Gam theoretic techniques for mechanism design

Gianluigi Greco

Social Choice Theory

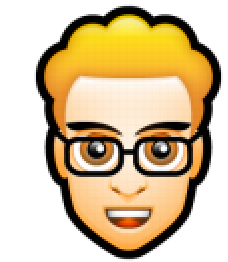
Rule for breaking ties: A > B > C

Alternatives

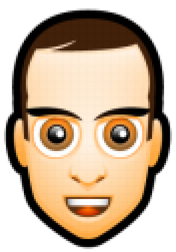
- { A, B, C }

***Social Choice Function*:**

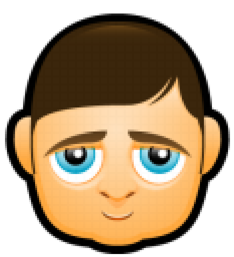
- Compute the alternative that is top-ranked by the majority



A > C > B



B > A > C



C > B > A

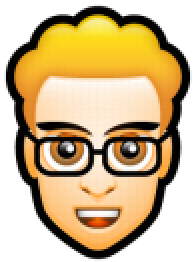
A

Social Choice Theory → Mechanism Design

Rule for breaking ties: A > B > C

Alternatives
- { A, B, C }

***Social Choice Function**:*
- Compute the alternative that is top-ranked by the majority



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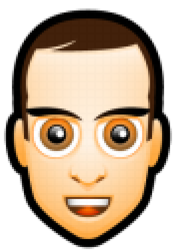
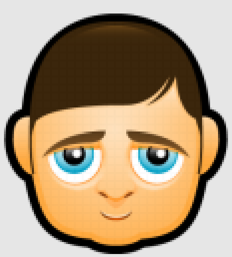


B > A > C



~~C~~ B > B > A

~~A~~ B

Strategic issues!

Mechanism Design

- Social Choice Theory is *non-strategic*
- In practice, agents **declare** their preferences
  - They are self interested
  - They might not reveal their true preferences
- We want to find optimal outcomes w.r.t. true preferences
- Optimizing w.r.t. the declared preferences might not achieve the goal

How to build a mechanism where agents find convenient to report their true preferences?

Outline

Game Theory

Mechanism Design

Mechanisms with Verification

Mechanisms and Allocation Problems

Complexity Analysis

Basic Concepts (1/2)

- Each agent i is associated with a **type** $\theta_i \in \Theta_i$

private knowledge, preferences, …

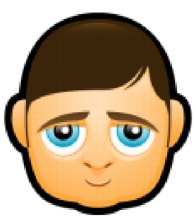
C > B > A

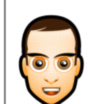
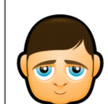
Basic Concepts (2/2)

- Consider the vector of the **joint strategies** $s = (s_1, \ldots, s_I)$

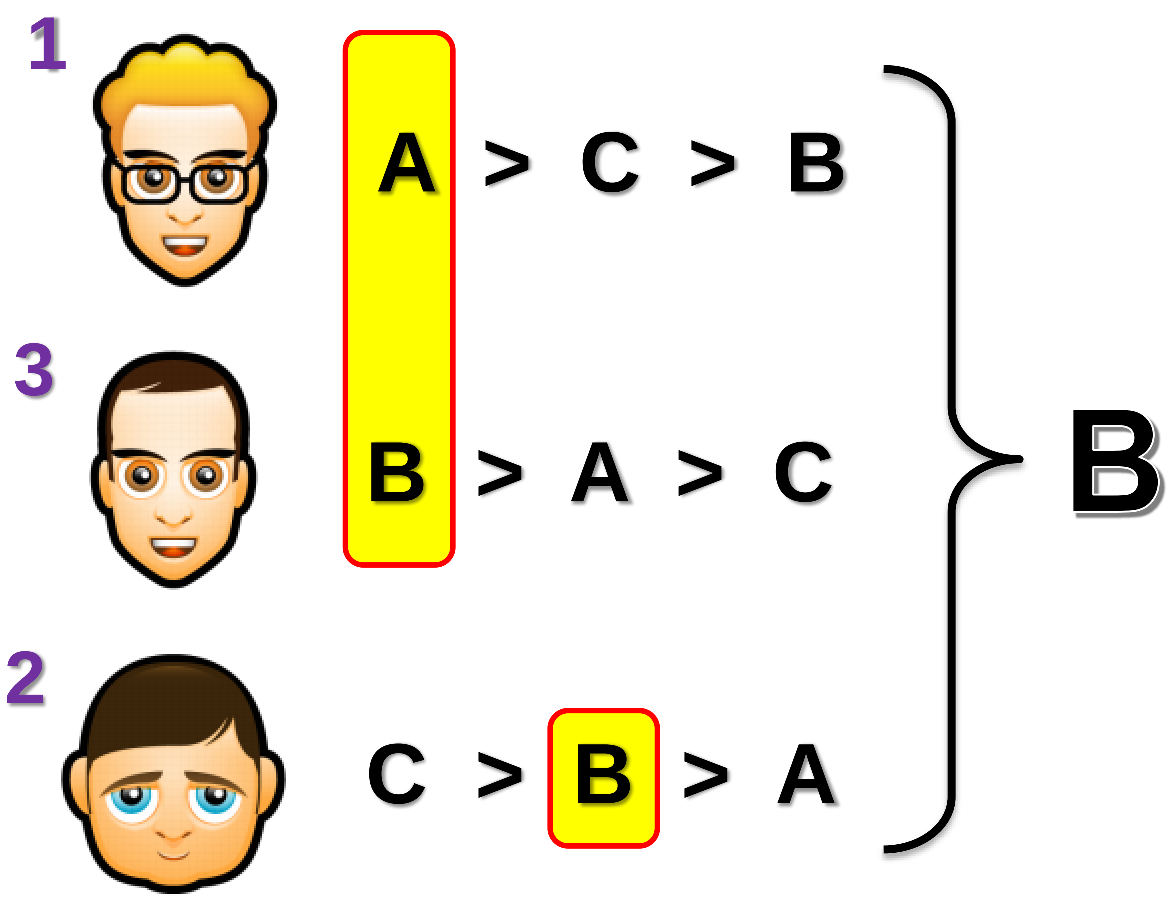
Game Theory (by Example)

- Consider the utility function of agent  $\left(C_3 > B_2 > A_1 \right)$
- Let us reason on the case where
  -  selects **A**
  -  selects **B**

 will select **B**

			Outcome	Utility
A	B	A	➡ A	➡ 1
A	B	B	➡ B	➡ 2
A	B	C	➡ A	➡ 1

Game Theory (by Example)



No agents can benefit by deviating!

Solution Concepts

- A **Nash equilibrium** is a strategy profile $s = (s_1, \ldots, s_I)$ such that, for every agent i and for every $s_i' \neq s_i$,

$$u_i(s_i, s_{-i}, \theta_i) \geq u_i(s_i', s_{-i}, \theta_i)$$

The strategies of the other agents are fixed…

Solution Concepts

- A **Nash equilbrium** is a strategy profile $s = (s_1, \ldots, s_I)$ such that, for every agent i and for every $s_i' \neq s_i$,

$$u_i(s_i, s_{-i}, \theta_i) \geq u_i(s_i', s_{-i}, \theta_i)$$

Bob	John goes *out*	John stays at *home*
out	2	0
home	0	1

John	Bob goes *out*	Bob stays at *home*
out	1	1
home	0	0

A Closer Look

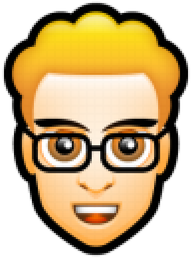
- To play a Nash equilibrium,
  - every agent must have perfect information
  - rationality is common knowledge
  - all agents must select the same Nash equilibrium

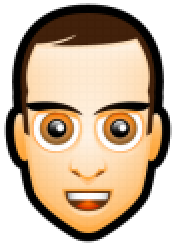
Bob	John goes *out*	John stays at *home*
out	2	0
home	0	1

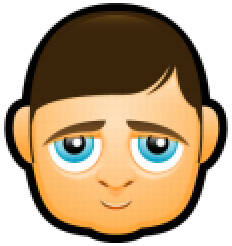
Dominant strategy

John	Bob goes *out*	Bob stays at *home*
out	1	1
home	0	0

Dominant Strategies (by Example)

 A > C > B

 B > A > C

 C > B > A

For , A is a dominant strategy. Why?

Solution Concepts

- A **Nash equilibrium** is a strategy profile $s = (s_1, \ldots, s_I)$ such that, for every agent i and for every $s_i' \neq s_i$,

$$u_i(s_i, s_{-i}, \theta_i) \geq u_i(s_i', s_{-i}, \theta_i)$$

- A strategy s_i is **dominant** for agent i, if for every $s_i' \neq s_i$ and for every s_{-i},

$$u_i(s_i, s_{-i}, \theta_i) \geq u_i(s_i', s_{-i}, \theta_i)$$

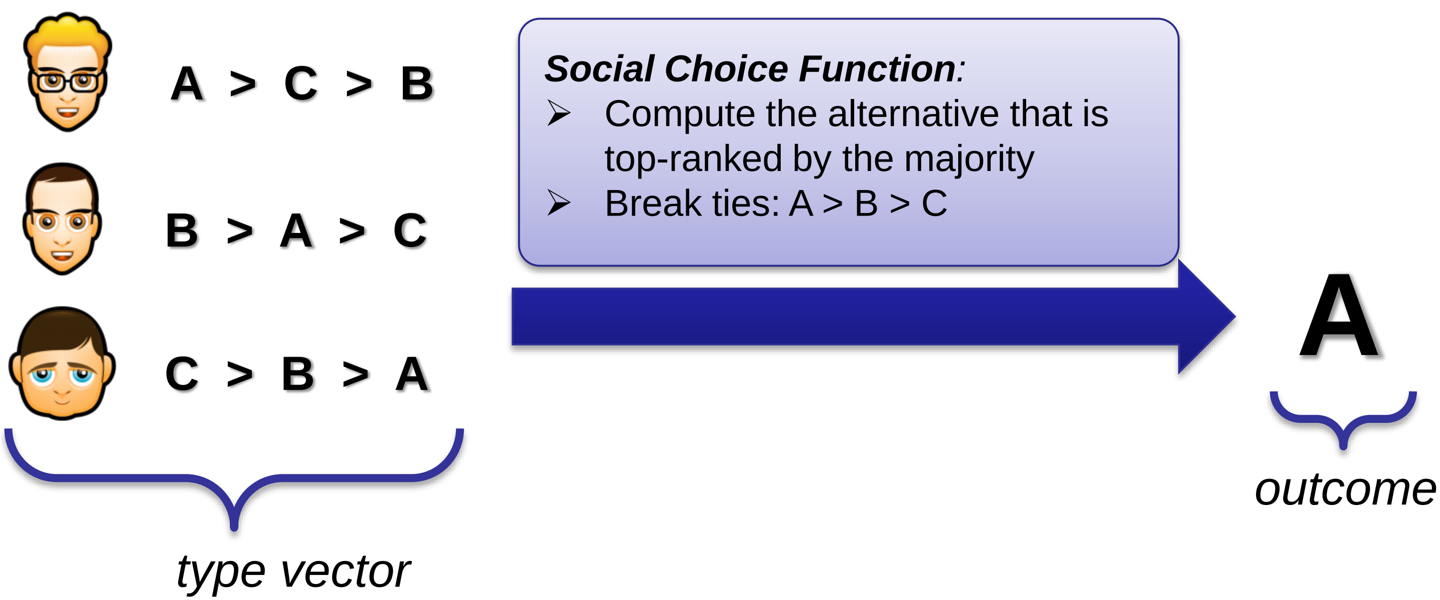
Independently on the other agents…

Outline

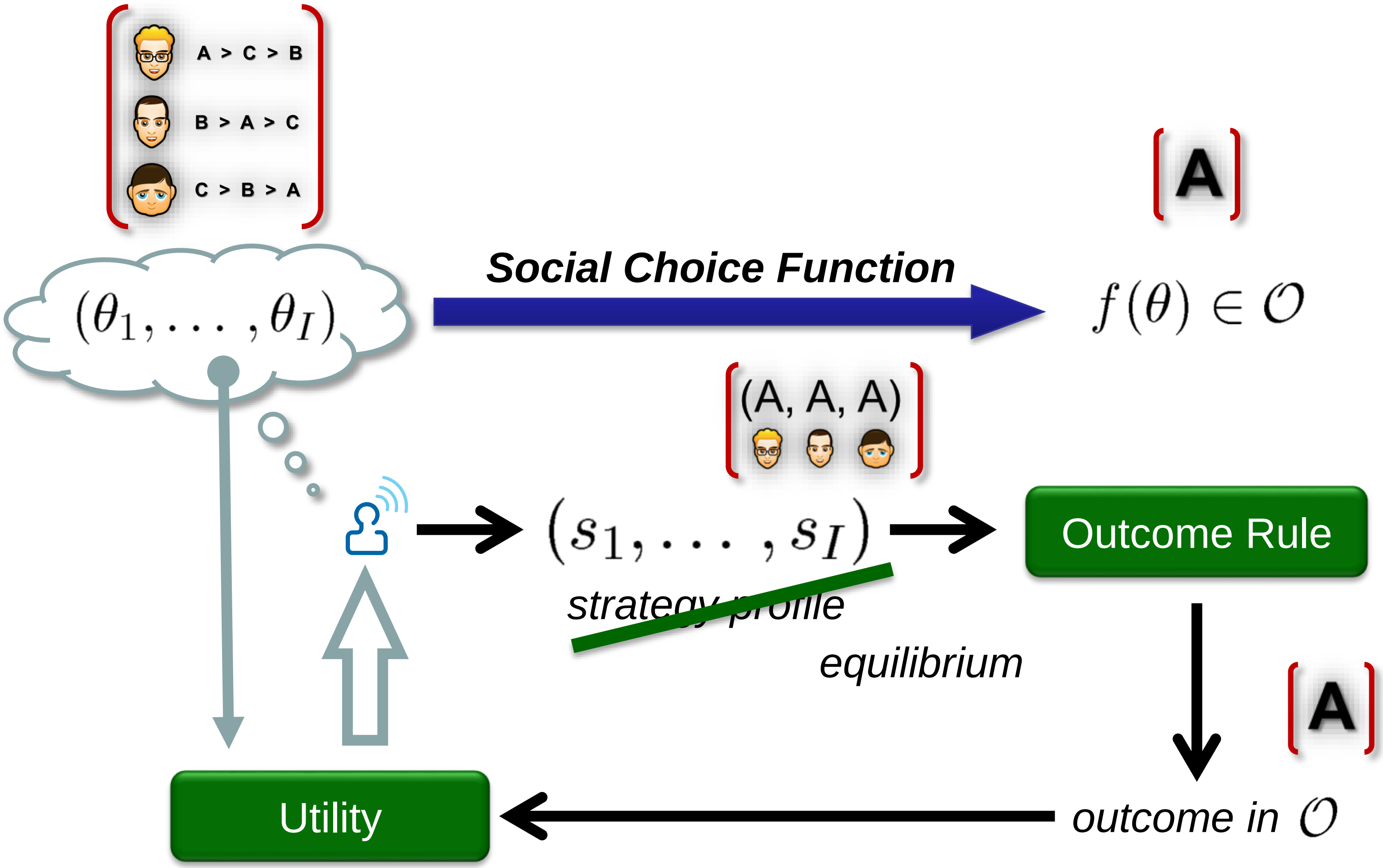
- Game Theory
- Mechanism Design
- Mechanisms with Verification
- Mechanisms and Allocation Problems
- Complexity Analysis

Social Choice Functions

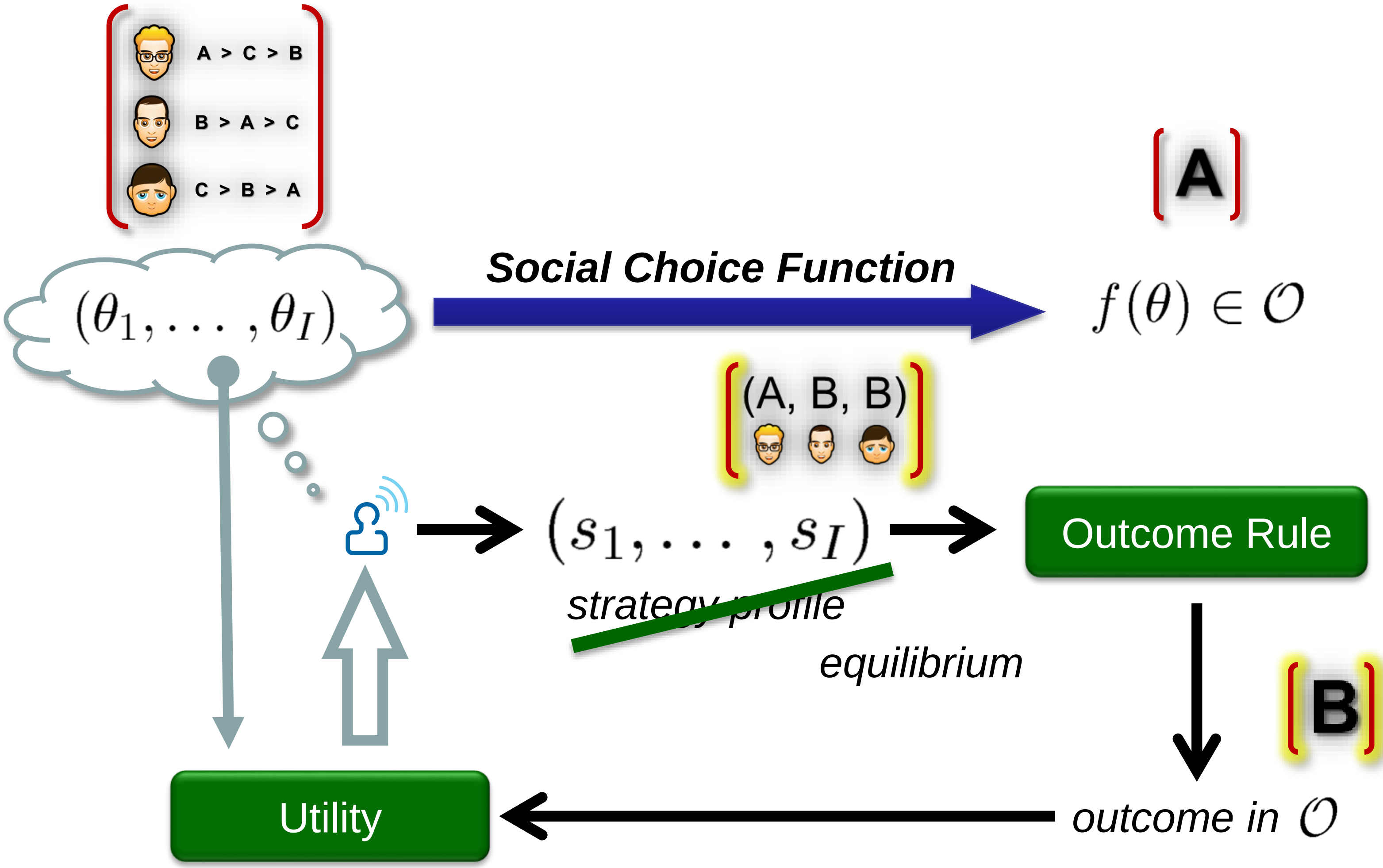
- A **social choice function** $f\ :\Theta_1 \times \ldots \times \Theta_I\ \rightarrow \mathcal{O}$
  - given a type vector $\theta = (\theta_1, \ldots, \theta_I)$
  - selects an outcome $f(\theta) \in \mathcal{O}$



