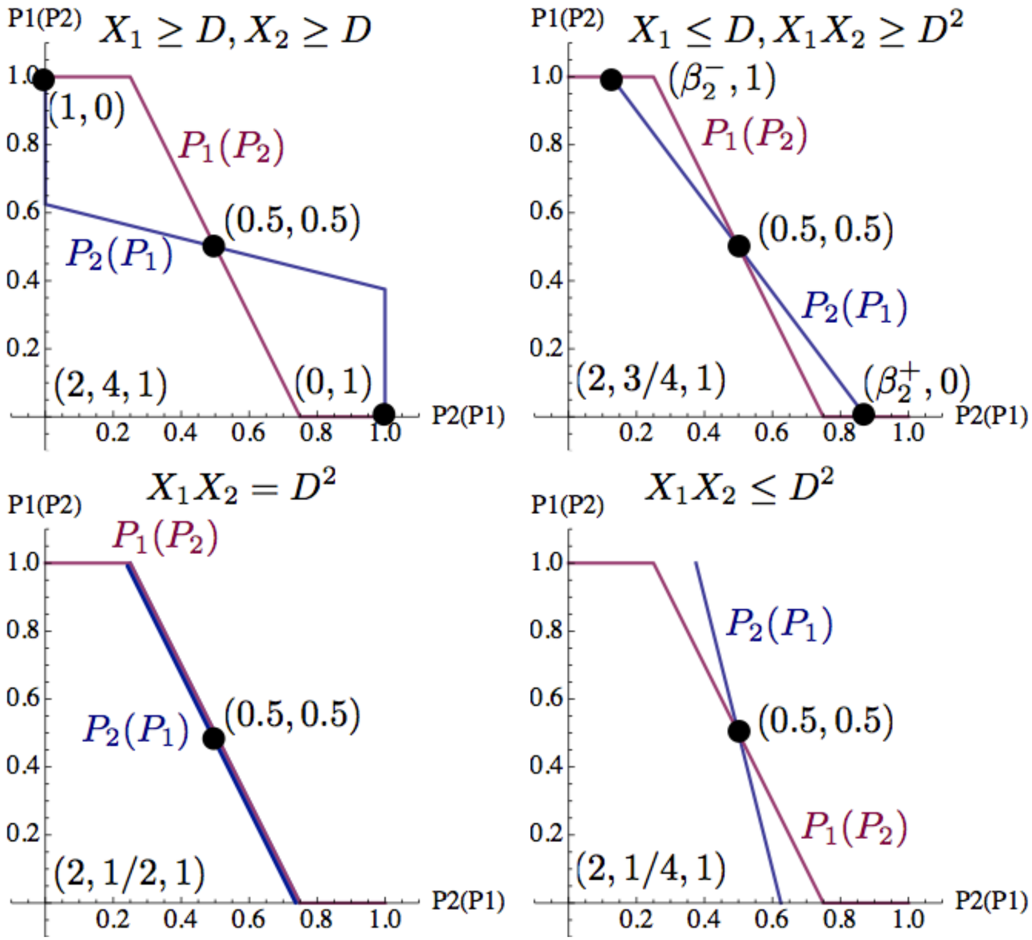
Nash Equilibrium?

$$U_i = B\log_2\left(1+\frac{P_i d^{-\alpha}}{\eta + P_j x_i^{-\alpha}}\right) + B\log_2\left(1+\frac{(P-P_i)d^{-\alpha}}{\eta+(P-P_j)x_i^{-\alpha}}\right)$$

