Survey of bankruptcy problems with non-standard features

Vito Fragnelli
Università del Piemonte Orientale
vito.fragnelli@mfn.unipmn.it

Joint work with:
Stefano Gagliardo
stefano.gagliardo@ge.imati.cnr.it
Fabio Gastaldi
fabio.gastaldi@mfn.unipmn.it

Compromesso di Lussemburgo - 1966

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Summary

Standard bankruptcy problems

Axioms

Non-standard bankruptcy problems
Standard bankruptcy problems

A bankruptcy problems $BP$ arises when an agent has several (monetary) debts with other agents and her/his (monetary) availability is not enough for cover all of them. The creditors have the same rights on the available estate.

Similar situations are the allocation of a scarce resource, or the collection of taxes.

Two very good surveys may be found in Thomson (2003 and 2015)
Formally

A $BP$ is a triple

$$\mathcal{B} = (N, E, c)$$

where $N = \{1, \ldots, n\}$ is the set of claimants, $E \in \mathbb{R}_\geq$ is the estate and $c = (c_1, \ldots, c_n) \in \mathbb{R}_\geq^n$ is the vector of claims, with $E \leq \sum_{i \in N} c_i = C$


A solution is a vector $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$ s.t.

$$0 \leq x_i \leq c_i, i \in N \quad \text{(rationality)}$$

$$\sum_{i \in N} x_i = E \quad \text{(efficiency)}$$

A rule is a map $f$ that associates to each $BP$ a solution
Basic rules

Proportional

\[ PROP(N, E, c)_i = c_i \frac{E}{C}, \quad i \in N \]

Constrained Equal Awards

\[ CEA(N, E, c)_i = \min\{c_i, \alpha\}, \quad i \in N \]

where \( \alpha \in \mathbb{R}_+ \) is s.t. \( \sum_{i \in N} CEA(N, E, c)_i = E \)

Constrained Equal Losses

\[ CEL(N, E, c)_i = \max\{c_i - \beta, 0\}, \quad i \in N \]

where \( \beta \in \mathbb{R}_+ \) is s.t. \( \sum_{i \in N} CEL(N, E, c)_i = E \)

Talmud

\[ TAL(N, E, c)_i = \begin{cases} 
CEA(N, c/2, E)_i & \text{if } E \leq C/2 \\
\frac{c_i}{2} + CEL(N, c/2, E - C/2)_i & \text{if } E > C/2
\end{cases} \]

Herrero, Villar (2001)
Axioms

How to select the “best” solution?
Each agent prefers the solution that allows to obtain the largest amount

GOOD PROPERTIES $\rightarrow$ AXIOMATIC CHARACTERIZATIONS

In the literature there exist more than 100 properties, perhaps more than 200

- Strong properties: Order preservation, Equal treatment of equals, Monotonicity, etc
- Context properties: Claim truncation, Merging, Splitting, etc
- Ad hoc properties: Exemption, Full compensation, etc

Claim truncation $\rightarrow$ game theoretical rules (Curiel, Pederzoli, Tijs, 1987)
Non-standard bankruptcy problems

MORE DATA
Each agent is represented by a unique datum, the claim

- Weights. A non-negative real number is associated to each agent, and this influence the rules
  Casas-Méndez, Fragnelli, García-Jurado (2011)

- Minimal rights and bounds. It is possible to introduce the minimal right of an agent as the amount of the estate nobody else claims, or bounds on the amount that has to be assigned to the agent (What happens when the sum of the lower bounds is strictly larger than the estate?)

- Multiclaims. Each agent may have more than one claim for the unique estate (How to aggregate the the claims of each agent? Conversely, how to split the estate?)
  Calleja, Borm, Hendrickx (2005) and Hinojosa, Mármol, Sánchez (2012)
DIFFERENT RIGHTS

- Types (Young, 1998)
- Priority (Moulin, 2000)
REPRESENTATION

- Communicating vessels (Kaminski, 2000)
  Extremely easy to adapt to different situations, thanks to shapes, sections, heights, levels
• Network

  – Minimum cost flow problem (Branzei, Ferrari, Fragnelli, Tijs, 2006)

  ![Diagram of minimum cost flow problem](image)

  – Flow problem (Bjørndal, Jörnsten, 2010)

  – In a multiple $BP$, the network describes the connections with each source

    Ilkılıç, Kayı (2014) and Moulin, Sethuraman (2013)
DIFFERENT UTILITY FUNCTIONS
Carpente, Casas, Gozalvez, Llorca, Pulido, Sanchez-Soriano (2008)

A PRIORI UNIONS
Borm, Carpente, Casas-Méndez, Hendrickx (2005)

SURPLUS
Moulin (1987) and Herrero, Maschler, Villar (1999)

INTEGER ESTATE
Wait some minutes ...
References

Hinojosa MA, Már mol AM, Sánchez F (2012) A consistent Talmud rule for division problems with multiple references. TOP 20, 661-678
Thanks!