Mechanism Design



Mechanism Design



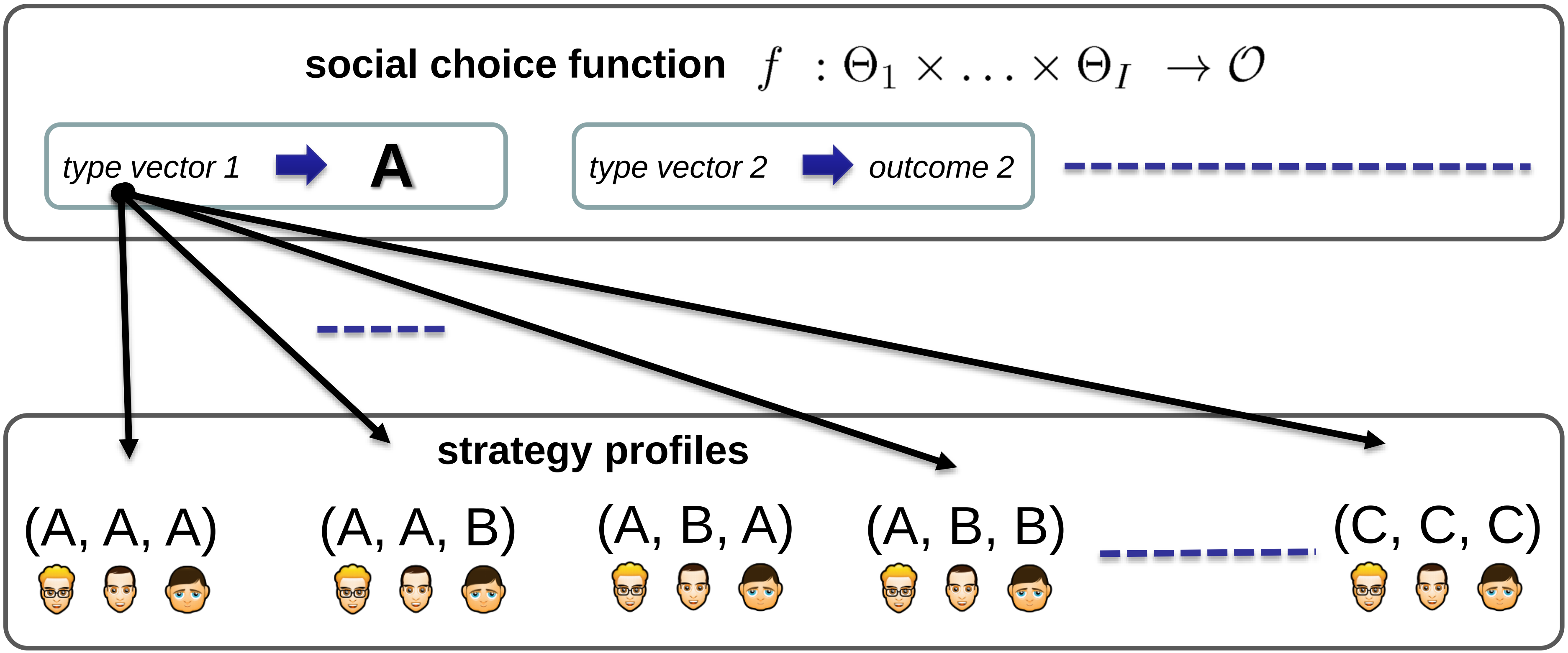
Mechanism and Implementation

social choice function $f : \Theta_1 \times \ldots \times \Theta_I \rightarrow \mathcal{O}$

type vector 1 → *outcome 1*

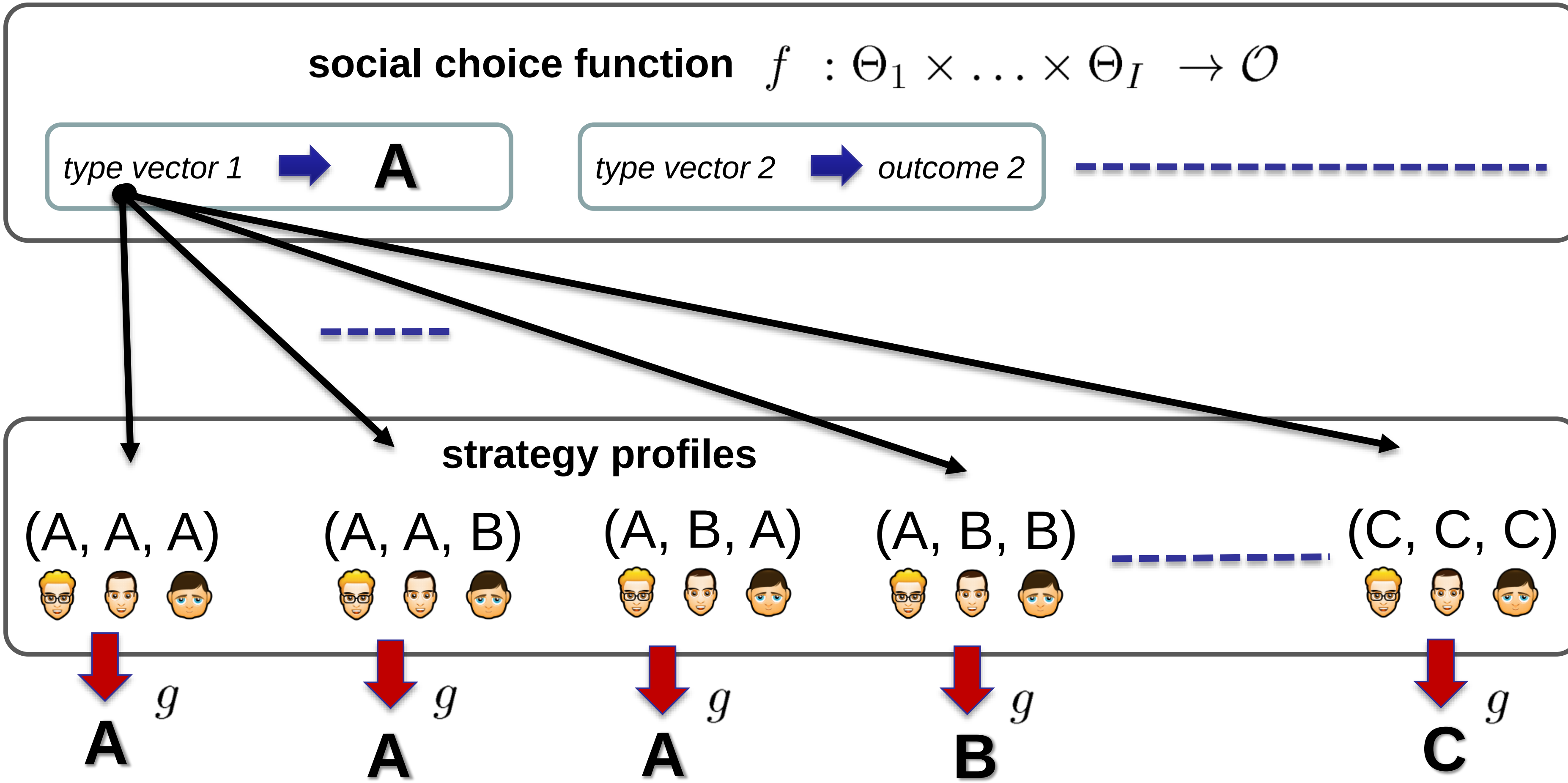
type vector 2 → *outcome 2*

Mechanism and Implementation



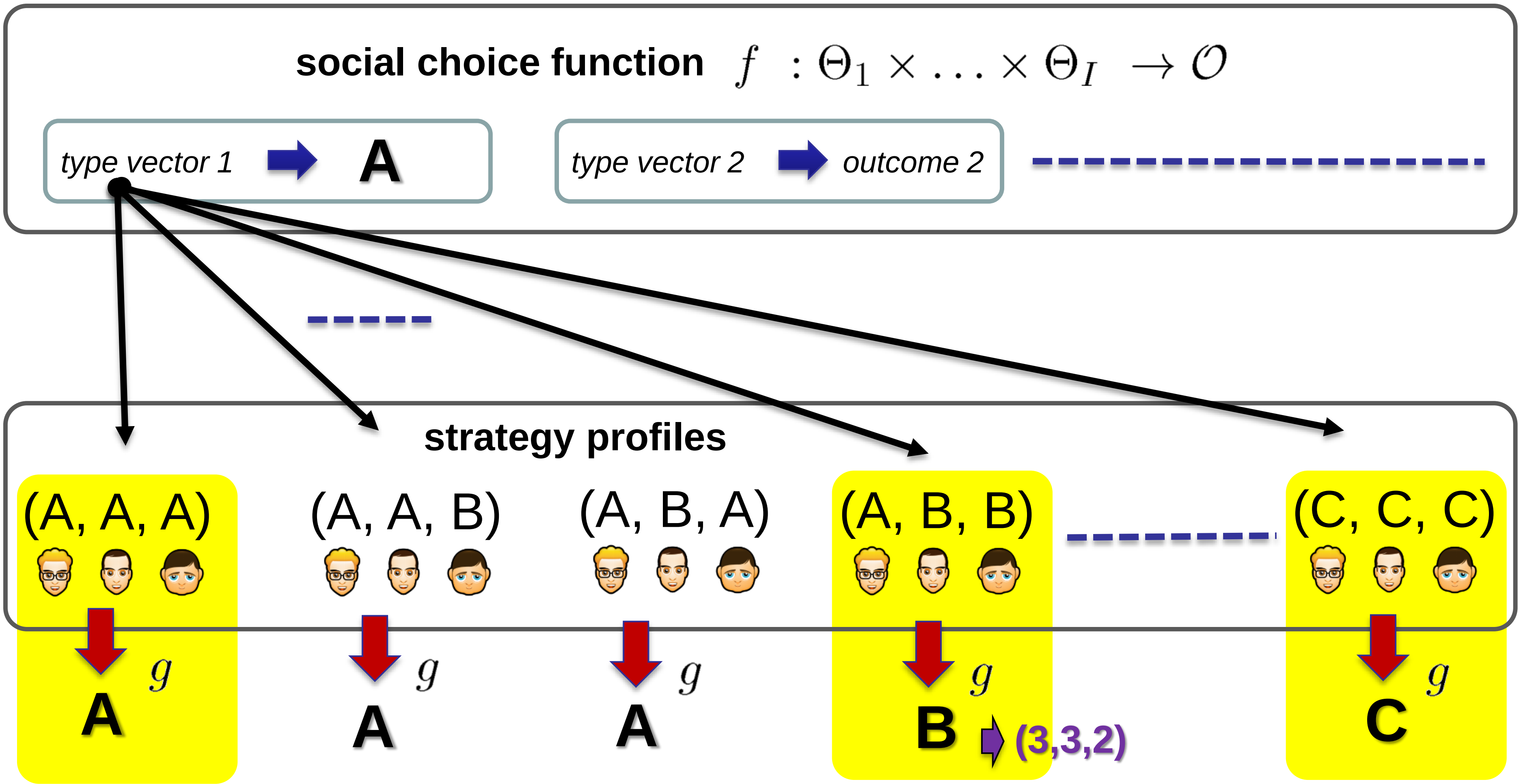
- *For a given type vector, all startegy profiles are in principle admissible*

Mechanism and Implementation



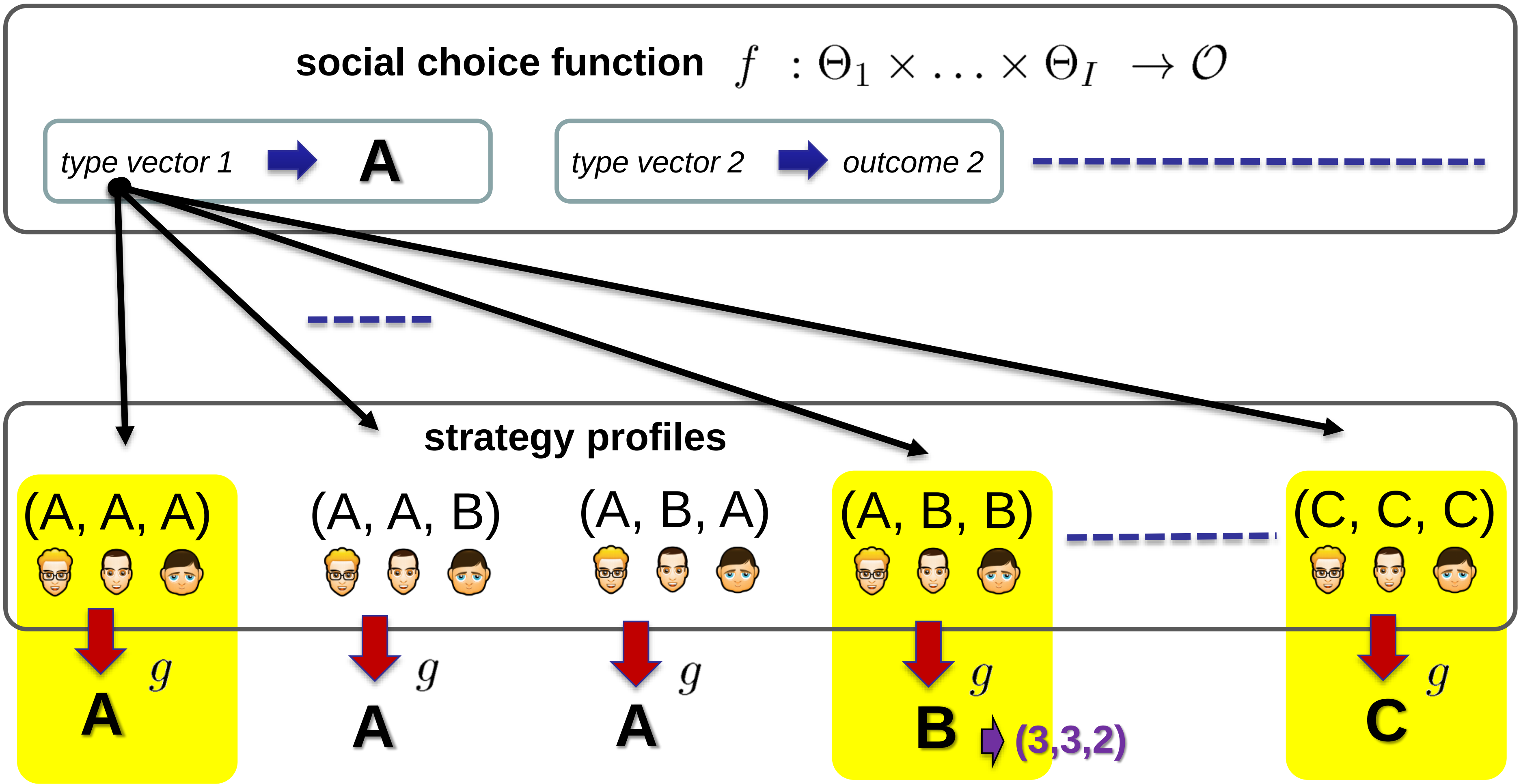
- *For a given type vector, all startegy profiles are in principle admissible*
- *An outcome rule is applied*

Mechanism and Implementation



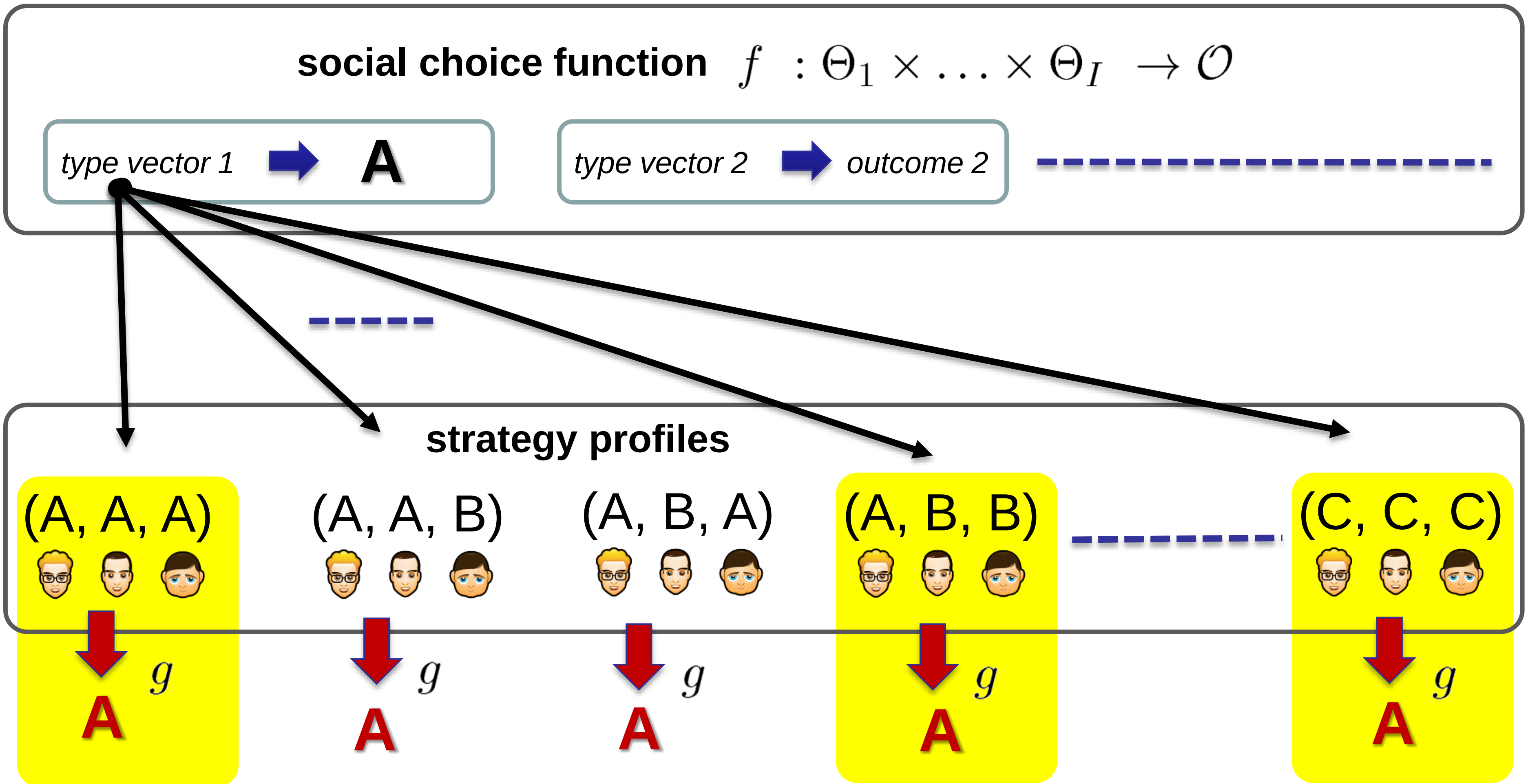
- *For a given type vector, all startegy profiles are in principle admissible*
- *An outcome rule is applied*
- *So, utilities can be computed and equilibria can be selected*

Mechanism and Implementation



GOAL: In all equlibria, the rule must select the outcome of the social choice function

Mechanism and Implementation



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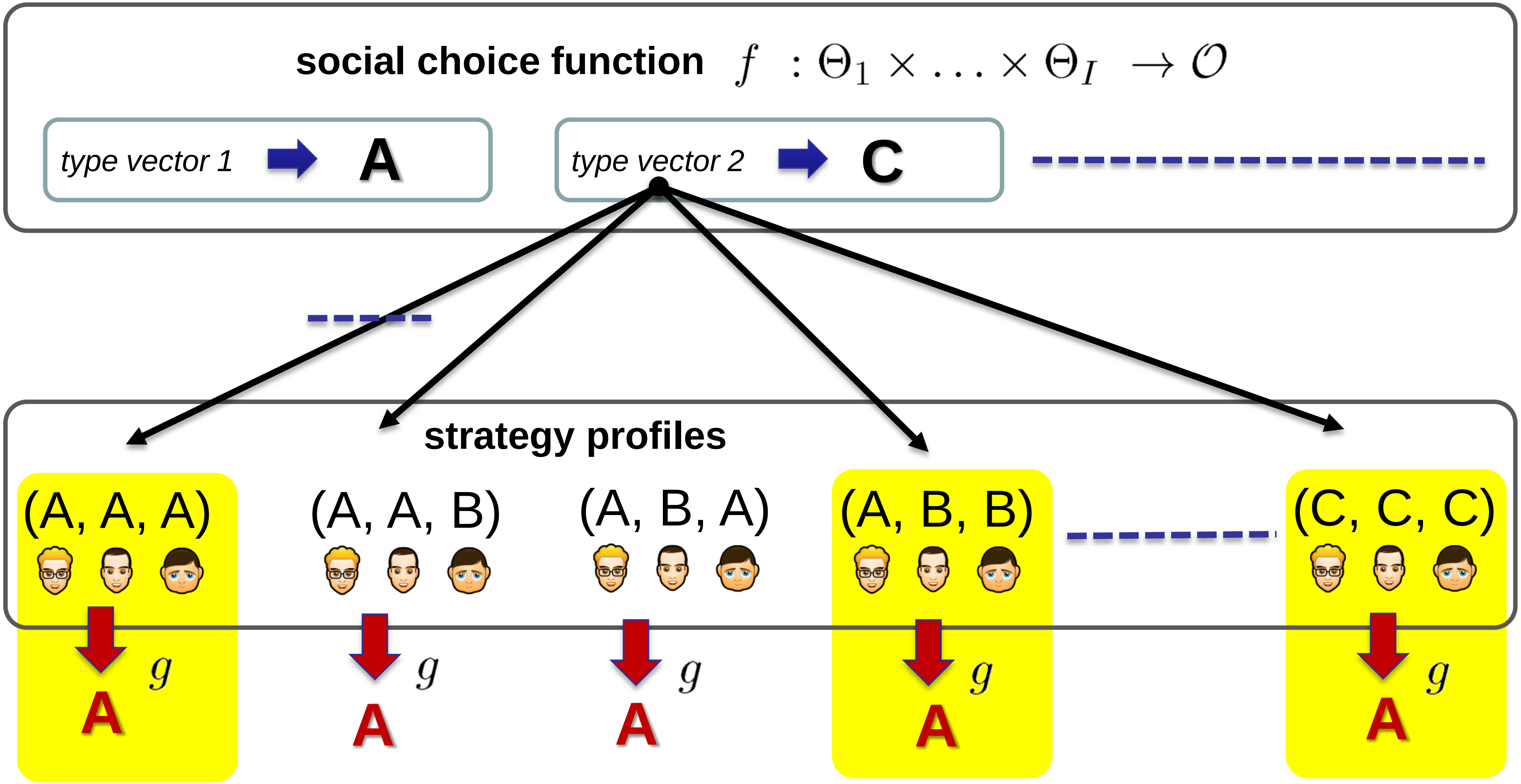
Mechanism and Implementation



GOAL: and this must happen with any type vector!

Mechanism and Implementation

- A **mechanism** is a tuple $\mathcal{M} = (\Sigma_1, \ldots, \Sigma_I, g(\cdot))$, where
  - for each agent i, Σ_i is the set of available strategies
  - $g : \Sigma_1 \times \ldots \times \Sigma_I \rightarrow \mathcal{O}$ is an outcome rule that
    - given a strategy profile $s = (s_1, \ldots, s_I)$
    - selects an outcome $g(s)$

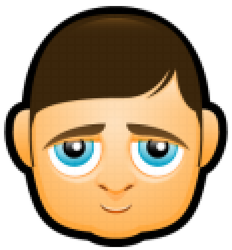
$\mathcal{M}$ **implements** in dominant strategy the social choice function f if,

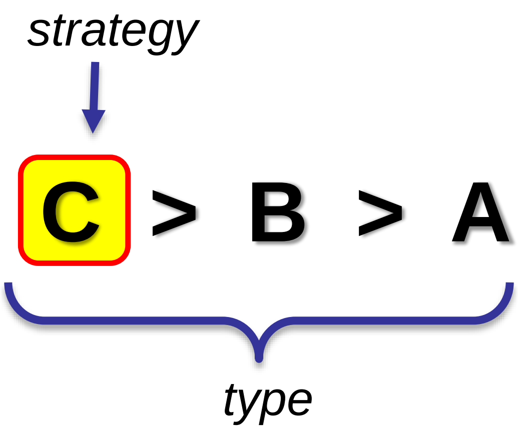
for each type vector $\theta = (\theta_1, \ldots, \theta_I)$,

$$g(s_1^*(\theta_1), \ldots, s_I^*(\theta_I)) = f(\theta)$$

where $(s_1^*, \ldots, s_I^*)$ is a dominant strategy.