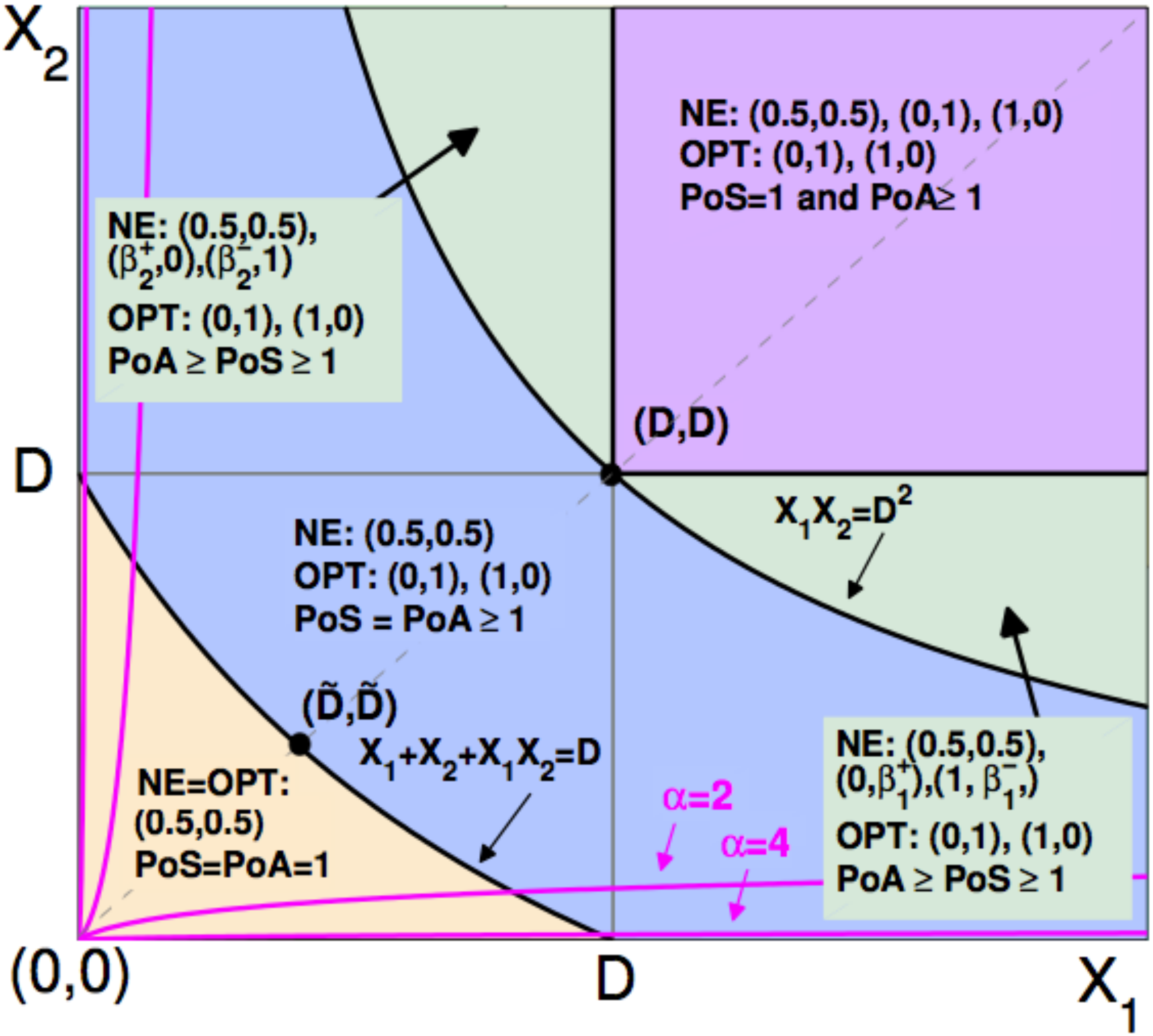
$$U_i = \log_2\left(1+\frac{P_i D}{1+P_j X_i}\right) + \log_2\left(1+\frac{\bar{P}_i D}{1+\bar{P}_j X_i}\right)$$

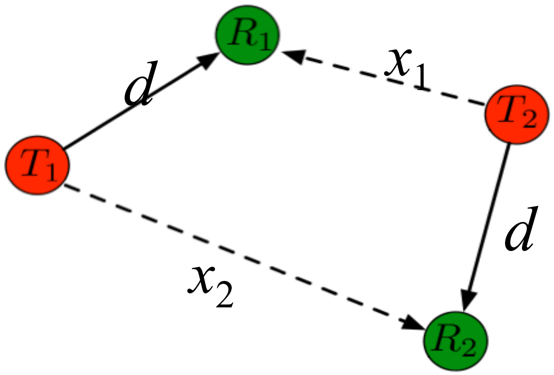
Best response and Nash Equilibria

- Best response: $P_i^*(P_j) = \left[\frac{1}{2} + \frac{X_i}{D}\left(\frac{1}{2} - P_j\right)\right]_0^1$
- Different NE according to scenario parameters