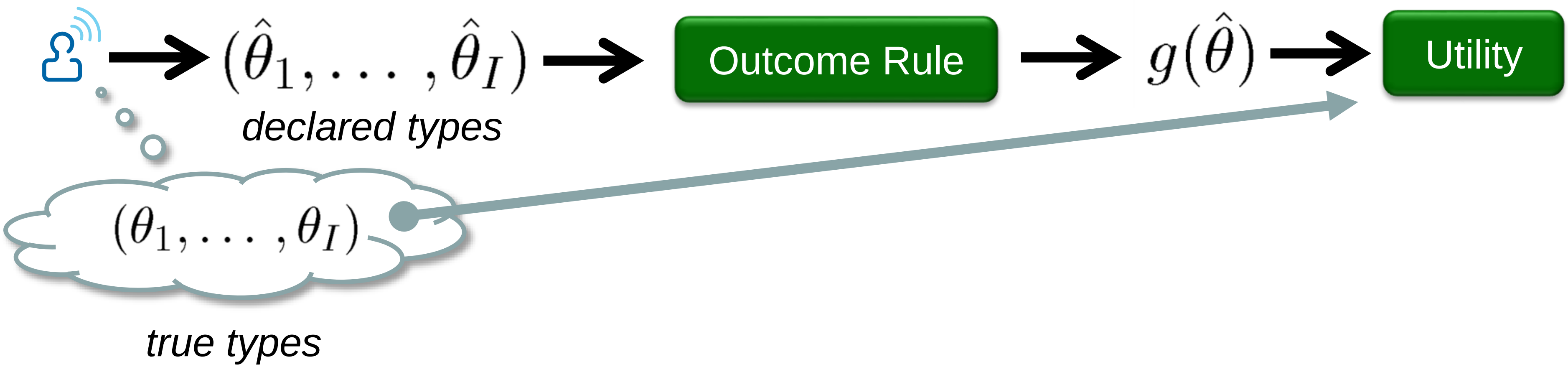
Types VS Strategies





- In a **direct revelation** mechanism, each strategy is restricted to a declaration about the private type

Types VS Strategies



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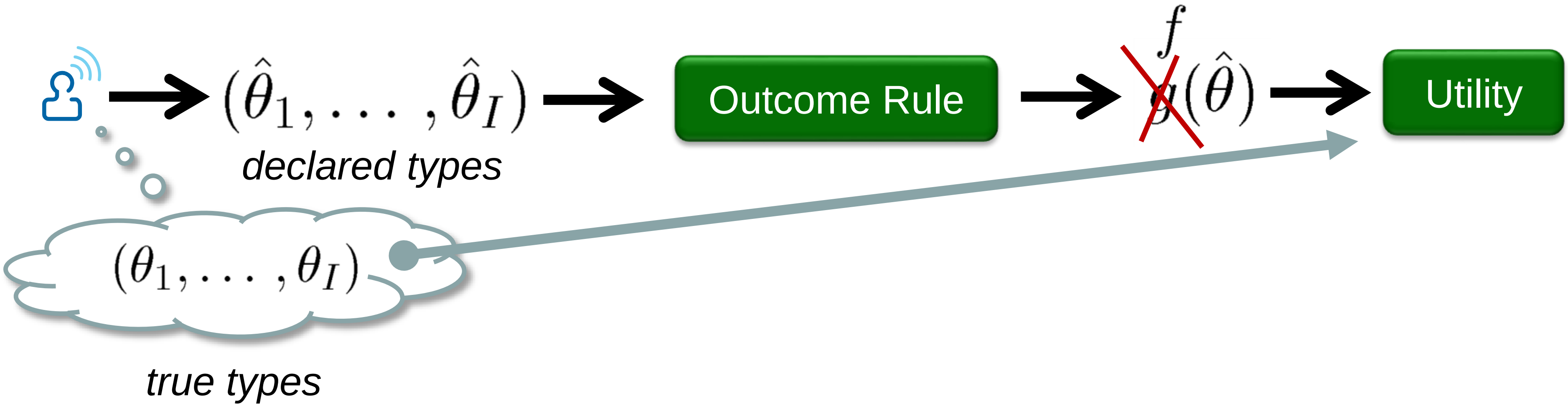
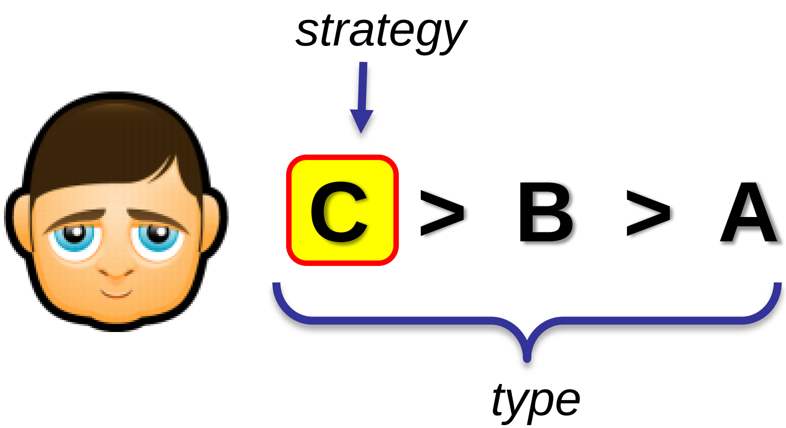


Definition. A direct-revelation mechanism is **strategy-proof** (dominant-strategy incentive-compatible) if truth-revelation is a dominant strategy for each agent.

- If the mechanism implements a function f, then $g = f$

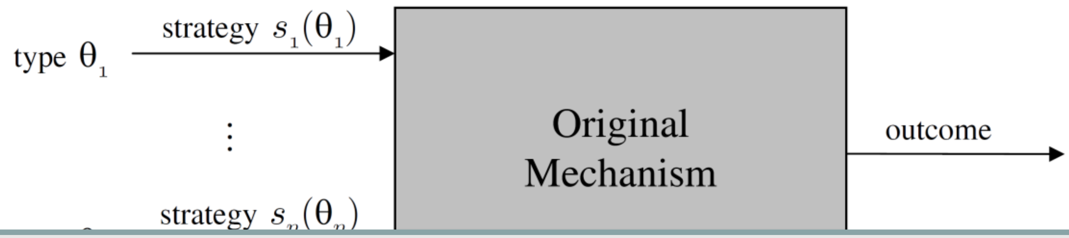
Revelation Principle



THEOREM. If a social choice function can be implemented in dominant strategies, then it can be implemented by a strategy-proof **direct-revelation** mechanism.

- It is a central theoretical tool in mechanism design
  - [Gibbard, 1973]
  - [Green and Laffont, 1977]
  - [Mayerson, 1979]

Revelation Principle: Proof Idea



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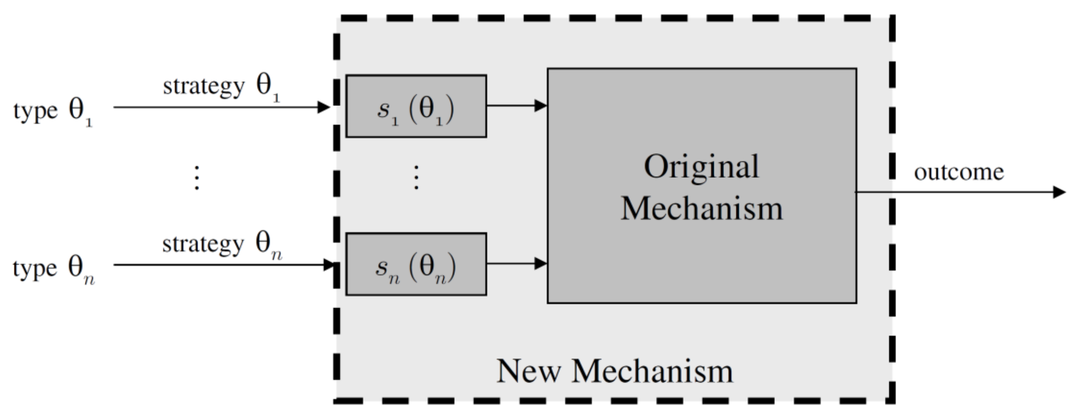

Impossibility Result

- A social choice function is **dictatorial** if one agent always receives one of its most preferred alternatives

A > C > B

B > A > C

C > B > A

Which functions can be implemented in dominant strategies?

Impossibility Result

- A social choice function is **dictatorial** if one agent always receives one of its most preferred alternatives

- A preference relation is **general** when it defines a complete and transitive ordering over the alternatives

Which functions can be implemented in dominant strategies?

Impossibility Result

THEOREM. Assume general preferences, at least two agents, and at least three optimal outcomes. A social choice function can be **implemented in dominant strategies** if, and only if, it is **dictatorial**.

- Very bad news...
  - [Gibbard, 1973] and [Satterthwaite, 1975]
- …, but must be interpreted with care

The result does not necessarily hold in restricted environments

Which functions can be implemented in dominant strategies?

Payments

Monetary compensation to induce **truthfulness**

- A utility is **quasi-linear** if it has the following form

$$u_i(o, \theta_i) = v_i(o, \theta_i) - p_i$$

valuation function
cardinal preferences

payment by the agent

Payments

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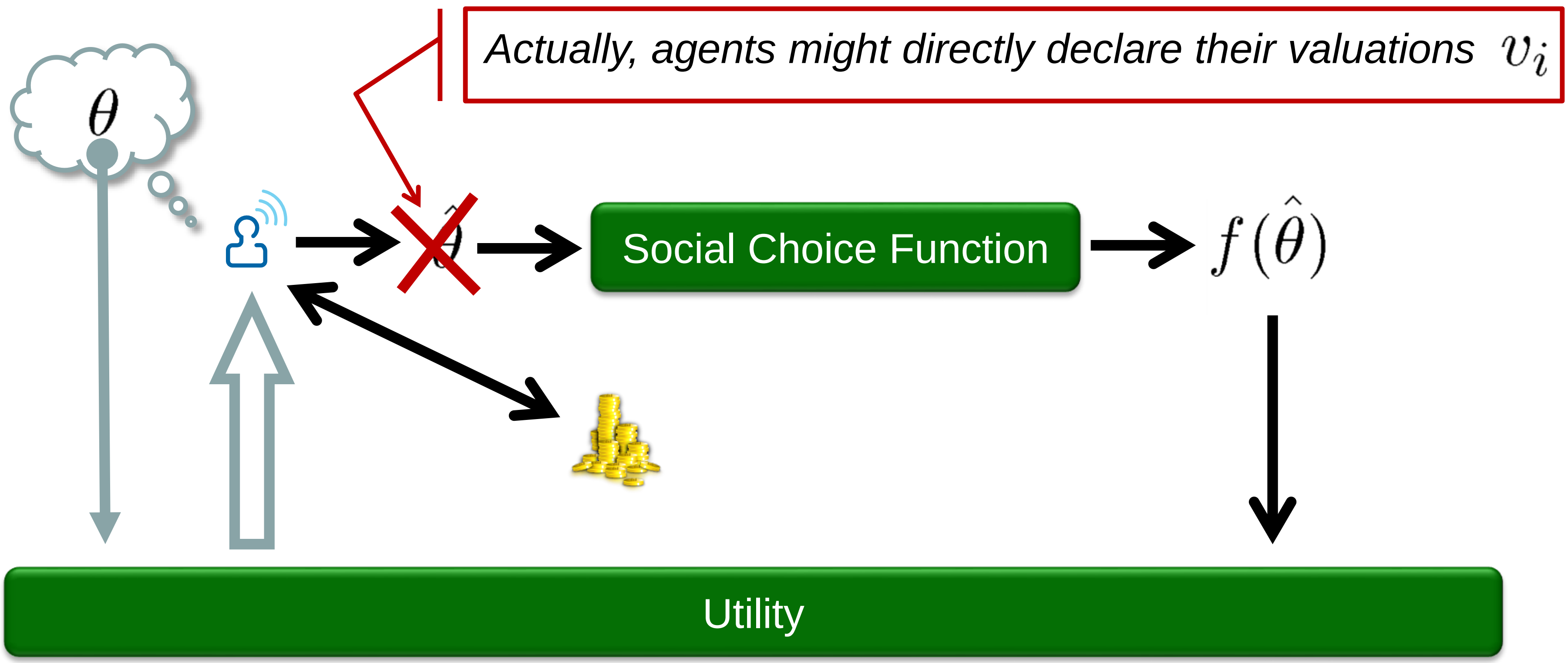
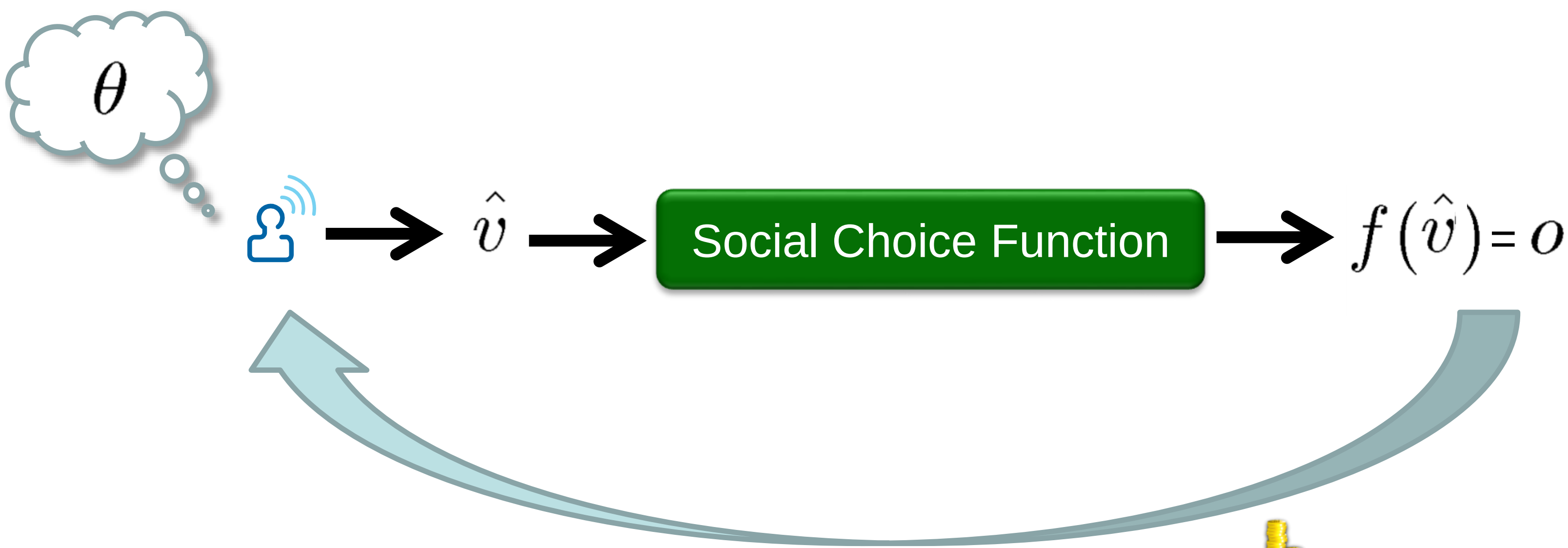
- Payments are defined by the mechanism

Direct Mechanisms with Payments



Direct Mechanisms with Payments



$$u_i(o, \theta_i) = v_i(o, \theta_i) - p_i$$

Vickrey-Clarke-Groves (VCG) Mechanisms

(1) **The mechanism selects the outcome** o^* **maximizing** $\sum_i \hat{v}_i(o, \theta_i)$.

(2) **Payments are such that** $p_i = h_i - \sum_{j \neq i} \hat{v}_j(o^*, \theta_j)$

Family of mechanisms (e.g., the value of the optimal outcome without the agent)

Vickrey-Clarke-Groves (VCG) Mechanisms

- An auction with one item
- We have bids: $b_1 > b_2 > \cdots > b_n$

Agent *1* receives the item

Agent *1* pays b_2

(1) **The r** **ne o^* maximizing** $\sum_i \hat{v}_i(o, \theta_i)$.

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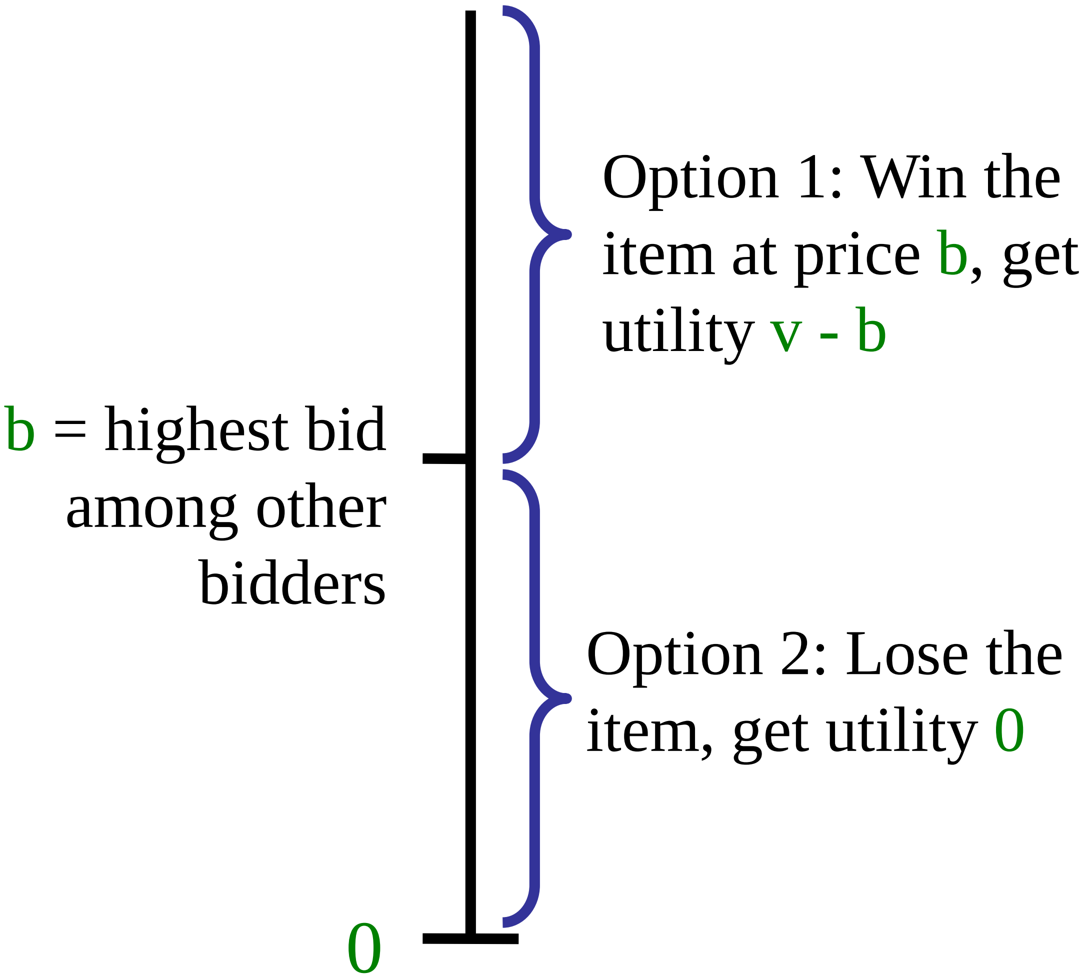
Family of mechanisms (e.g., the value of the optimal outcome without the agent)

Vickrey Auction is Strategy-Proof

What should a bidder with value v bid?



Would like to win if and only if v - b > 0

Payment Rules (Again...)

- Monetary compensation to induce **truthfulness**

see, e.g., [Shoham, Leyton-Brown; 2009]

Payment Rules (Again…)

- Monetary compensation to induce **truthfulness**

GOAL: Budget Balance

- ✓ The algebraic sum of the monetary transfers is zero
- ✓ In particular, mechanisms cannot run into deficit

see, e.g., [Shoham, Leyton-Brown; 2009]

Payment Rules (Again…)

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GOAL: Budget Balance

✓ The algebraic sum of the monetary transfers is zero
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- Monetary compensation to induce **fairness**

✓ For instance, it is desirable that ***no agent envies*** the allocation of any another agent, or that
✓ The outcome is ***Pareto efficient***, i.e., there is no different allocation such that every agent gets at least the same utility and one of them improves.