Comparison NE and Optimum





NE stability

- (0.5,0.5) is stable only if unique
- Deviation

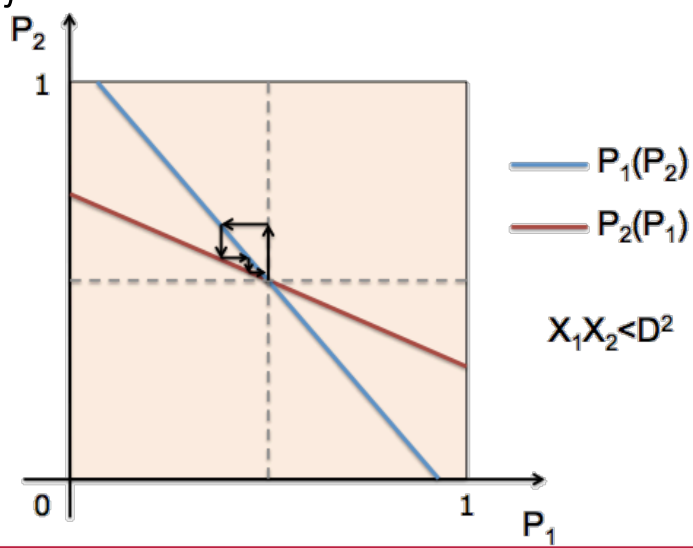
$$P_j = \frac{1}{2} + \epsilon \quad \Rightarrow \quad P_i(P_j) = \frac{1}{2} + \frac{X_i}{D}\left(\frac{1}{2} - \frac{1}{2} - \epsilon\right) = \frac{1}{2} - \frac{X_i}{D}\epsilon$$

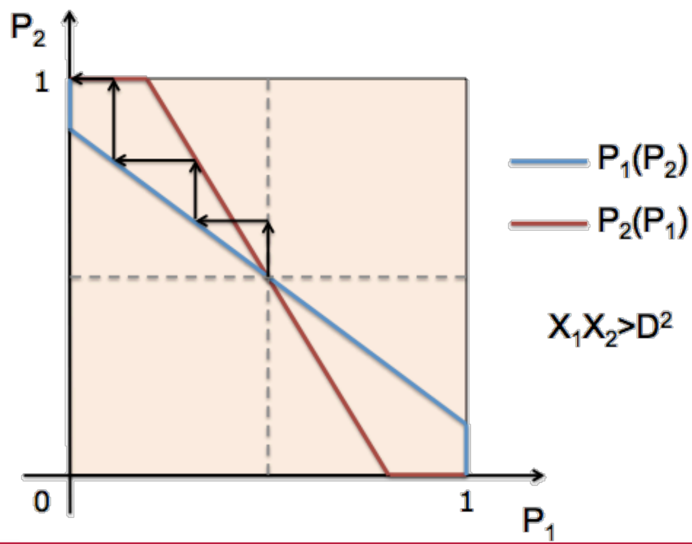
$$P_i = \frac{1}{2} - \frac{X_i}{D}\epsilon \quad \Rightarrow \quad P_j(P_i) = \frac{1}{2} + \frac{X_j}{D}\left(\frac{1}{2} - \frac{1}{2} + \frac{X_i}{D}\epsilon\right) = \frac{1}{2} + \frac{X_i X_j}{D^2}\epsilon$$

- After N moves

$$P_i = \frac{1}{2} - \frac{X_i}{D}\left(\frac{X_i X_j}{D^2}\right)^N \epsilon \qquad P_j = \frac{1}{2} + \left(\frac{X_i X_j}{D^2}\right)^N \epsilon$$

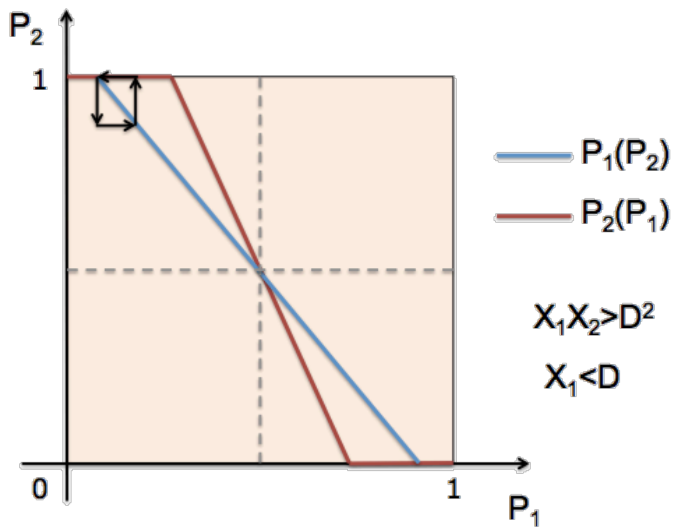
- If $X_i X_j < D^2$ we have stability in (0.5,0.5), otherwise instability

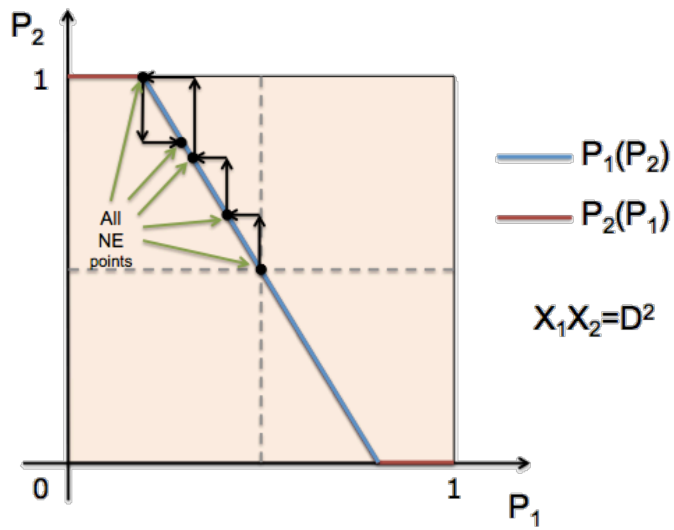




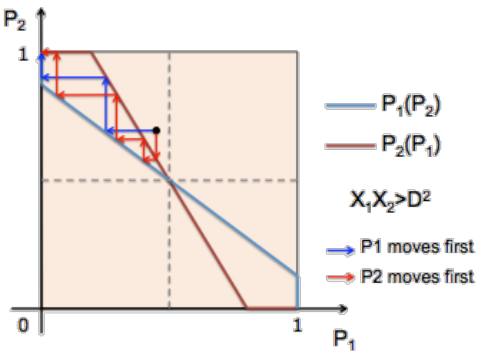
NE stability (cont'd)