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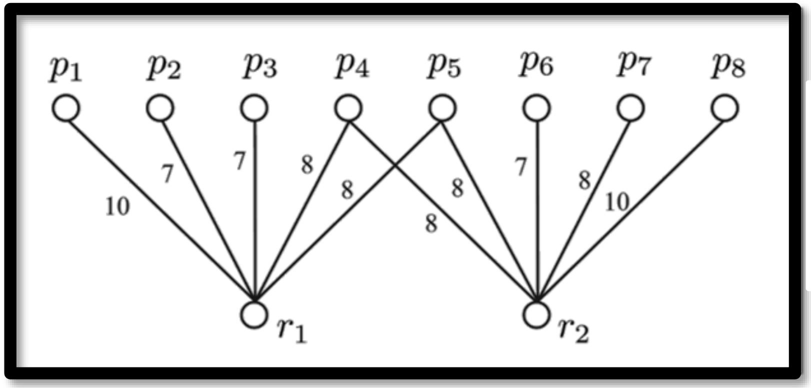
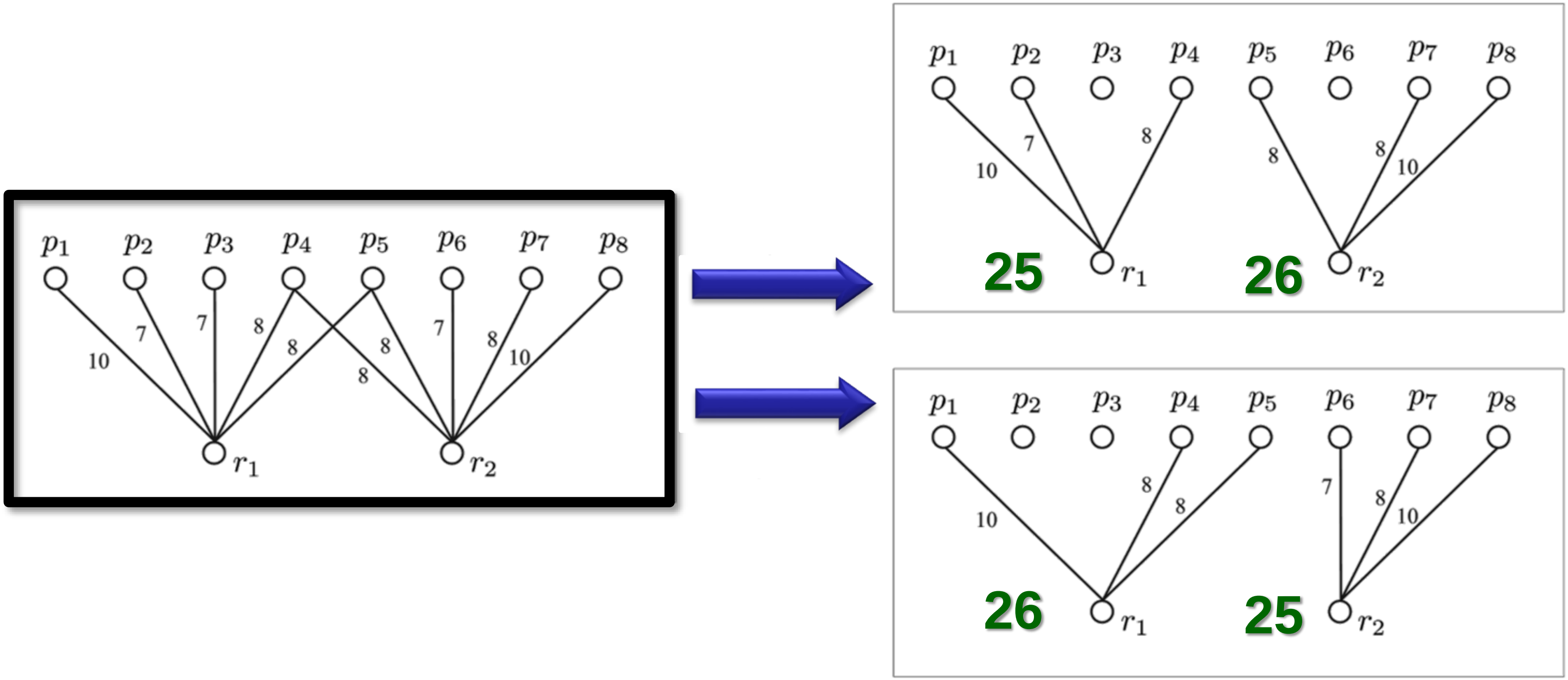
Fairness vs Efficiency

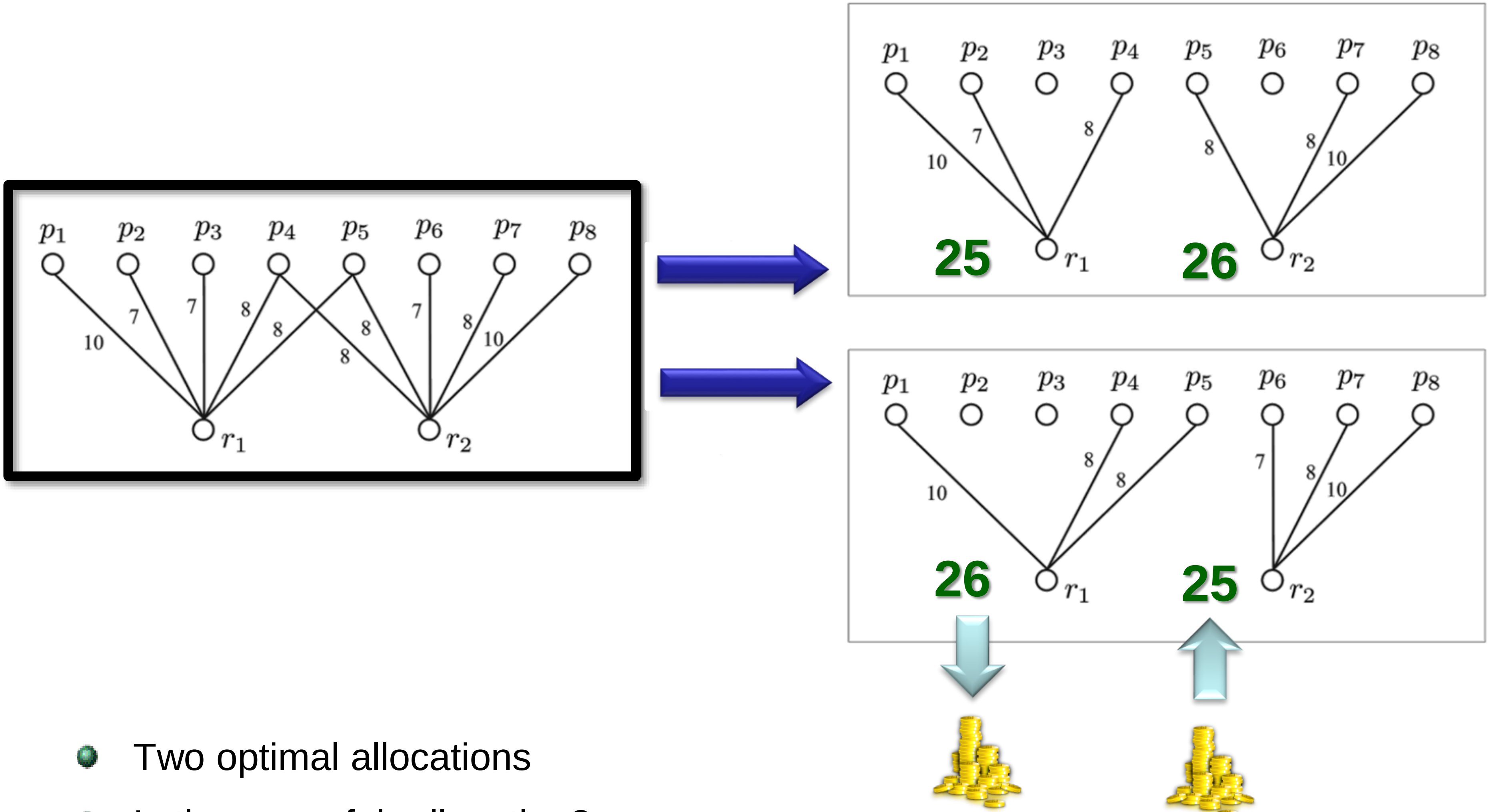


Fairness vs Efficiency



- Two optimal allocations
- Is there any fair allocation?

Fairness vs Efficiency



- Two optimal allocations
- Is there any fair allocation?

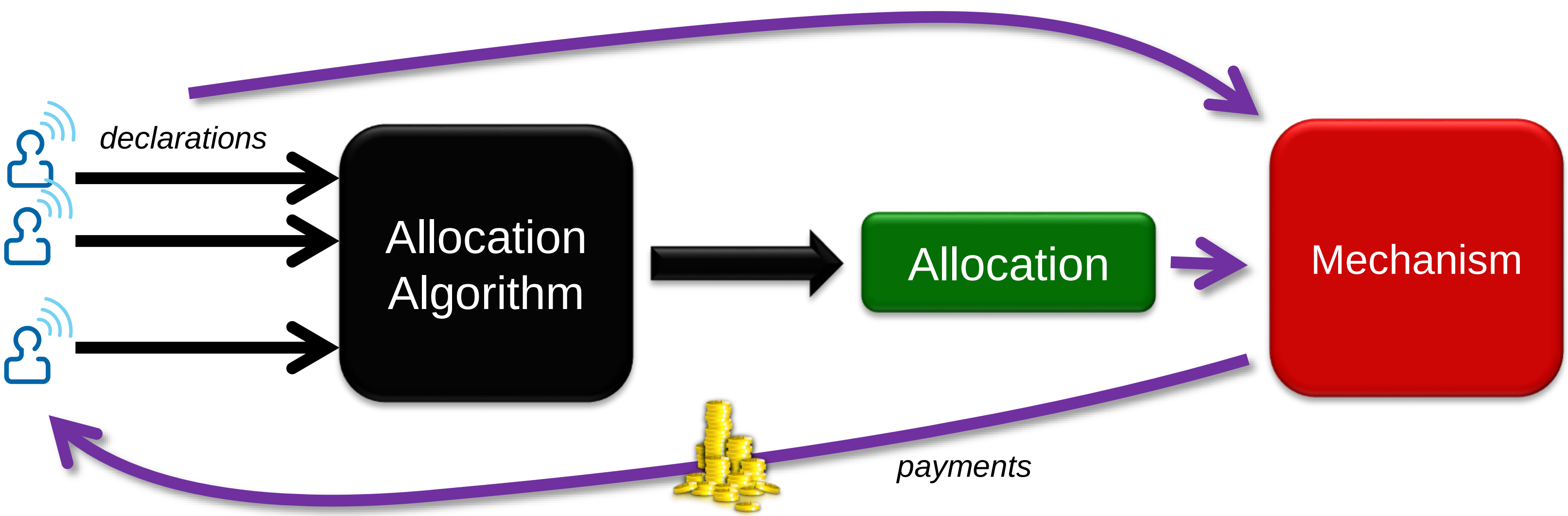
(A Few…) Impossibility Results

Efficiency **+** Truthfulness **+** Budget Balance

[Green, Laffont; 1977]
[Hurwicz; 1975]

Fairness **+** Truthfulness **+** Budget Balance

[Tadenuma, Thomson;1995]
[Alcalde, Barberà; 1994]
[Andersson, Svensson, Ehlers; 2010]



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(A Few…) Impossibility Results

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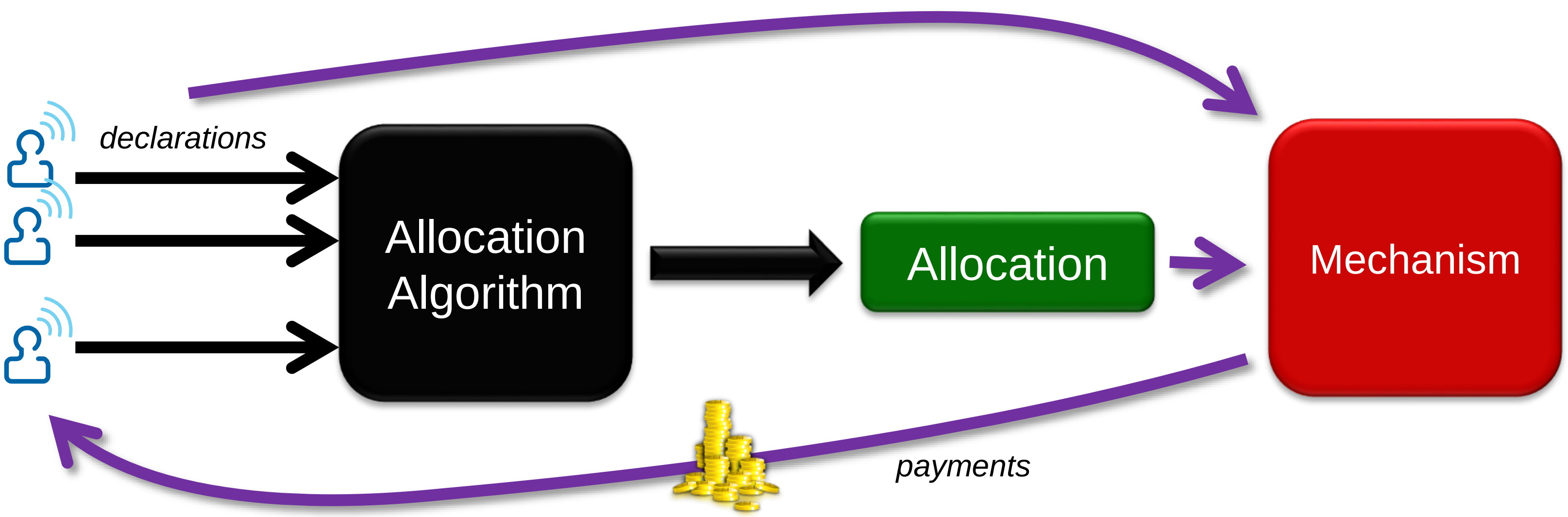
Fairness **+** Truthfulness **+** Budget Balance

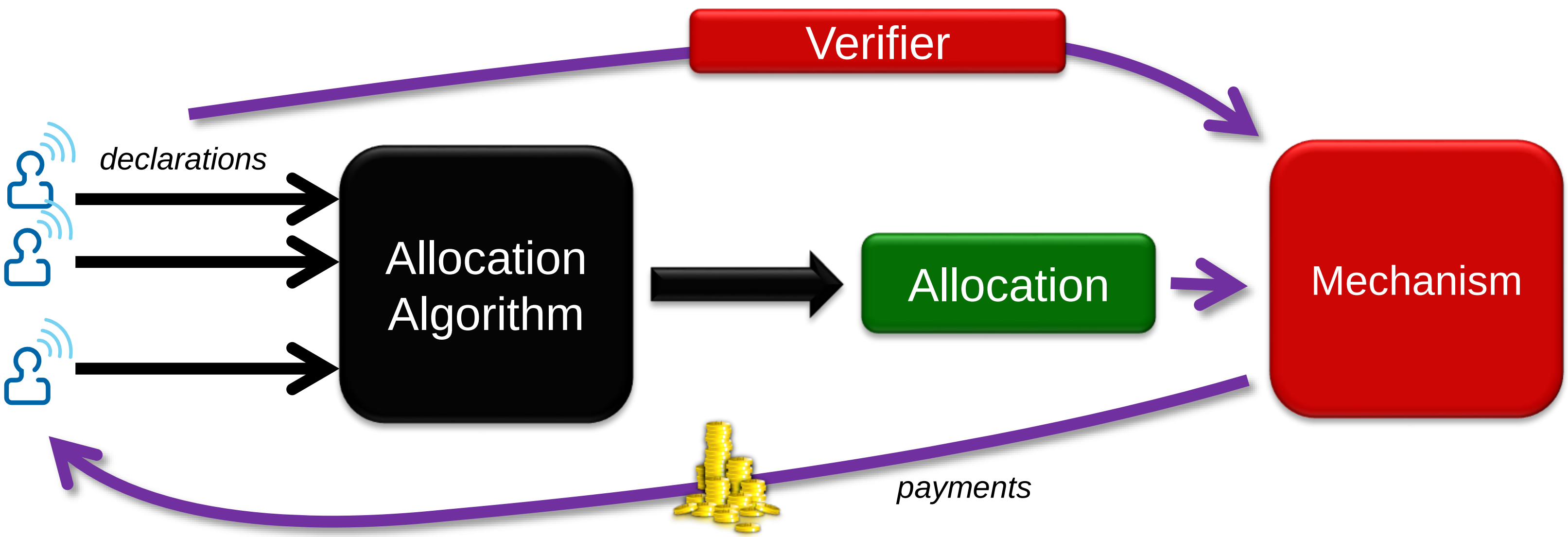


(A Few…) Impossibility Results

Efficiency **+** Truthfulness **+** Budget Balance

Fairness **+** Truthfulness **+** Budget Balance

- Verification on «selected» declarations



Approaches to Verification

(1) Partial Verification

(2) Probabilistic Verification

Approaches to Verification

(1) Partial Verification

[Green, Laffont; 1986]
[Nisan, Ronen; 2001]

(2) Probabilistic Verification

Approaches to Verification

(1) Partial Verification

[Auletta, De Prisco, Ferrante, Krysta, Parlato, Penna, Persiano, Sorrentino, Ventre]

(2) Probabilistic Verification

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(2) Probabilistic Verification

[Caragiannis, Elkind, Szegedy, Yu; 2012]

Approaches to Verification

(1) Partial Verification

(2) Probabilistic Verification

Punishments are used to enforce truthfulness

Approaches to Verification

(1) Partial Verification

(2) Probabilistic Verification

Punishments are used to enforce truthfulness

- Verification is performed via **sensing**
  - Hence, it is subject to errors; for instance, because of the limited precision of the measurement instruments.
  - It might be problematic to decide whether an observed discrepancy between verified values and declared ones is due to a strategic behavior or to such sensing errors.

[Greco, Scarcello; 2014]

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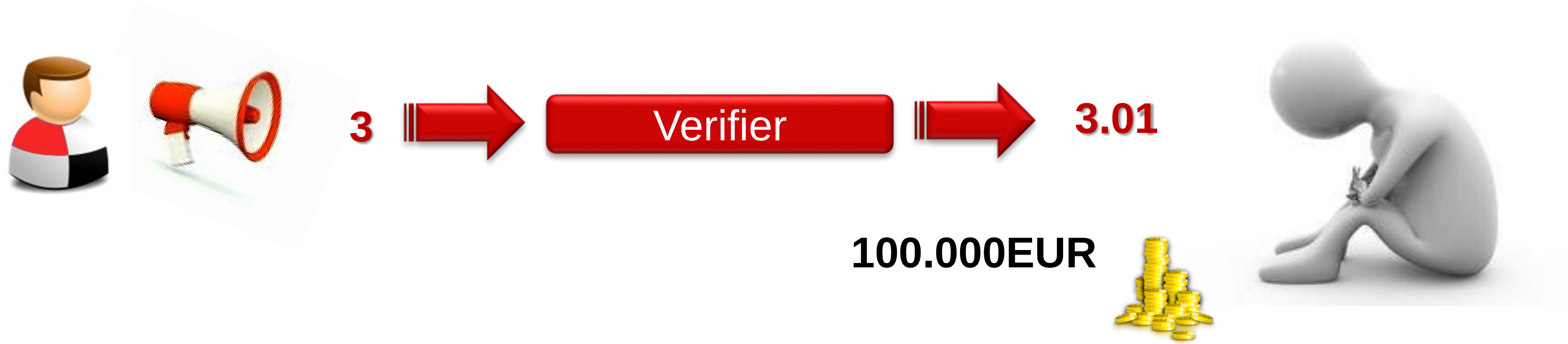
  - It might be problematic to decide whether an observed discrepancy between verified values and declared ones is due to a strategic behavior or to such sensing errors.

Approaches to Verification (bis)

- Agents might be uncertain of their private features; for instance, due to limited computational resources
  - There might be no strategic issues

Approaches to Verification (ter)



- Punishments enforce truthfulness
  - They might be disproportional to the harm done by misreporting
  - Inappropriate in real life situations in which uncertainty is inherent due to measurements errors or uncertain inputs.

[Feige, Tennenholtz; 2011]

Approaches to Verification

(1) Partial Verification

(2) Probabilistic Verification

Punishments are used to enforce truthfulness

(3) Full Verification

The verifier returns a value.

Approaches to Verification

(1) Partial Verification

(2) Probabilistic Verification

Punishments are used to enforce truthfulness

(3) Full Verification

The verifier returns a value. But,…

- ***no punishment***
  - payments are always computed under the presumption of innocence, where incorrect declared values do not mean manipulation attempts by the agents
- **error tolerance**
  - the consequences of errors in the declarations produce a linear "distorting effect" on the various properties of the mechanism

Payment Rules

- Monetary compensation to induce **truthfulness**

GOAL: Budget Balance

✓ The algebraic sum of the monetary transfers is zero
✓ In particular, mechanisms cannot run into deficit

- Monetary compensation to induce **fairness**

✓ For instance, it is desirable that ***no agent envies*** the allocation of any another agent, or that
✓ The outcome is ***Pareto efficient***, i.e., there is no different allocation such that every agent gets at least the same utility and one of them improves.