- Stable (1,0) or (0,1), while if $X_1X_2=D^2$, infinite NE

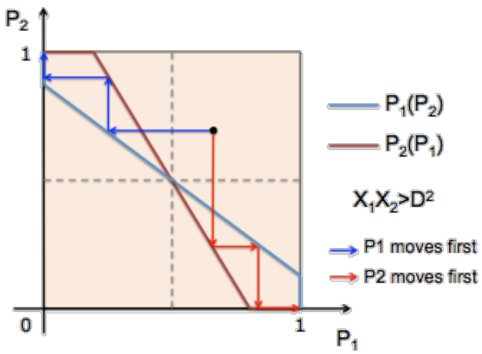




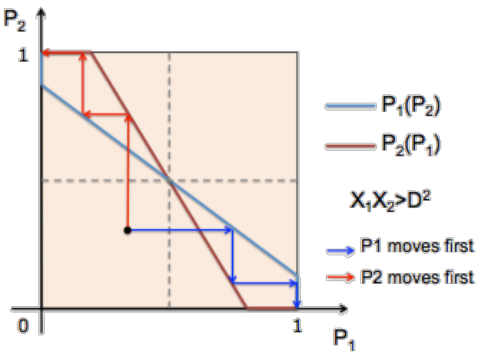
Attraction regions

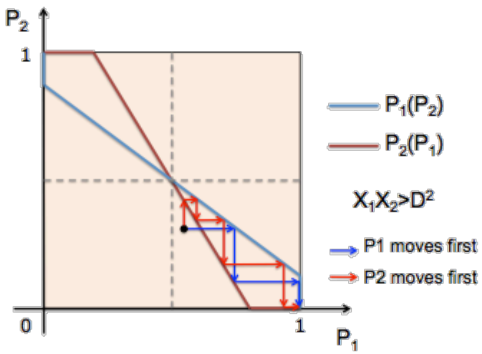


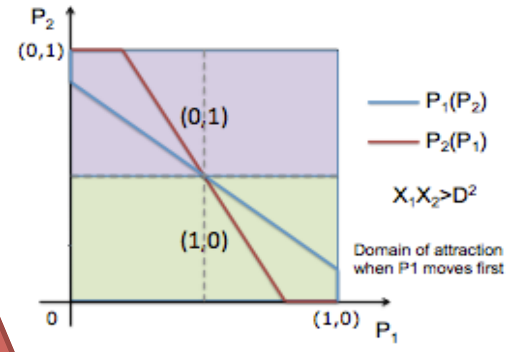
(a)



(b)







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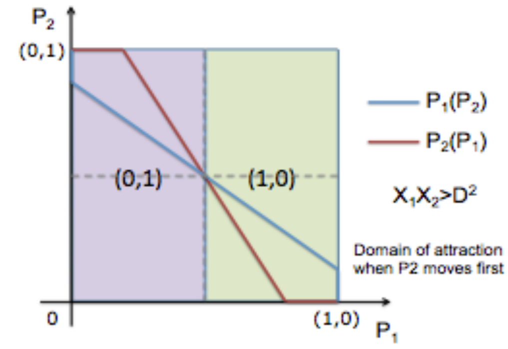
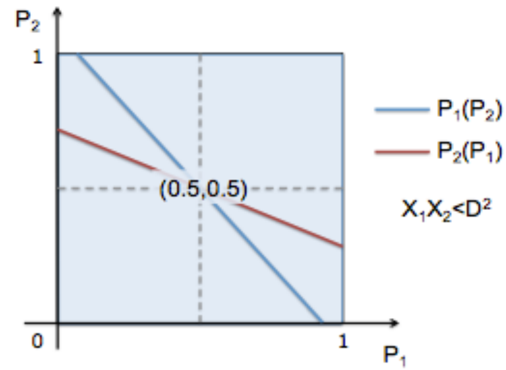


(b)