Payment Rules & Full Verification

- Monetary compensation to induce **truthfulness**

GOAL: Budget Balance

- ✓ The algebraic sum of the monetary transfers is zero
- ✓ In particular, mechanisms cannot run into deficit

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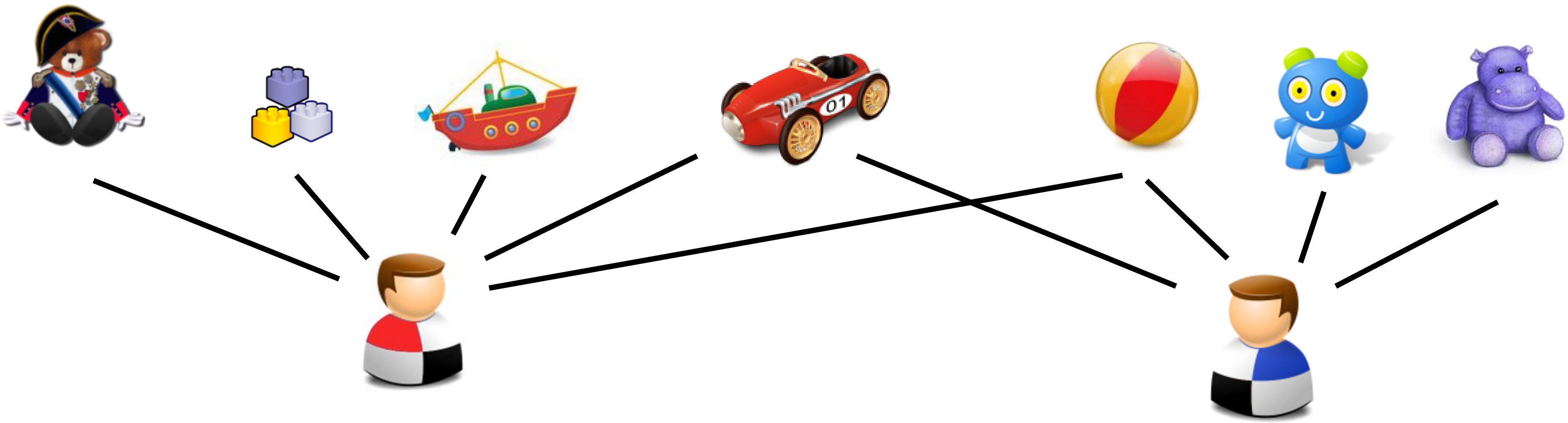
  - ✓ For instance, it is desirable that ***no agent envies*** the allocation of any another agent, or that
  - ✓ The outcome is ***Pareto efficient***, i.e., there is no different allocation such that every agent gets at least the same utility and one of them improves.

Outline

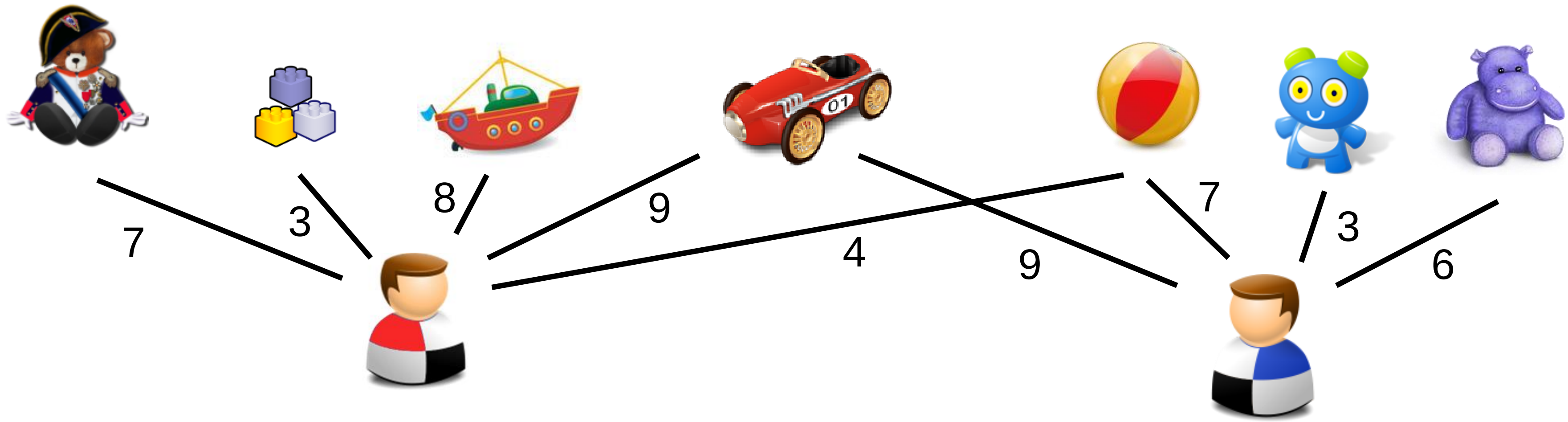
- Game Theory
- Mechanism Design
- Mechanisms with Verification
- **Mechanisms and Allocation Problems**
- **Complexity Analysis**

The Model



- Goods are indivisible and non-sharable
- Constraints on the maximum number of goods to be allocated to each agent
- Cardinal preferences: *Utility functions*

The Model



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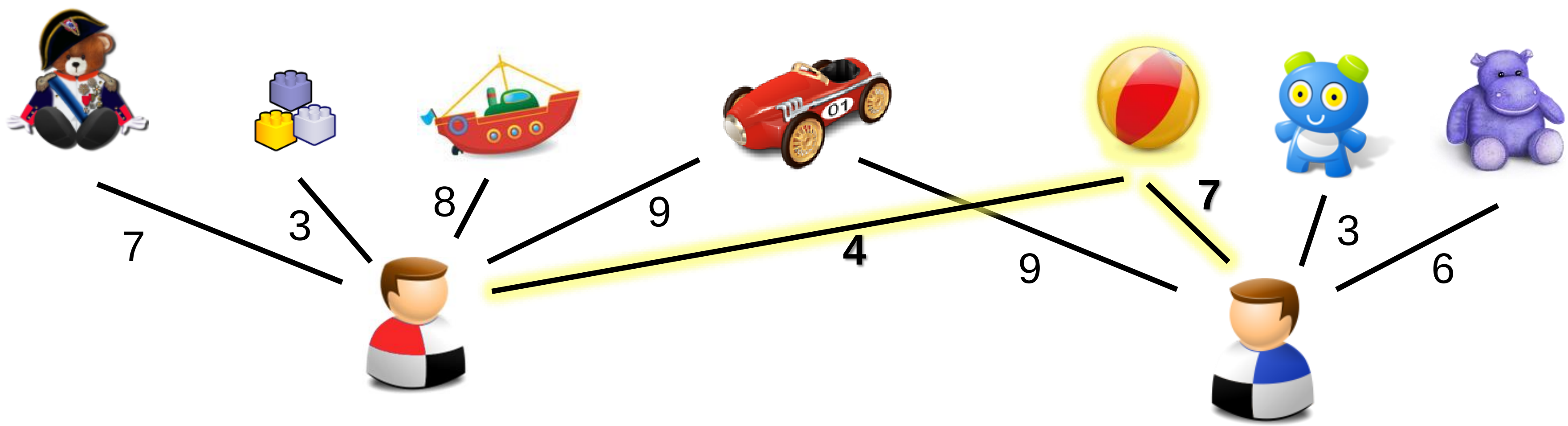
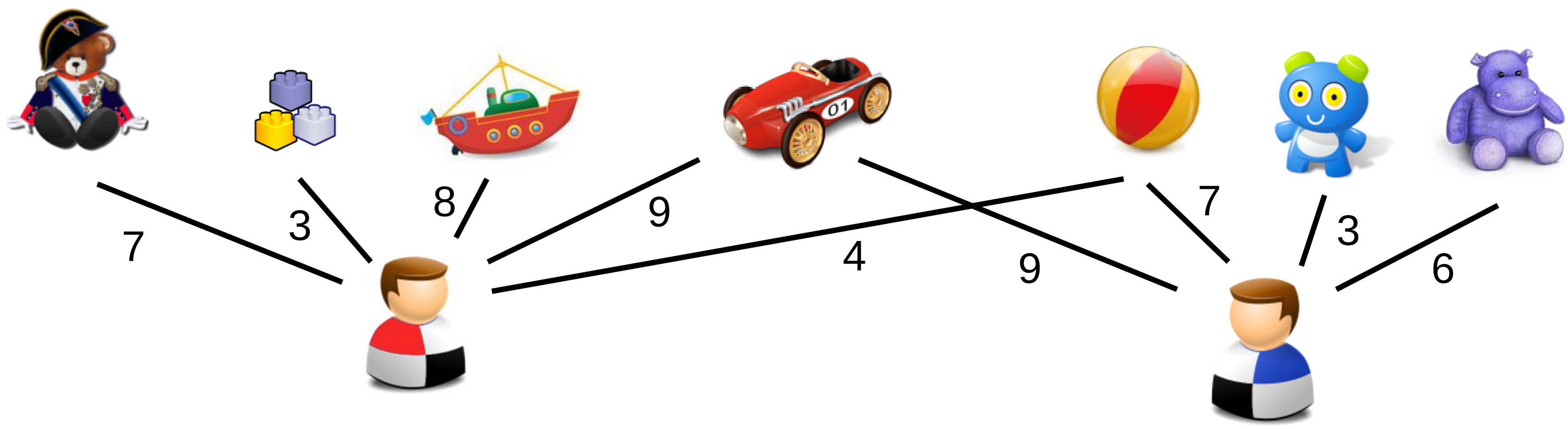


- Goods are indivisible and non-sharable
- Constraints on the maximum number of goods to be allocated to each agent
- Cardinal preferences: *Utility functions*

Different agents might have different valuations for the same good

The Model



- Goods are indivisible and non-sharable
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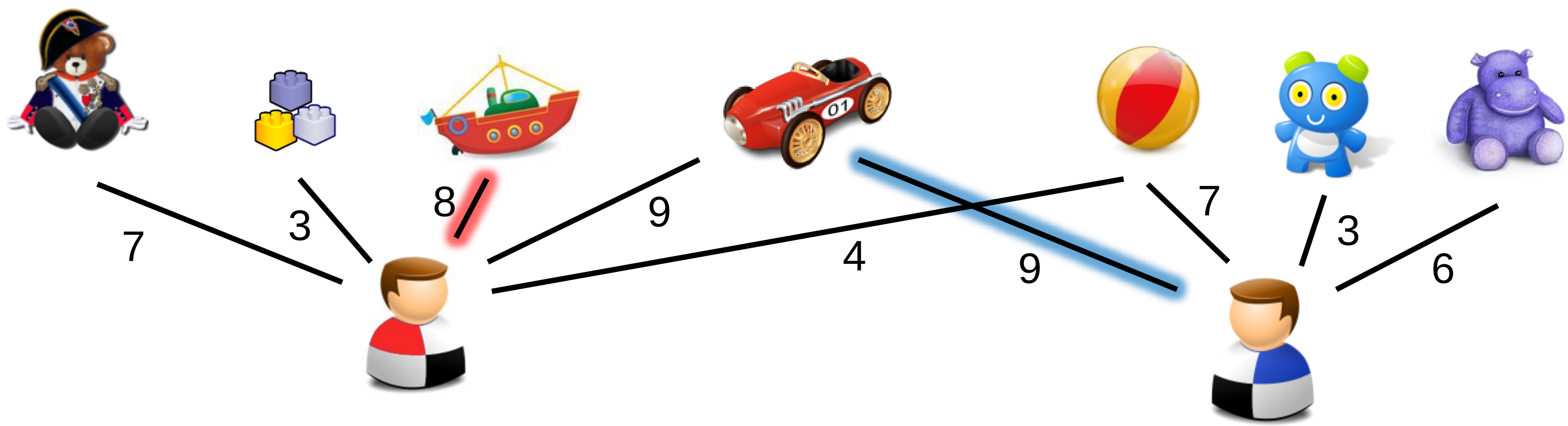
GOAL: Optimal Allocations

✓ Social Welfare
✓ Efficiency

The Model



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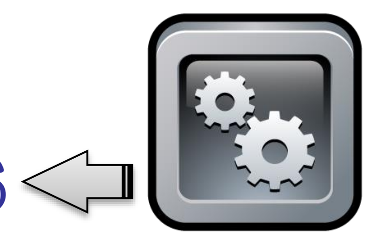
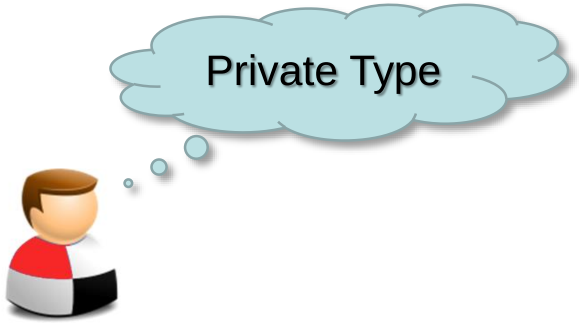
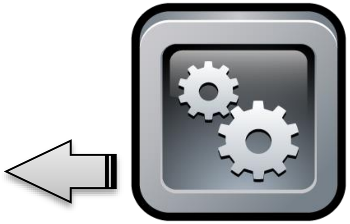


✓ Social Welfare
✓ Efficiency

Strategic Issues

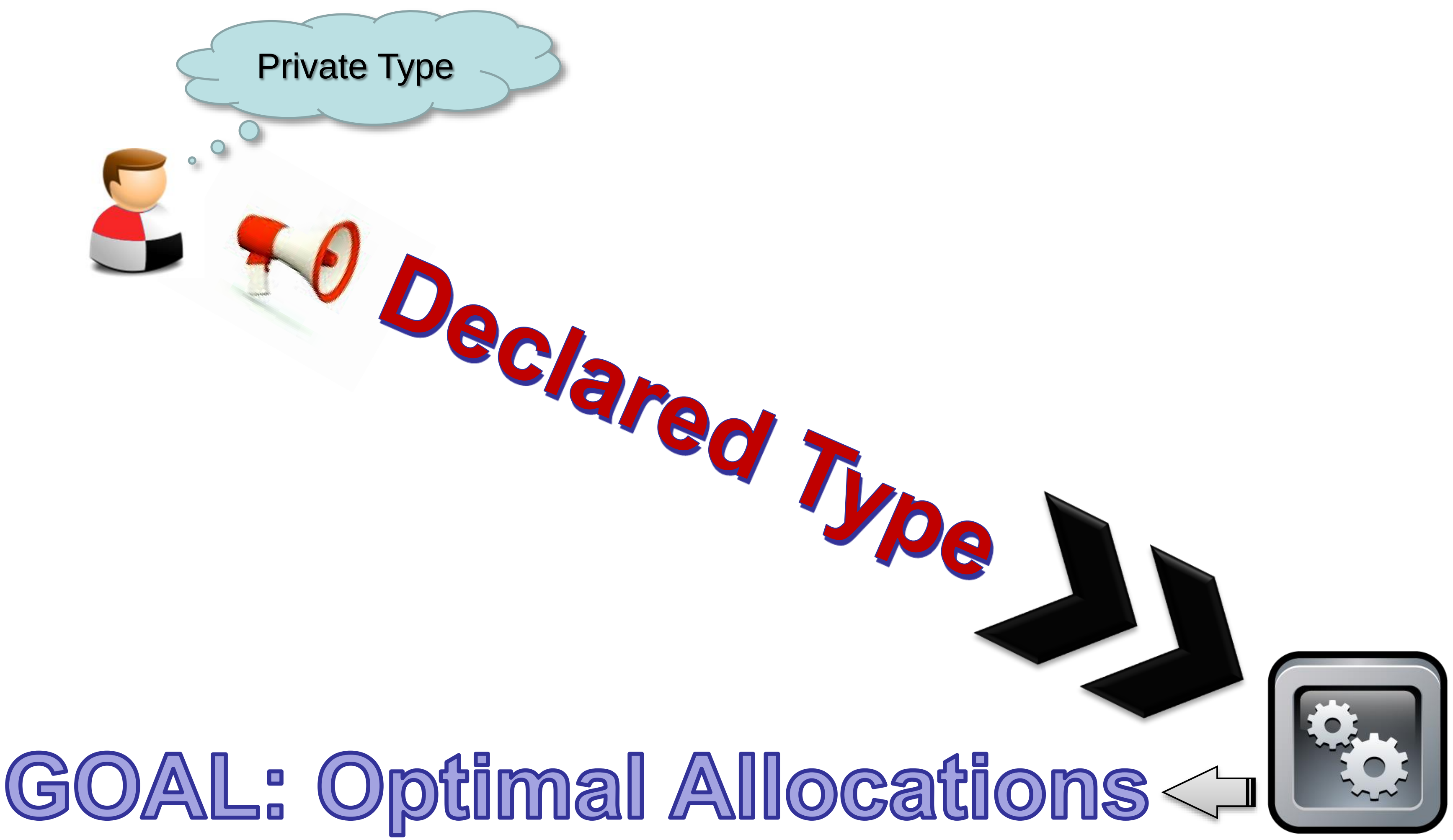


GOAL: Optimal Allocations



- ✓ Social Welfare
- ✓ Efficiency

Strategic Issues



- ✓ Social Welfare
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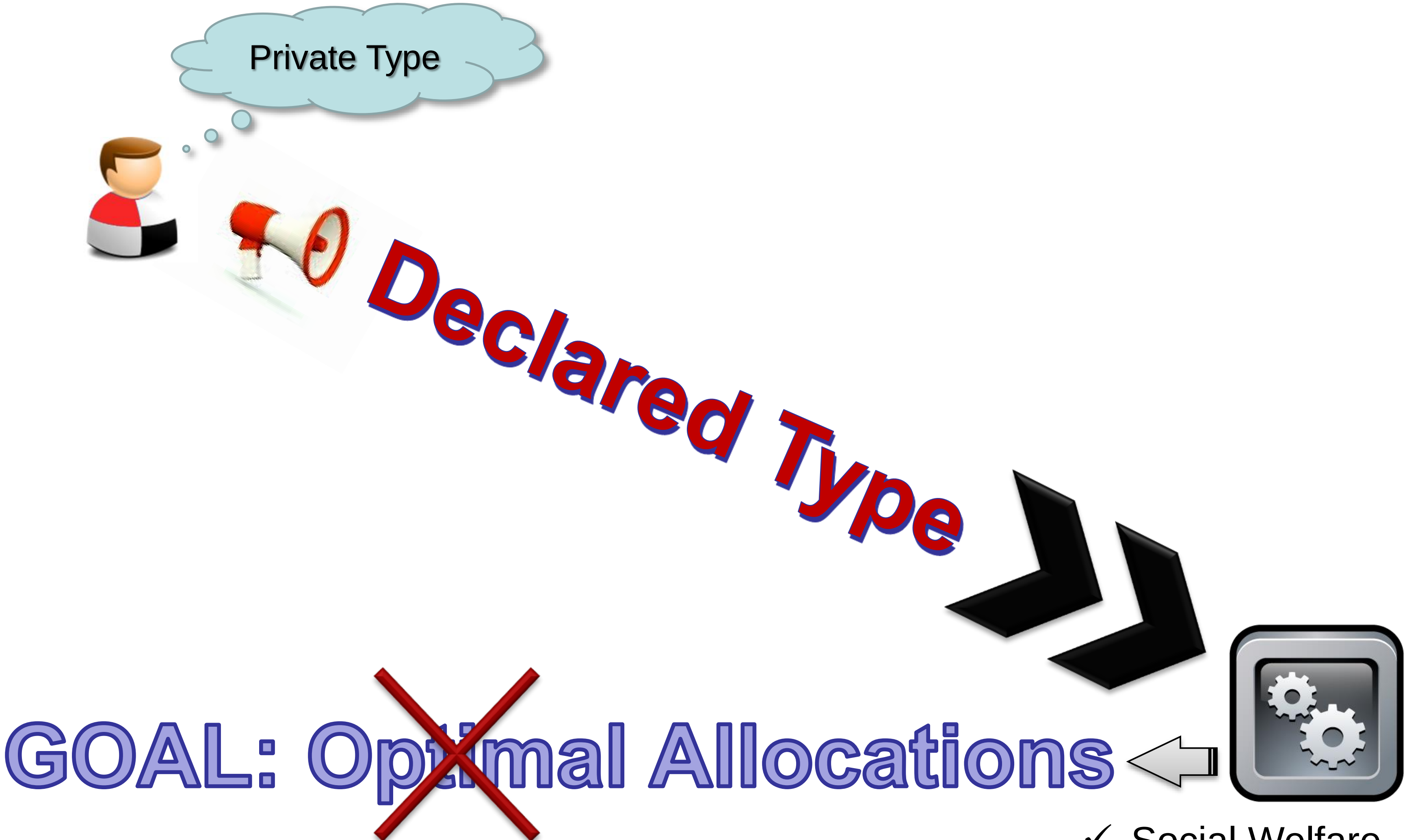
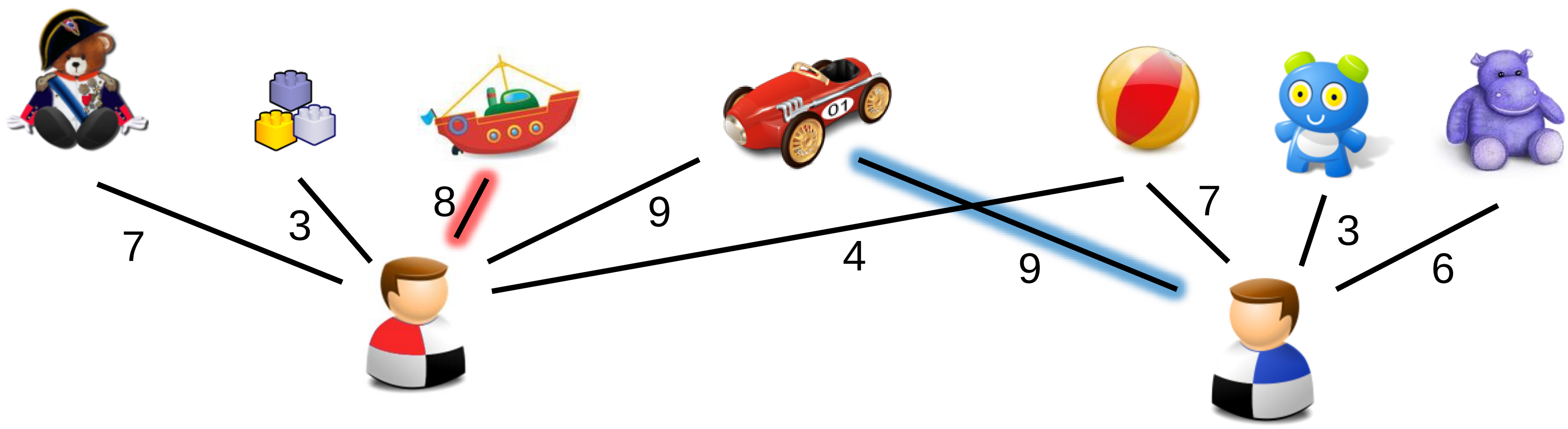
Strategic Issues