Stochastic characterization

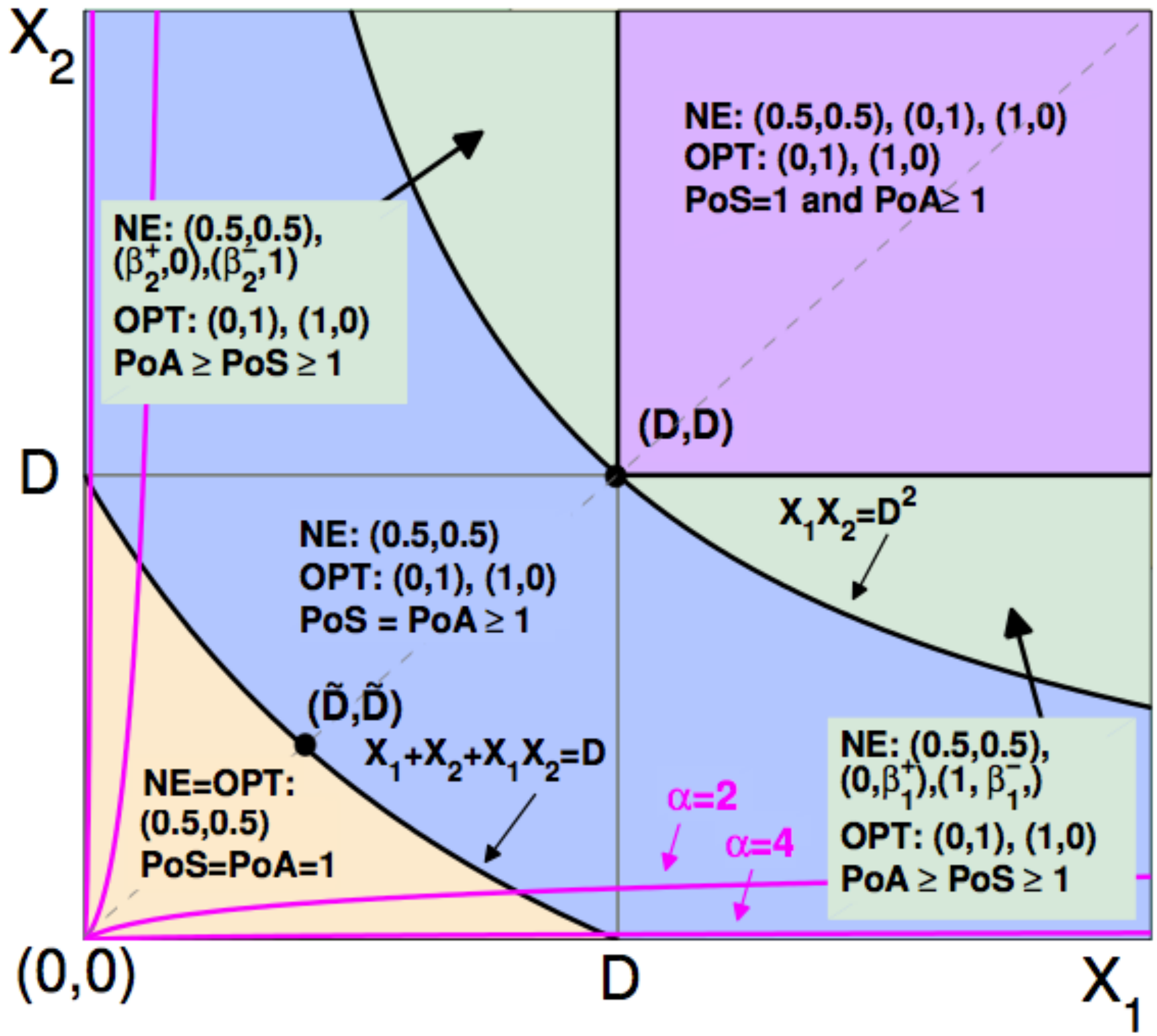
- Why?
  - Stochastic description of the game on the basis of the distances between TXs and RXs, assuming **uniform placement** of the users
- How?
  - Derive the **joint probability density function** of the distances between each transmitter/receiver pair
- Goal:
  - Provide **probability distributions** on the different regions that characterize the equilibria