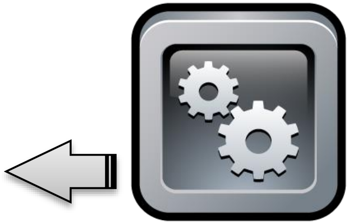
- ✓ Social Welfare
- ✓ Efficiency

Strategic Issues: Example



Before: 8+9=17

GOAL: Optimal Allocations



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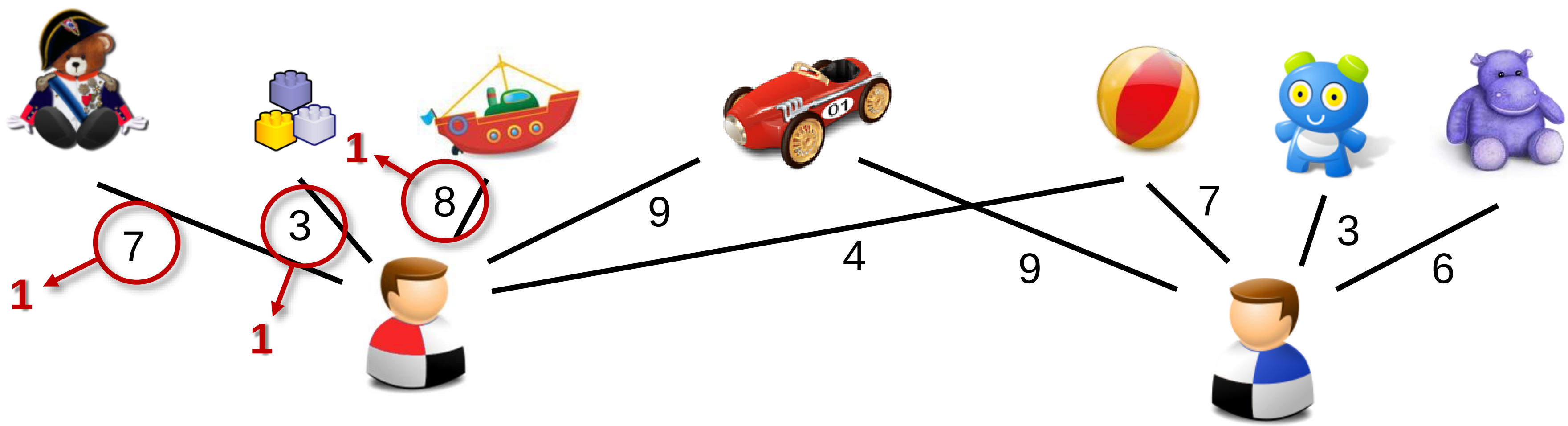

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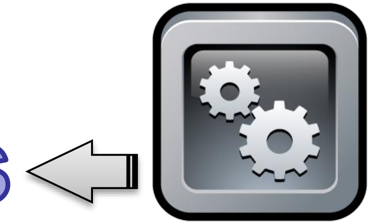
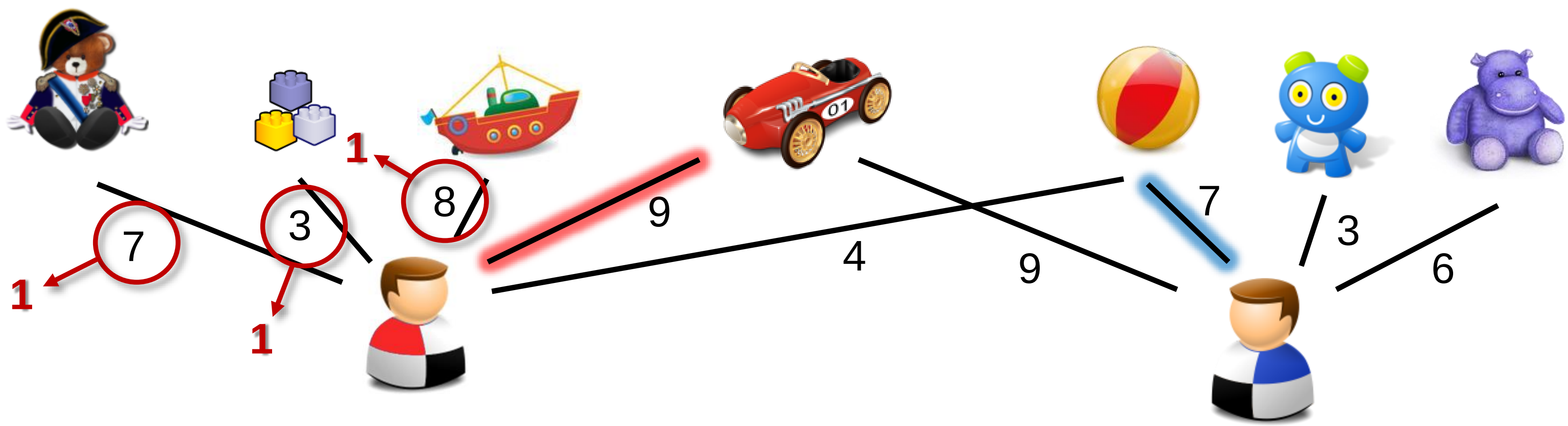
GOAL: Optimal Allocations



- ✓ Social Welfare
- ✓ Efficiency

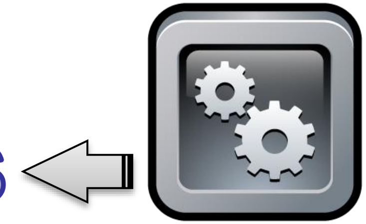
Strategic Issues: Example



Before: 8+9=17

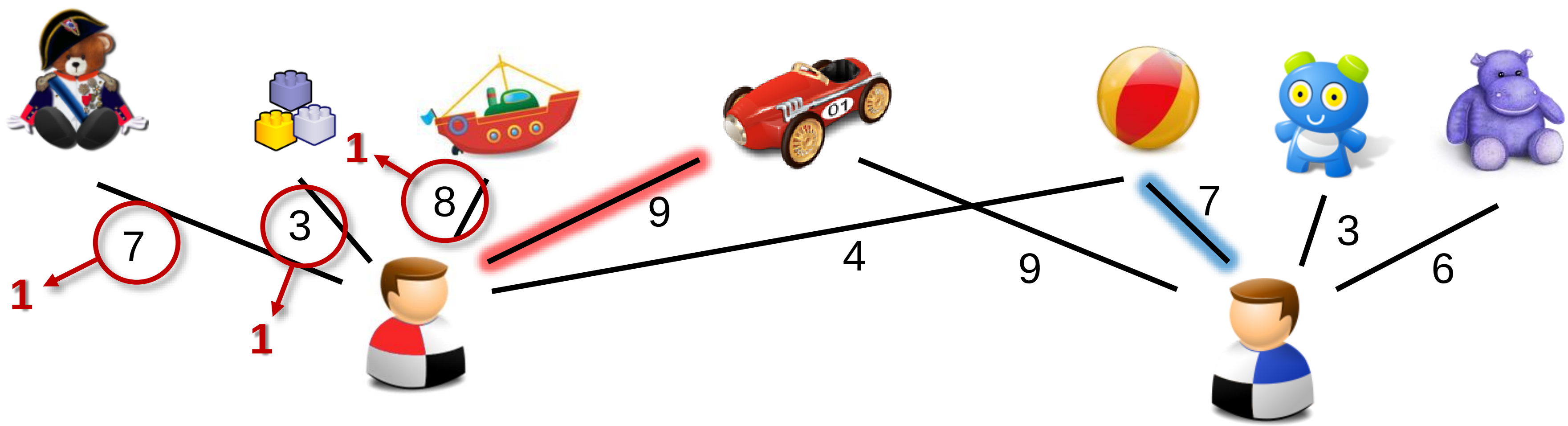
After: 9+7=16

GOAL: ~~Optimal~~ Allocations



✓ Social Welfare
✓ Efficiency

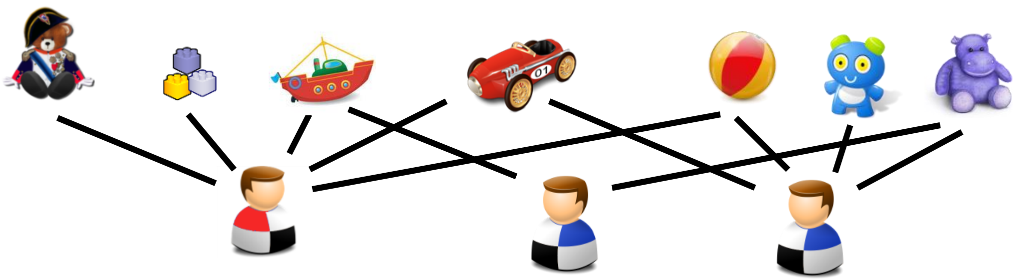
Strategic Issues: Verification



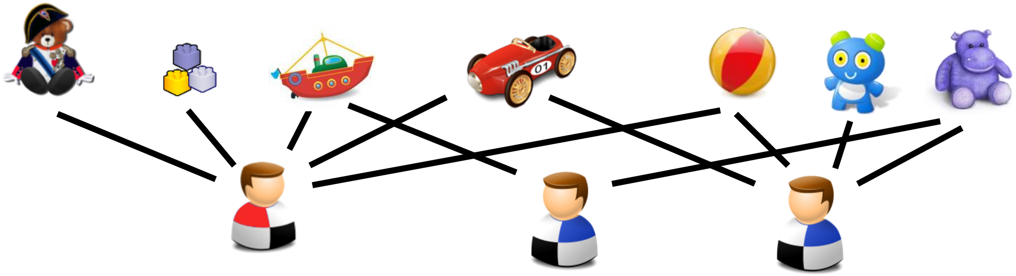
We assume full-verification.

But, of course, we can verify only the goods that are selected.

A Key Lemma

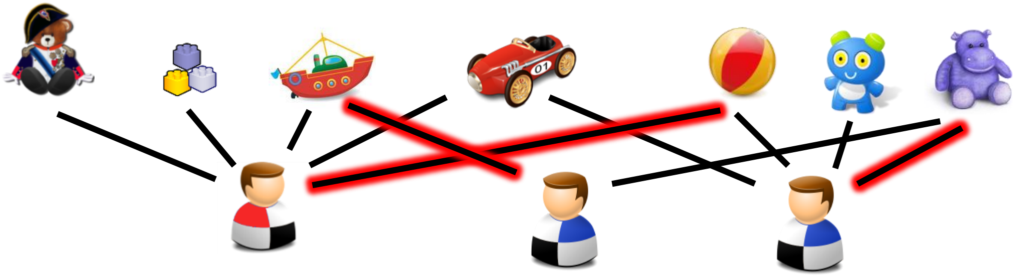


A Key Lemma



- Consider an optimal allocation (w.r.t. some declared types)

A Key Lemma



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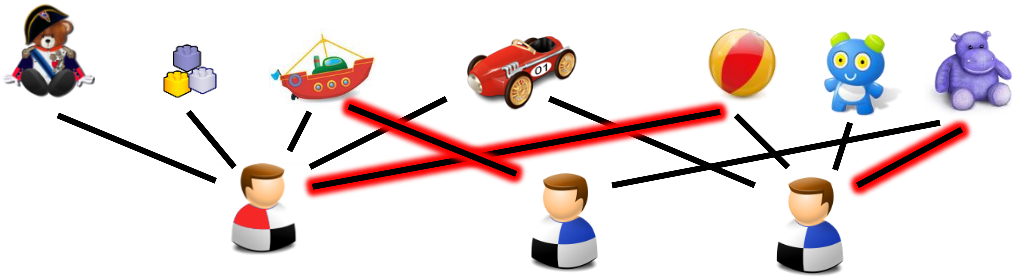
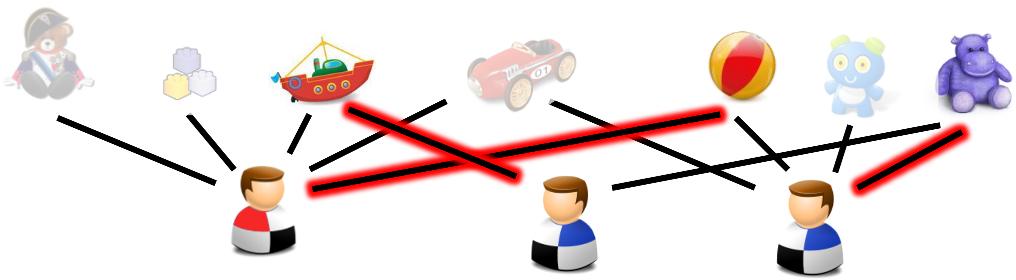


- Consider an optimal allocation (w.r.t. some declared types)
- Ignore the goods that are not allocated,
  - and hence that cannot be verified later…

A Key Lemma



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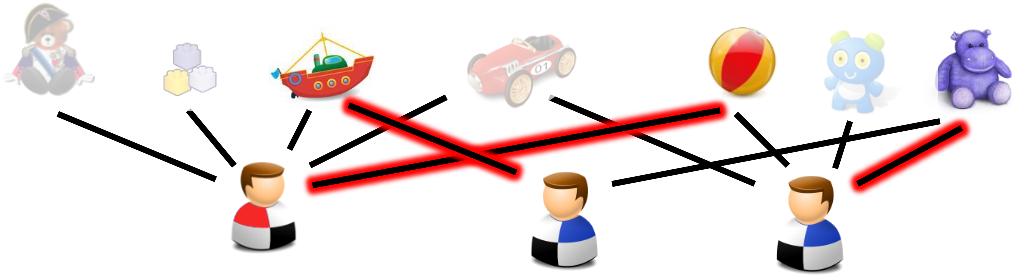

- Consider an optimal allocation (w.r.t. some declared types)
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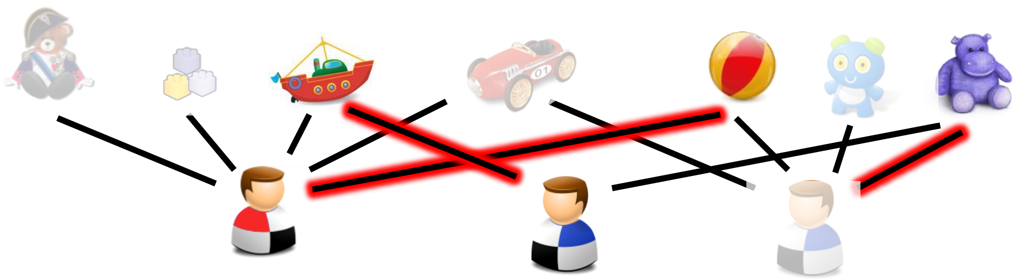
- Focus on an arbitrary coalition of agents

A Key Lemma



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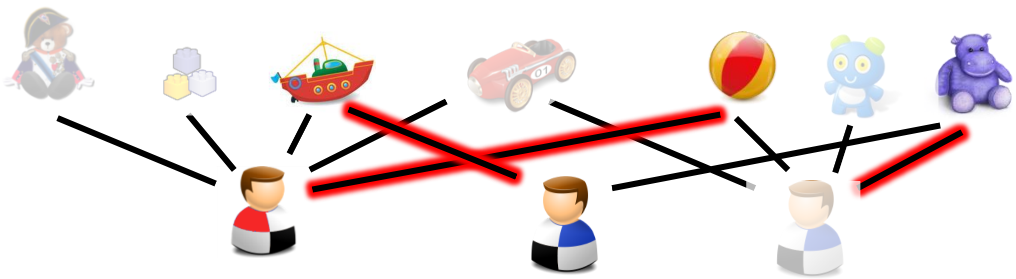
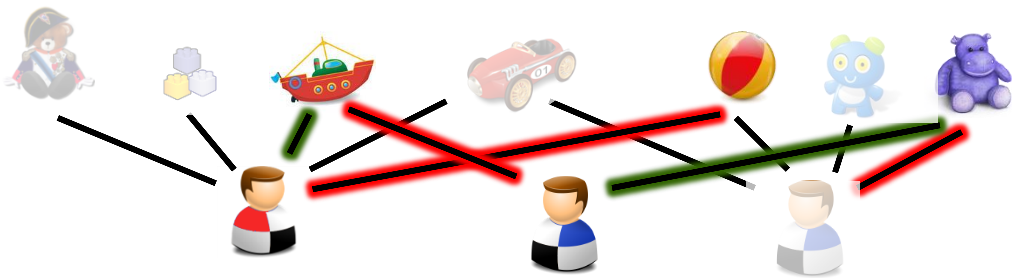
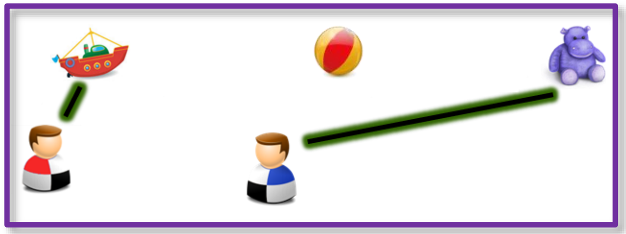
A Key Lemma



- Consider an optimal allocation (w.r.t. some declared types)
- Ignore the goods that are not allocated,
  - and hence that cannot be verified later…
- Focus on an arbitrary coalition of agents
- In this novel setting, compute an optimal allocation

A Key Lemma





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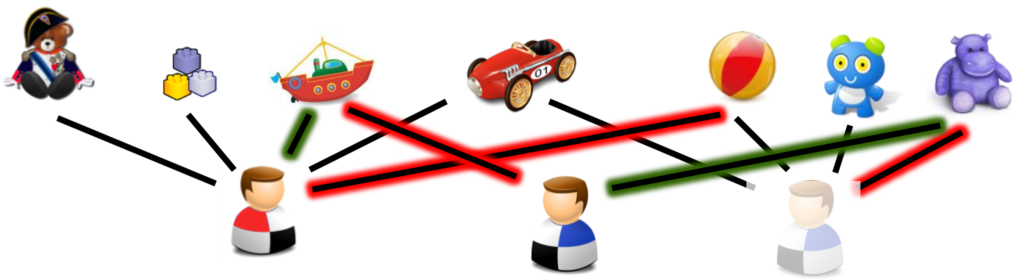

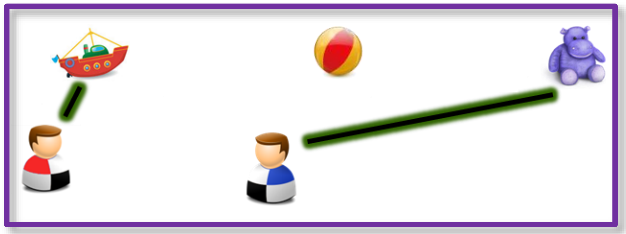

- Consider an optimal allocation (w.r.t. some declared types)
- Ignore the goods that are not allocated,
  - and hence that cannot be verified later…
- Focus on an arbitrary coalition of agents
- In this novel setting, compute an optimal allocation

❖ **The allocation is also optimal for that coalition, even if all goods were actually available**

The Mechanism…

Input: An allocation π for $\langle \mathcal{A}, G, \omega \rangle$, and a vector $\mathbf{w} \in \mathbf{D}$;

Assumption: A verifier $\mathbf{v}$ is available. Let $\mathbf{v}(\pi) = (v_1, ..., v_n)$;

Let $\mathbb{C}$ denote the set of all possible subsets of $\mathcal{A}$;

For each set $\mathcal{C} \in \mathbb{C}$,

  Compute an optimal allocation $\pi_{\mathcal{C}}$ for $\langle \mathcal{C}, \texttt{img}(\pi), \omega \rangle$ w.r.t. $\mathbf{w}$;

For each agent $i \in \mathcal{A}$,

  For each set $\mathcal{C} \in \mathbb{C}$,

    Let $\Delta^1_{\mathcal{C},i}(\pi, \mathbf{w}) := \texttt{val}(\pi_{\mathcal{C}}, (v_i, \mathbf{w}_{-i}))$; $(= v_i(\pi_{\mathcal{C}}) + \sum_{j \in \mathcal{C} \setminus \{i\}} w_j(\pi_{\mathcal{C}}))$;

    Let $\Delta^2_{\mathcal{C},i}(\pi, \mathbf{w}) := \texttt{val}(\pi_{\mathcal{C} \setminus \{i\}}, \mathbf{w})$; $(= \sum_{j \in \mathcal{C} \setminus \{i\}} w_j(\pi_{\mathcal{C} \setminus \{i\}}))$;

  Let $\xi_i(\pi, \mathbf{w}) := \sum_{\mathcal{C} \in \mathbb{C}} \frac{(|\mathcal{A}| - |\mathcal{C}|)!(|\mathcal{C}| - 1)!}{|\mathcal{A}|!} (\Delta^1_{\mathcal{C},i}(\pi, \mathbf{w}) - \Delta^2_{\mathcal{C},i}(\pi, \mathbf{w}))$;

  Define $p^{\xi}_i(\pi, \mathbf{w}) := \xi_i(\pi, \mathbf{w}) - v_i(\pi)$;

The Mechanism…

Input:	An allocation π for $\langle \mathcal{A}, G, \omega \rangle$, and a vector $\mathbf{w} \in \mathbf{D}$;
Assumption:	A verifier $\mathbf{v}$ is available. Let $\mathbf{v}(\pi) = (v_1, ..., v_n)$;