Characterize the joint distribution

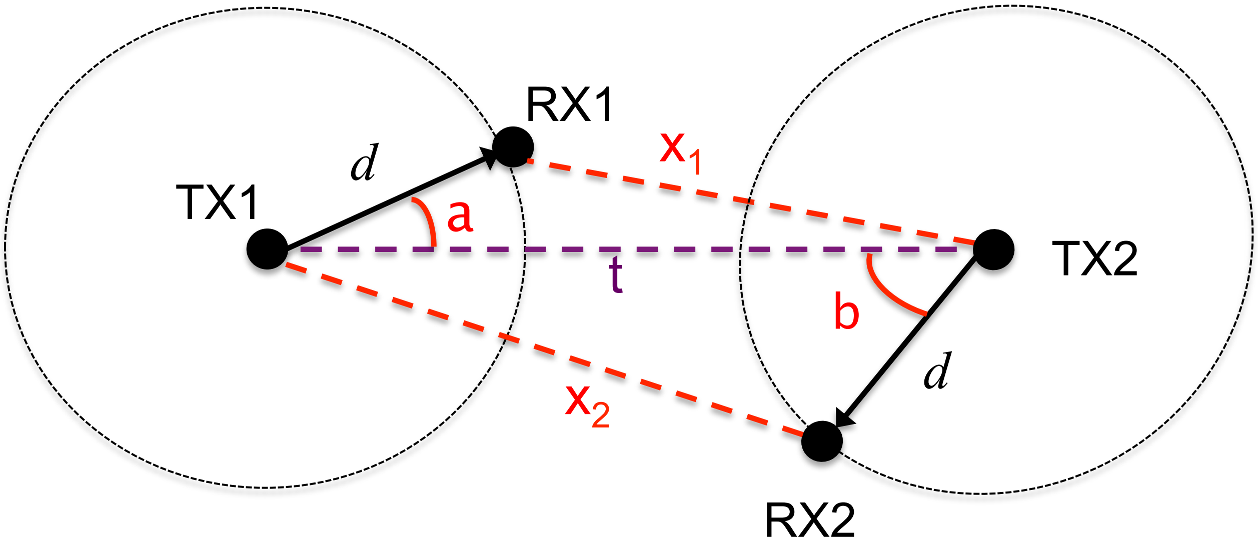
- Characterize

$$f_{x_1,x_2}(x_1, x_2)$$

- Derive the equilibria distribution for the different regions previously derived



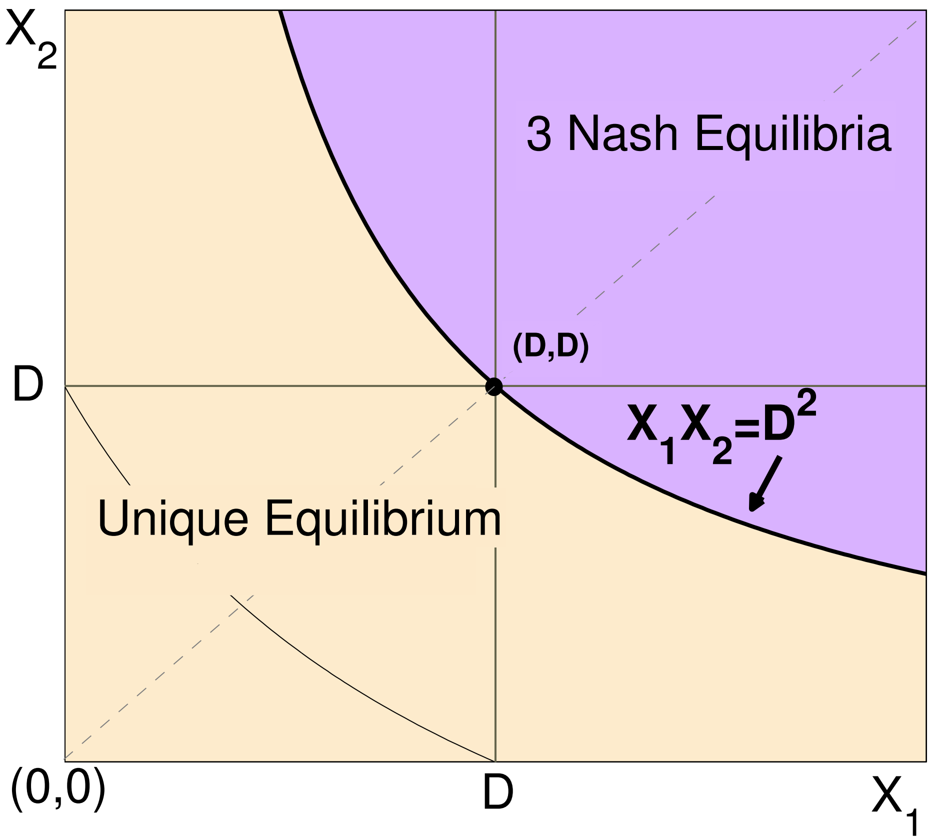
Conditional joint distribution



d is fixed

$$f_{x_1,x_2}(x_1,x_2) = \frac{2x_1x_2}{\pi^2 d^2 L^2} \int \frac{1}{t\sqrt{\left[1-\left(\frac{d^2+t^2-x_1^2}{2dt}\right)^2\right]\left[1-\left(\frac{d^2+t^2-x_2^2}{2dt}\right)^2\right]}}dt$$