1. Let $\mathbb{C}$ denote the set of all possible subsets of $\mathcal{A}$;
2. For each set $\mathcal{C} \in \mathbb{C}$,
3. $\lfloor$ Compute an optimal allocation $\pi_\mathcal{C}$ for $\langle \mathcal{C}, \mathtt{img}(\pi), \omega \rangle$ w.r.t. $\mathbf{w}$;
4. For each agent $i \in \mathcal{A}$,
5. $|$ For each set $\mathcal{C} \in \mathbb{C}$,
6. $|$ $|$ Let $\Delta^1_{\mathcal{C},i}(\pi, \mathbf{w}) := \mathtt{val}(\pi_\mathcal{C}, (v_i, \mathbf{w}_{-i}))$; $\quad (= v_i(\pi_\mathcal{C}) + \sum_{j \in \mathcal{C} \setminus \{i\}} w_j(\pi_\mathcal{C}))$;
7. $|$ $\lfloor$ Let $\Delta^2_{\mathcal{C},i}(\pi, \mathbf{w}) := \mathtt{val}(\pi_{\mathcal{C} \setminus \{i\}}, \mathbf{w})$; $\quad (= \sum_{j \in \mathcal{C} \setminus \{i\}} w_j(\pi_{\mathcal{C} \setminus \{i\}}))$;
8. $|$ Let $\xi_i(\pi, \mathbf{w}) := \sum_{\mathcal{C} \in \mathbb{C}} \frac{(|\mathcal{A}| - |\mathcal{C}|)!(|\mathcal{C}| - 1)!}{|\mathcal{A}|!} (\Delta^1_{\mathcal{C},i}(\pi, \mathbf{w}) - \Delta^2_{\mathcal{C},i}(\pi, \mathbf{w}))$;
9. $\lfloor$ Define $p^\xi_i(\pi, \mathbf{w}) := \xi_i(\pi, \mathbf{w}) - v_i(\pi)$;

Allocated goods are considered only

The Mechanism…

Input: An allocation π for $\langle \mathcal{A}, G, \omega \rangle$, and a vector $\mathbf{w} \in \mathbf{D}$;
Assumption: A verifier $\mathbf{v}$ is available. Let $\mathbf{v}(\pi) = (v_1, ..., v_n)$;

Let $\mathbb{C}$ denote the set of all possible subsets of $\mathcal{A}$;
For each set $\mathcal{C} \in \mathbb{C}$,
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For each agent $i \in \mathcal{A}$,
  For each set $\mathcal{C} \in \mathbb{C}$,
    Let $\Delta^1_{\mathcal{C},i}(\pi, \mathbf{w}) := \texttt{val}(\pi_{\mathcal{C}}, (v_i, \mathbf{w}_{-i}))$; $(= v_i(\pi_{\mathcal{C}}) + \sum_{j \in \mathcal{C} \setminus \{i\}} w_j(\pi_{\mathcal{C}}))$;
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  Let $\xi_i(\pi, \mathbf{w}) := \sum_{\mathcal{C} \in \mathbb{C}} \frac{(|\mathcal{A}| - |\mathcal{C}|)!(|\mathcal{C}|-1)!}{|\mathcal{A}|!} (\Delta^1_{\mathcal{C},i}(\pi, \mathbf{w}) - \Delta^2_{\mathcal{C},i}(\pi, \mathbf{w}))$;
  Define $p^{\xi}_i(\pi, \mathbf{w}) := \xi_i(\pi, \mathbf{w}) - v_i(\pi)$;

Allocated goods are considered only

By the previous lemma, this is without loss of generality.
In fact, allocated goods are the only ones that we verify.

The Mechanism…

Input: An allocation π for $\langle \mathcal{A}, G, \omega \rangle$, and a vector $\mathbf{w} \in \mathbf{D}$;
Assumption: A verifier $\mathbf{v}$ is available. Let $\mathbf{v}(\pi) = (v_1, ..., v_n)$;

1. Let $\mathbb{C}$ denote the set of all possible subsets of $\mathcal{A}$;
2. For each set $\mathcal{C} \in \mathbb{C}$,
3. Compute an optimal allocation $\pi_{\mathcal{C}}$ for $\langle \mathcal{C}, \texttt{img}(\pi), \omega \rangle$ w.r.t. $\mathbf{w}$;
4. For each agent $i \in \mathcal{A}$,
5. For each set $\mathcal{C} \in \mathbb{C}$,
6. Let $\Delta^1_{\mathcal{C},i}(\pi, \mathbf{w}) := \texttt{val}(\pi_{\mathcal{C}}, (v_i, \mathbf{w}_{-i}))$; $(= v_i(\pi_{\mathcal{C}}) + \sum_{j \in \mathcal{C} \setminus \{i\}} w_j(\pi_{\mathcal{C}}))$;
7. Let $\Delta^2_{\mathcal{C},i}(\pi, \mathbf{w}) := \texttt{val}(\pi_{\mathcal{C} \setminus \{i\}}, \mathbf{w})$; $(= \sum_{j \in \mathcal{C} \setminus \{i\}} w_j(\pi_{\mathcal{C} \setminus \{i\}}))$;
8. Let $\xi_i(\pi, \mathbf{w}) := \sum_{\mathcal{C} \in \mathbb{C}} \frac{(|\mathcal{A}| - |\mathcal{C}|)!(|\mathcal{C}|-1)!}{|\mathcal{A}|!} (\Delta^1_{\mathcal{C},i}(\pi, \mathbf{w}) - \Delta^2_{\mathcal{C},i}(\pi, \mathbf{w}))$;
9. Define $p_i^{\xi}(\pi, \mathbf{w}) := \xi_i(\pi, \mathbf{w}) - v_i(\pi)$;

Allocated goods are considered only

«Bonus and Compensation», by Nisan and Ronen (2001)

The Mechanism…

Input: An allocation π for $\langle \mathcal{A}, G, \omega \rangle$, and a vector $\mathbf{w} \in \mathbf{D}$;
Assumption: A verifier $\mathbf{v}$ is available. Let $\mathbf{v}(\pi) = (v_1, ..., v_n)$;

Let $\mathbb{C}$ denote the set of all possible subsets of $\mathcal{A}$;
For each set $\mathcal{C} \in \mathbb{C}$,
  Compute an optimal allocation $\pi_{\mathcal{C}}$ for $\langle \mathcal{C}, \mathtt{img}(\pi), \omega \rangle$ w.r.t. $\mathbf{w}$;
For each agent $i \in \mathcal{A}$,
  For each set $\mathcal{C} \in \mathbb{C}$,
    Let $\Delta^1_{\mathcal{C},i}(\pi, \mathbf{w}) := \mathtt{val}(\pi_{\mathcal{C}}, (v_i, \mathbf{w}_{-i}))$; $(= v_i(\pi_{\mathcal{C}}) + \sum_{j \in \mathcal{C} \setminus \{i\}} w_j(\pi_{\mathcal{C}}))$;
    Let $\Delta^2_{\mathcal{C},i}(\pi, \mathbf{w}) := \mathtt{val}(\pi_{\mathcal{C} \setminus \{i\}}, \mathbf{w})$; $(= \sum_{j \in \mathcal{C} \setminus \{i\}} w_j(\pi_{\mathcal{C} \setminus \{i\}}))$;
  Let $\xi_i(\pi, \mathbf{w}) := \sum_{\mathcal{C} \in \mathbb{C}} \frac{(|\mathcal{A}| - |\mathcal{C}|)!(|\mathcal{C}|-1)!}{|\mathcal{A}|!} (\Delta^1_{\mathcal{C},i}(\pi, \mathbf{w}) - \Delta^2_{\mathcal{C},i}(\pi, \mathbf{w}))$;
  Define $p^{\xi}_i(\pi, \mathbf{w}) := \xi_i(\pi, \mathbf{w}) - v_i(\pi)$;

Allocated goods are considered only

«Bonus and Compensation»,
by Nisan and Ronen (2001)

No punishments!

The Mechanism…

Input: An allocation π for $\langle \mathcal{A}, G, \omega \rangle$, and a vector $\mathbf{w} \in \mathbf{D}$;

Assumption: A verifier $\mathbf{v}$ is available. Let $\mathbf{v}(\pi) = (v_1, \ldots, v_n)$;

```
Let ℂ denote the set of all possible subsets of 𝒜;
For each set 𝒞 ∈ ℂ,
⌊   Compute an optimal allocation π_𝒞 for ⟨𝒞, img(π), ω⟩ w.r.t. w;
For each agent i ∈ 𝒜,
|   For each set 𝒞 ∈ ℂ,
|   |   Let Δ¹_{𝒞,i}(π, w) := val(π_𝒞, (v_i, w_{−i}));      (= v_i(π_𝒞) + Σ_{j∈𝒞∖{i}} w_j(π_𝒞));
|   ⌊   Let Δ²_{𝒞,i}(π, w) := val(π_{𝒞∖{i}}, w);      (= Σ_{j∈𝒞∖{i}} w_j(π_{𝒞∖{i}}));
|   Let ξ_i(π, w) := Σ_{𝒞∈ℂ} ((|𝒜|−|𝒞|)!(|𝒞|−1)! / |𝒜|!) (Δ¹_{𝒞,i}(π, w) − Δ²_{𝒞,i}(π, w));
⌊   Define p^ξ_i(π, w) := ξ_i(π, w) − v_i(π);
```

Line 6: $\Delta^1_{\mathcal{C},i}(\pi,\mathbf{w}) := \mathtt{val}(\pi_{\mathcal{C}}, (v_i, \mathbf{w}_{-i}))$; $\quad (= v_i(\pi_{\mathcal{C}}) + \sum_{j \in \mathcal{C}\setminus\{i\}} w_j(\pi_{\mathcal{C}}))$;

Line 7: $\Delta^2_{\mathcal{C},i}(\pi,\mathbf{w}) := \mathtt{val}(\pi_{\mathcal{C}\setminus\{i\}}, \mathbf{w})$; $\quad (= \sum_{j \in \mathcal{C}\setminus\{i\}} w_j(\pi_{\mathcal{C}\setminus\{i\}}))$;

Line 8: $\xi_i(\pi,\mathbf{w}) := \sum_{\mathcal{C} \in \mathbb{C}} \frac{(|\mathcal{A}|-|\mathcal{C}|)!(|\mathcal{C}|-1)!}{|\mathcal{A}|!} (\Delta^1_{\mathcal{C},i}(\pi,\mathbf{w}) - \Delta^2_{\mathcal{C},i}(\pi,\mathbf{w}))$;

Line 9: $p^{\xi}_i(\pi,\mathbf{w}) := \xi_i(\pi,\mathbf{w}) - v_i(\pi)$;

Allocated goods are considered only

«Bonus and Compensation», by Nisan and Ronen (2001)

❖ **Truth-telling is a dominant strategy for each agent**

The Mechanism…

Input: An allocation π for $\langle \mathcal{A}, G, \omega \rangle$, and a vector $\mathbf{w} \in \mathbf{D}$;
Assumption: A verifier $\mathbf{v}$ is available. Let $\mathbf{v}(\pi) = (v_1, ..., v_n)$;

1. Let $\mathbb{C}$ denote the set of all possible subsets of $\mathcal{A}$;
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3. Compute an optimal allocation $\pi_{\mathcal{C}}$ for $\langle \mathcal{C}, \mathtt{img}(\pi), \omega \rangle$ w.r.t. $\mathbf{w}$;
4. For each agent $i \in \mathcal{A}$,
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6. Let $\Delta^1_{\mathcal{C},i}(\pi, \mathbf{w}) := \mathtt{val}(\pi_{\mathcal{C}}, (v_i, \mathbf{w}_{-i}))$; $\quad (= v_i(\pi_{\mathcal{C}}) + \sum_{j \in \mathcal{C} \setminus \{i\}} w_j(\pi_{\mathcal{C}}))$;
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Allocated goods are considered only

«Bonus and Compensation», by Nisan and Ronen (2001)

Does not depend on *i*

Is maximized when the declared type coincides with the verified one

❖ **Truth-telling is a dominant strategy for each agent**

The Mechanism…

Input: An allocation π for $\langle \mathcal{A}, G, \omega \rangle$, and a vector $\mathbf{w} \in \mathbf{D}$;
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Let $\mathbb{C}$ denote the set of all possible subsets of $\mathcal{A}$;
For each set $\mathcal{C} \in \mathbb{C}$,
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Coalitional Games

- Players form *coalitions*
- Each coalition is associated with a *worth*
- A *total worth* has to be distributed

$$\mathcal{G} = \langle N, \varphi \rangle,\ \varphi : 2^N \mapsto \mathbb{R}$$

- **Solution Concepts** characterize outcomes in terms of
  - Fairness
  - Stability

Coalitional Games: Shapley Value

$$\phi_i(\mathcal{G}) = \sum_{C \subseteq N} \frac{(|N| - |C|)!(|C| - 1)!}{|N|!}(\varphi(C) - \varphi(C \setminus \{i\}))$$

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Relevant Properties of the Shapley Value

(I) $\sum_{i \in N} \phi_i(\mathcal{G}) = \varphi(N)$;

(II) If φ is *supermodular* (resp., *submodular*), then $\sum_{i \in R} \phi_i(\mathcal{G}) \geq \varphi(R)$ (resp., $\sum_{i \in R} \phi_i(\mathcal{G}) \leq \varphi(R)$), for each coalition $R \subseteq N$.

(III) If $\mathcal{G}' = \langle N, \varphi' \rangle$ is a game such that $\varphi'(R) \geq \varphi(R)$, for each $R \subseteq N$, then $\phi_i(\mathcal{G}') \geq \phi_i(\mathcal{G})$, for each agent $i \in N$.

Core Allocation

$$\varphi(R \cup T) + \varphi(R \cap T) \geq \varphi(R) + \varphi(T) \text{ (resp., } \varphi(R \cup T) + \varphi(R \cap T) \leq \varphi(R) + \varphi(T))$$

The Mechanism

$$\mathcal{G} = \langle N, \varphi \rangle,\ \varphi : 2^N \mapsto \mathbb{R}$$

- $\varphi(C)$ is the *contribution* of the coalition **w.r.t.** $\left\{ \begin{array}{l} \textbf{selected products} \\ \qquad \textit{and} \\ \textbf{verified values} \end{array} \right.$

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⬇

Best possible allocation,
assuming that agents in C are the only ones in the game

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Properties

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The game is supermodular;
so the Shapley value is stable

Further Observations for Fairness

- Let π be an optimal allocation
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$$\mathcal{G} \longleftarrow \pi$$

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As π is optimal, then $\varphi(C)$ is in fact optimal even by considering all possible products as available

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By the monotonicity of the Shapley value, $\phi_i \geq \phi_i'$

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$$\pi \geq \pi'$$

❖ **Optimal allocations are always preferred by ALL agents**
❖ **There is no difference between two different optimal allocations**

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Efficiency  **Fairness**

Outline

- Game Theory
- Mechanism Design
- Mechanisms with Verification
- Mechanisms and Allocation Problems
- **Complexity Analysis**

Complexity Issues

- For many classes of «compact games» (e.g., *graph games*), the Shapley-value can be efficiently calculated
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- #P is the class the class of all functions that can be computed by *counting Turing machines* in polynomial time.
- A counting Turing machine is a standard nondeterministic Turing machine with an auxiliary output device that prints in binary notation the number of accepting computations induced by the input.
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Reduction from the problem of counting the number of perfect matchings in certain bipartite graphs [Valiant, 1979]

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Complexity Issues

- #P-complete
- However…

Probabilistic Computation

- #P-complete
- However…

Fully Polynomial-Time Randomized Approximation Scheme

- Always Efficient and Budget Balanced
- All other properties in expectation (with high probability)

Coupling of the algorithm with a sampling strategy for the coalitions by [Liben-Nowell,Sharp, Wexler, Woods; 2012]

Probabilistic Computation

Input: An allocation π for $\langle \mathcal{A}, G, \omega \rangle$, and a vector $\mathbf{w} \in \mathbf{D}$;
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9. Define $p_i^{\xi}(\pi, \mathbf{w}) := \xi_i(\pi, \mathbf{w}) - v_i(\pi)$;

Use sampling, rather than exaustive search.

Coupling of the algorithm with a sampling strategy for the coalitions by [Liben-Nowell,Sharp, Wexler, Woods; 2012]

Back to Exact Computation: Islands of Tractability

- Can we find classes of instances for «allocation games» over which the Shapley value can be efficiently computed?

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Restrictions [G., Lupia and Scarcello; 2015]

- Utility functions
  - Values taken from specific domains
  - For instance, use *k* values at most

#P-complete, even for k=2

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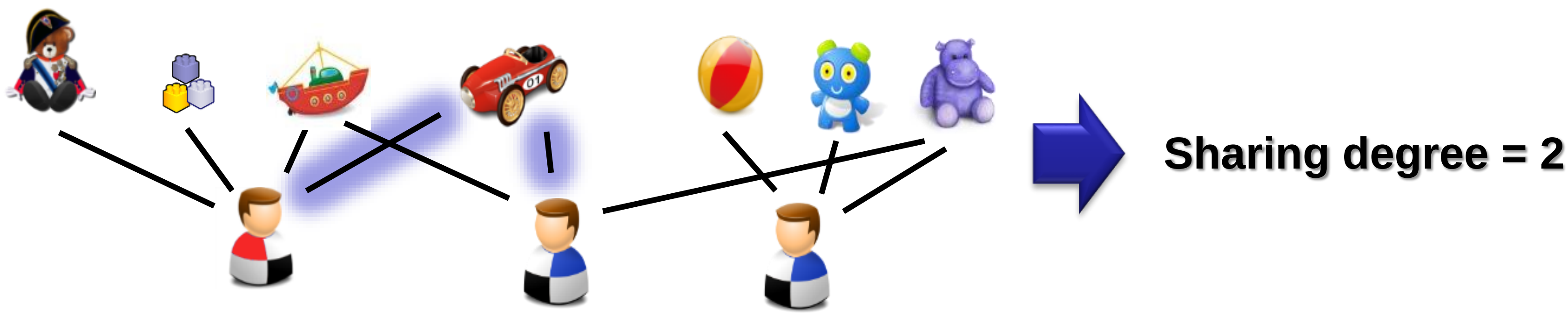
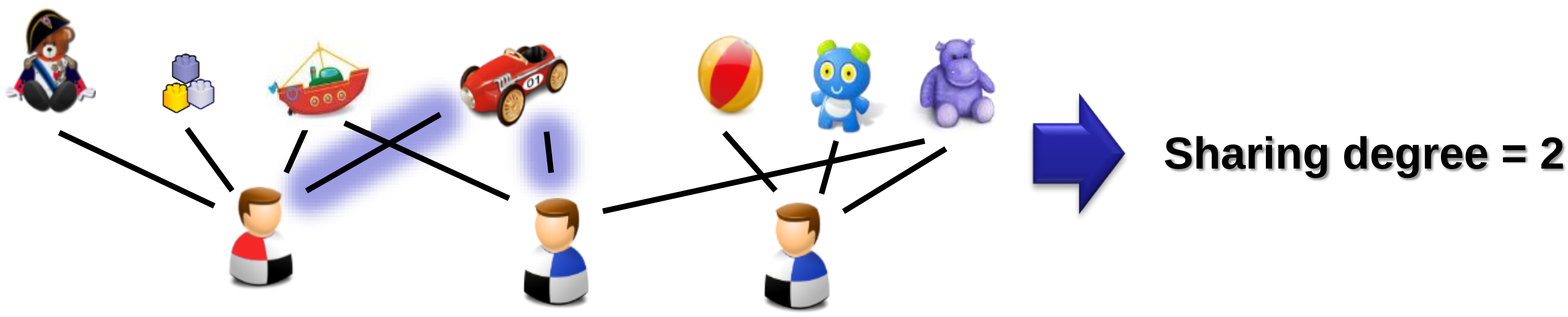
- Structural restrictions…

Bounded Sharing Degree



- Sharing degree
  - Maximum number of agents competing for the same good

Bounded Sharing Degree



- Sharing degree
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The Shapley value can be computed in polynomial time whenever the sharing degree is 2 at most.