Application to game theory



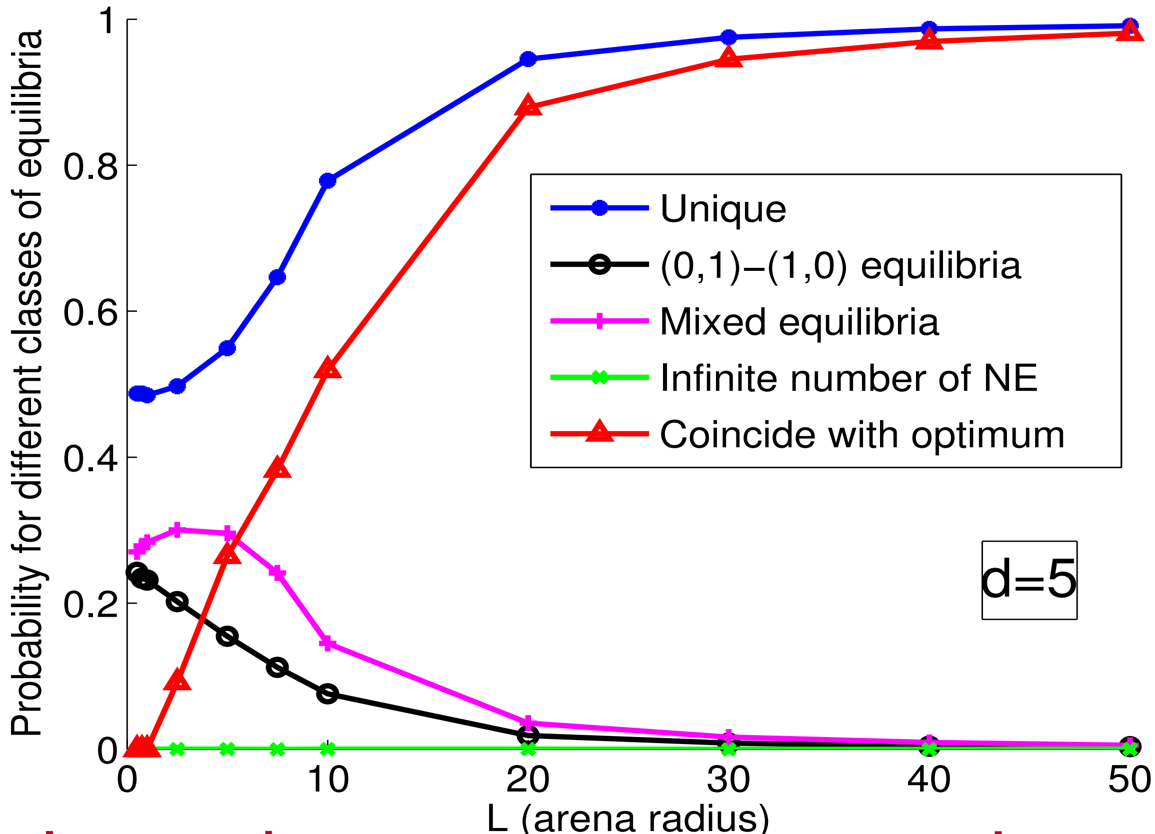
- *What is the probability that, given L, the 2-player game admits a unique equilibrium?*
- Condition in terms of pure distances (uniqueness):

$$x_1 x_2 > d^2$$

- numerical evaluation of the integral:

$$\mathbb{P}(\text{unique}) = \int_{(x_1,x_2):x_1x_2>d^2} f_{\mathsf{x}_1,\mathsf{x}_2}(x_1,x_2)dx_1dx_2$$

Numerical Results (2-player game)



Small playground

- Interference could not be negligible
- Sometimes at NE users select one channel

Large playground

- Probability of having close pairs is low
- Full spectrum utilization (0.5,0.5)

What next?

- How to extend 2-player Power Game to general case N-player Power Game?
- How to design a real protocol that implements a game without wasting transmission time?
- How to design an hybrid system with regulator and incentives to overcome NE with high social costs?

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