Bounded Interactions

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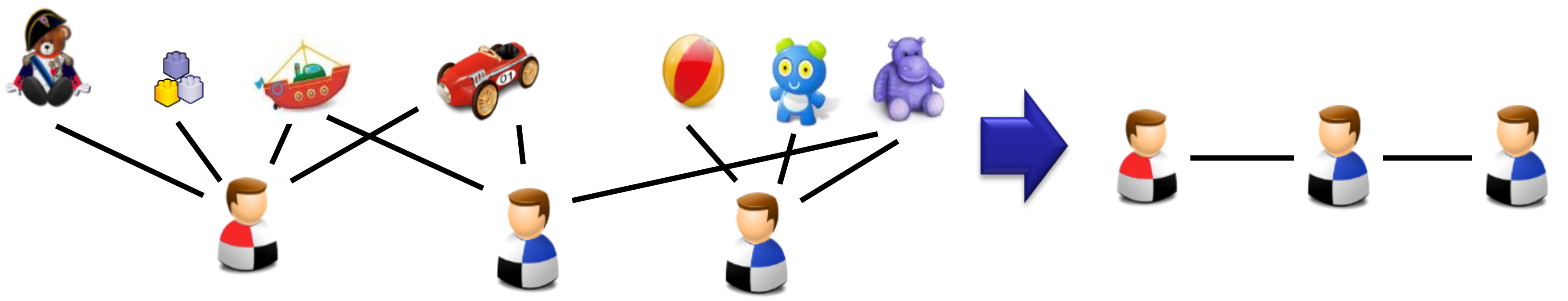
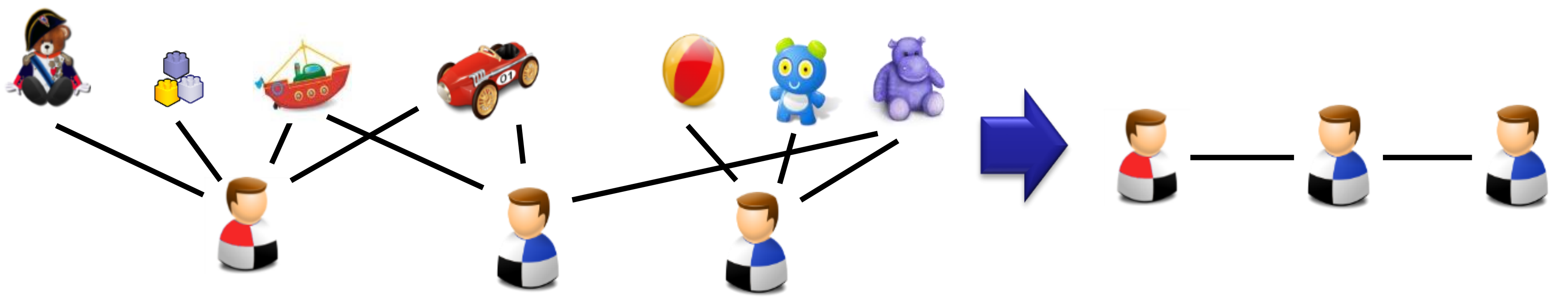


- Interaction graph
  - There is an edge between any pair of agents competing for the same good

Bounded Interactions



- Interaction graph
  - There is an edge between any pair of agents competing for the same good

The Shapley value can be computed in polynomial time whenever the interaction graph is a tree.

or, more generally, if it has bounded treewidth

Application

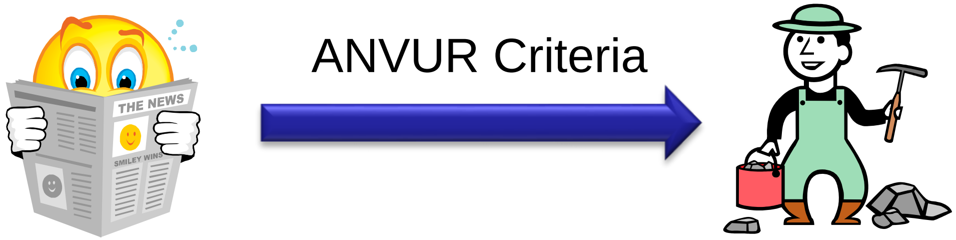
The Italian Research Assessment Program

Case study: Italian Research Assessment Program

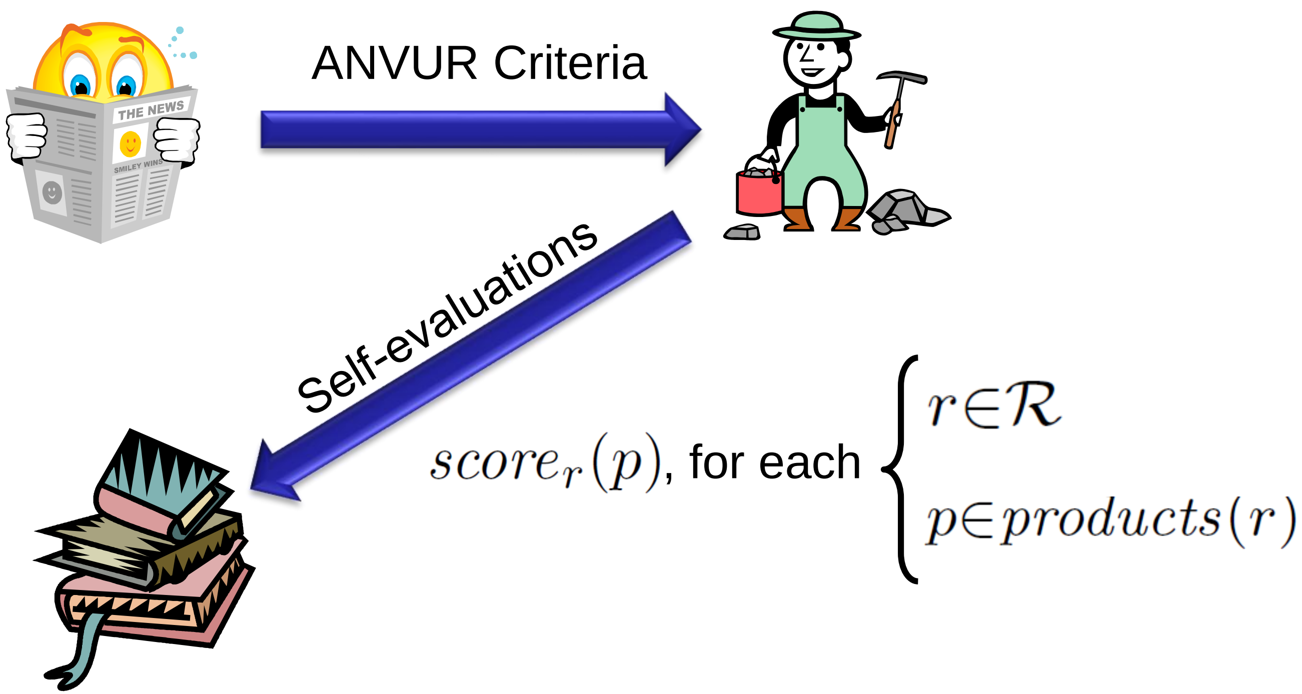
- VQR: ANVUR should evaluate the quality of research of all Italian research structures
- Funds for the structures in the next years depend on the outcome of this evaluation
- Substructures will be also evaluated (departments)

(1) 2004-2010
(2) 2011-2014

ANVUR Evaluation



ANVUR Evaluation



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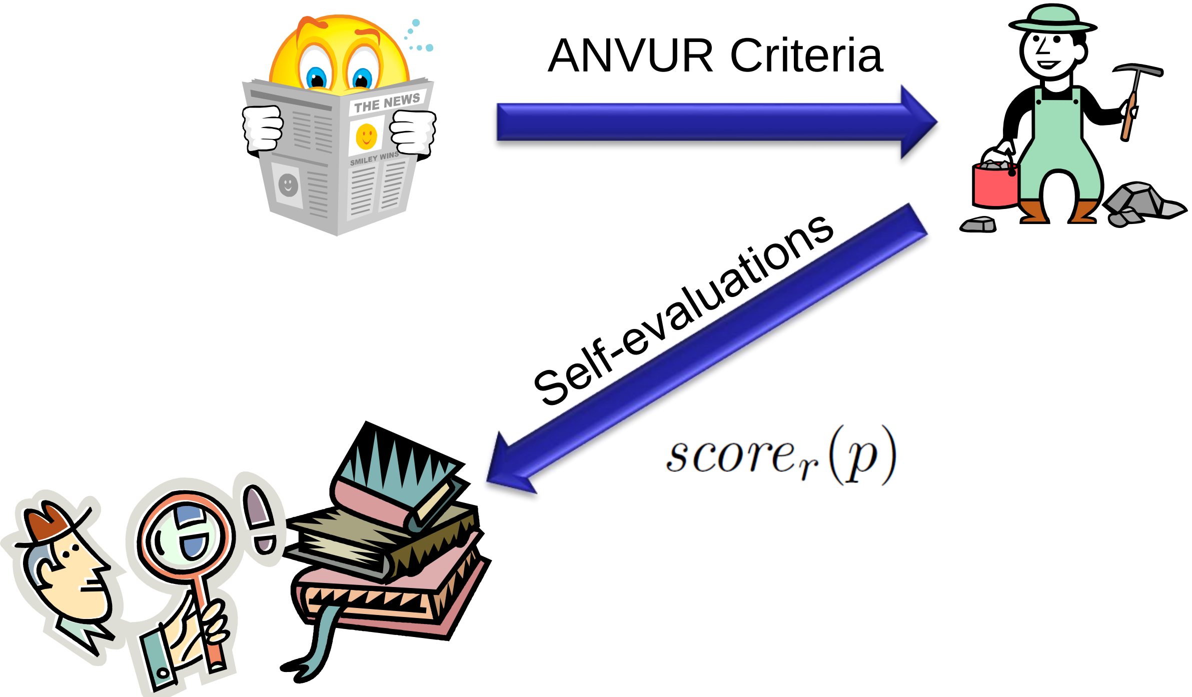
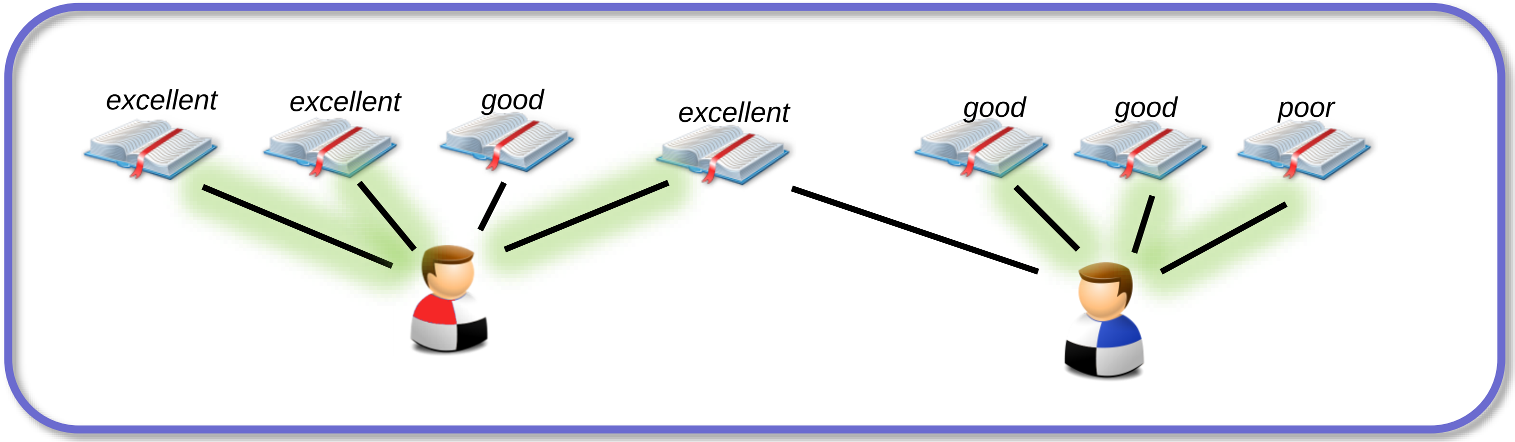


Structures are in charge of selecting the products to submit

Constraints

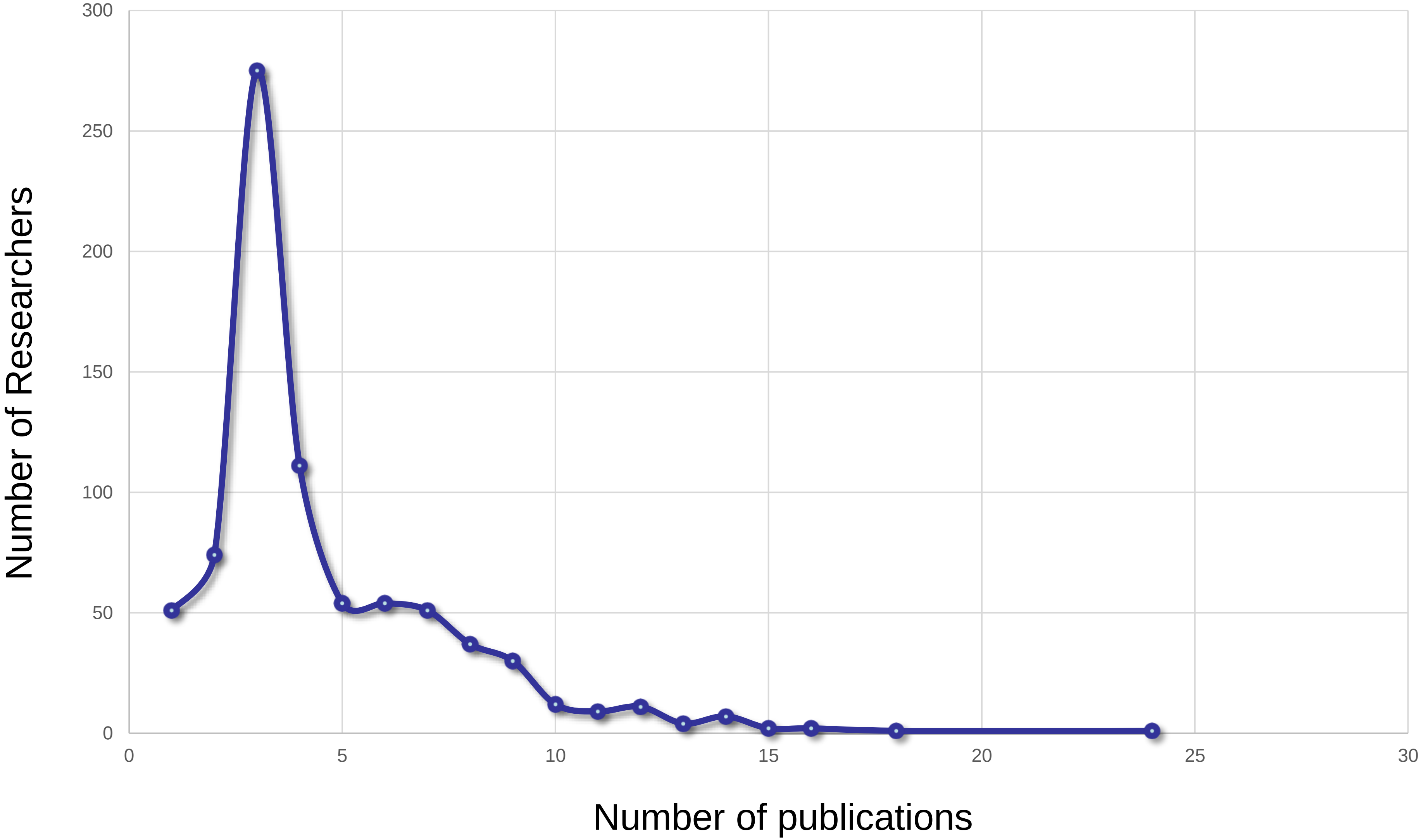
- Every researcher has to submit 3 publications
- A publication cannot be allocated to two researchers

Allocation Problem



(based on **declared** values, i.e., not necessarily true!)

Co-Autorships at University of Calabria



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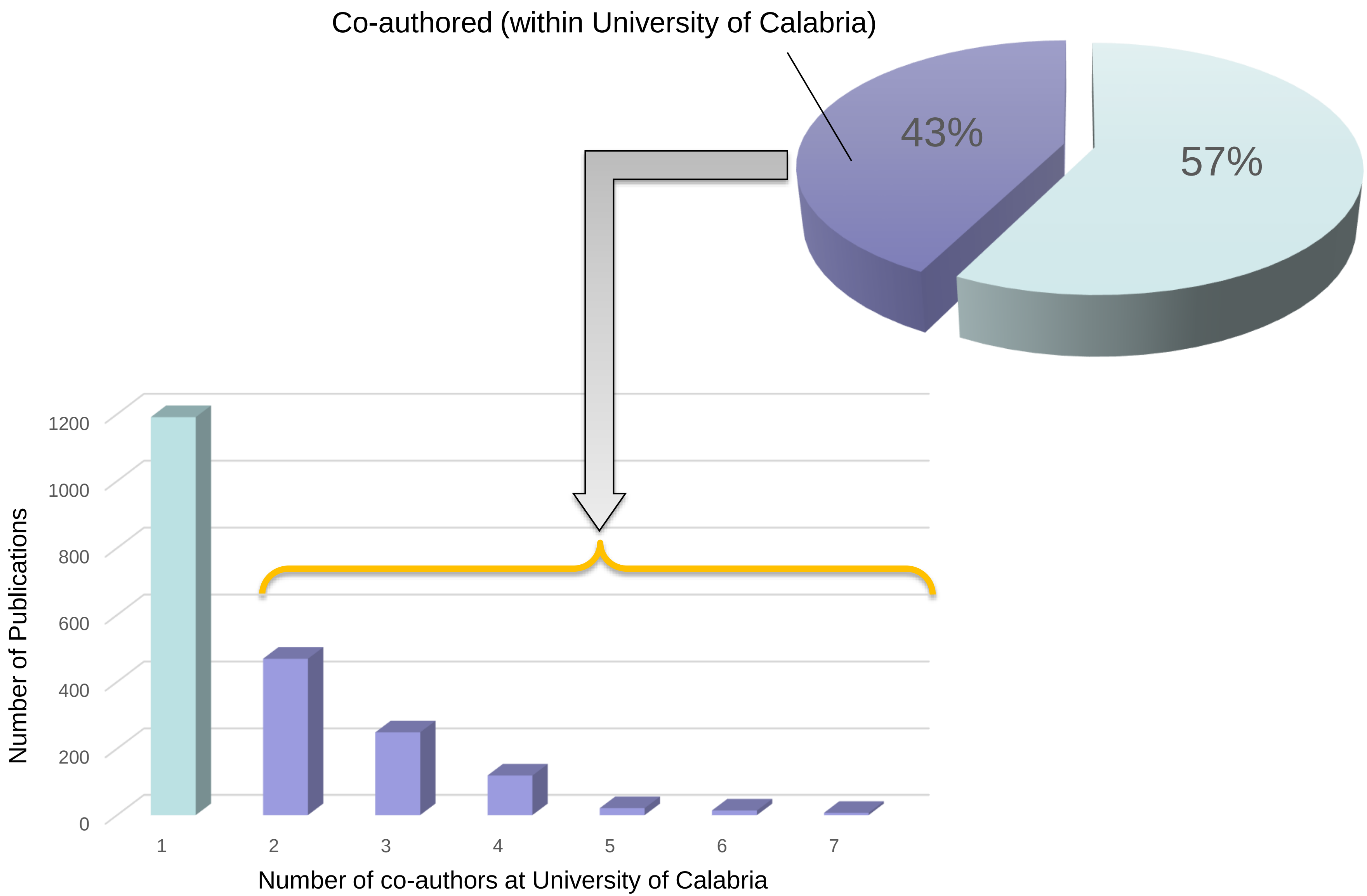

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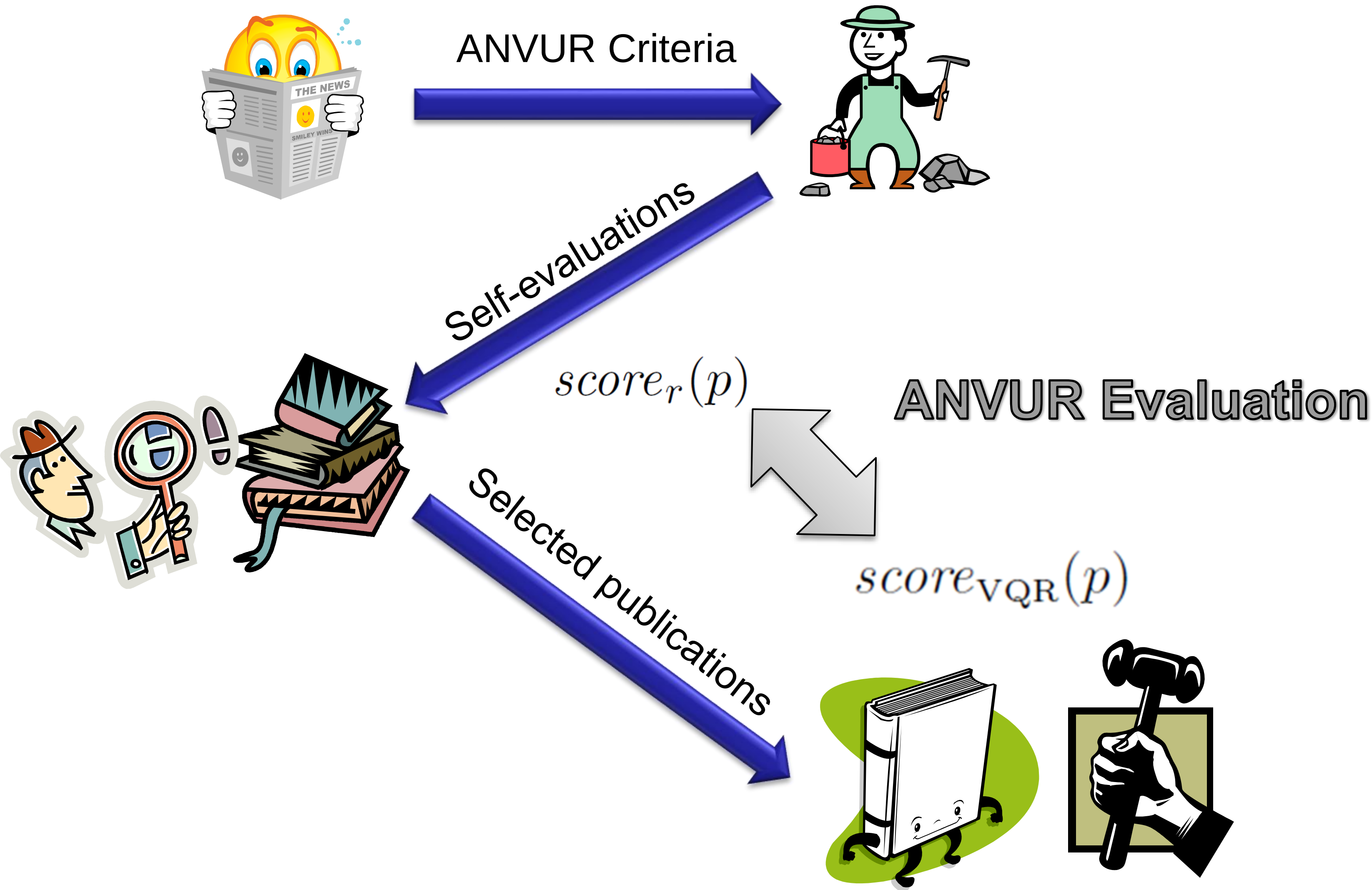

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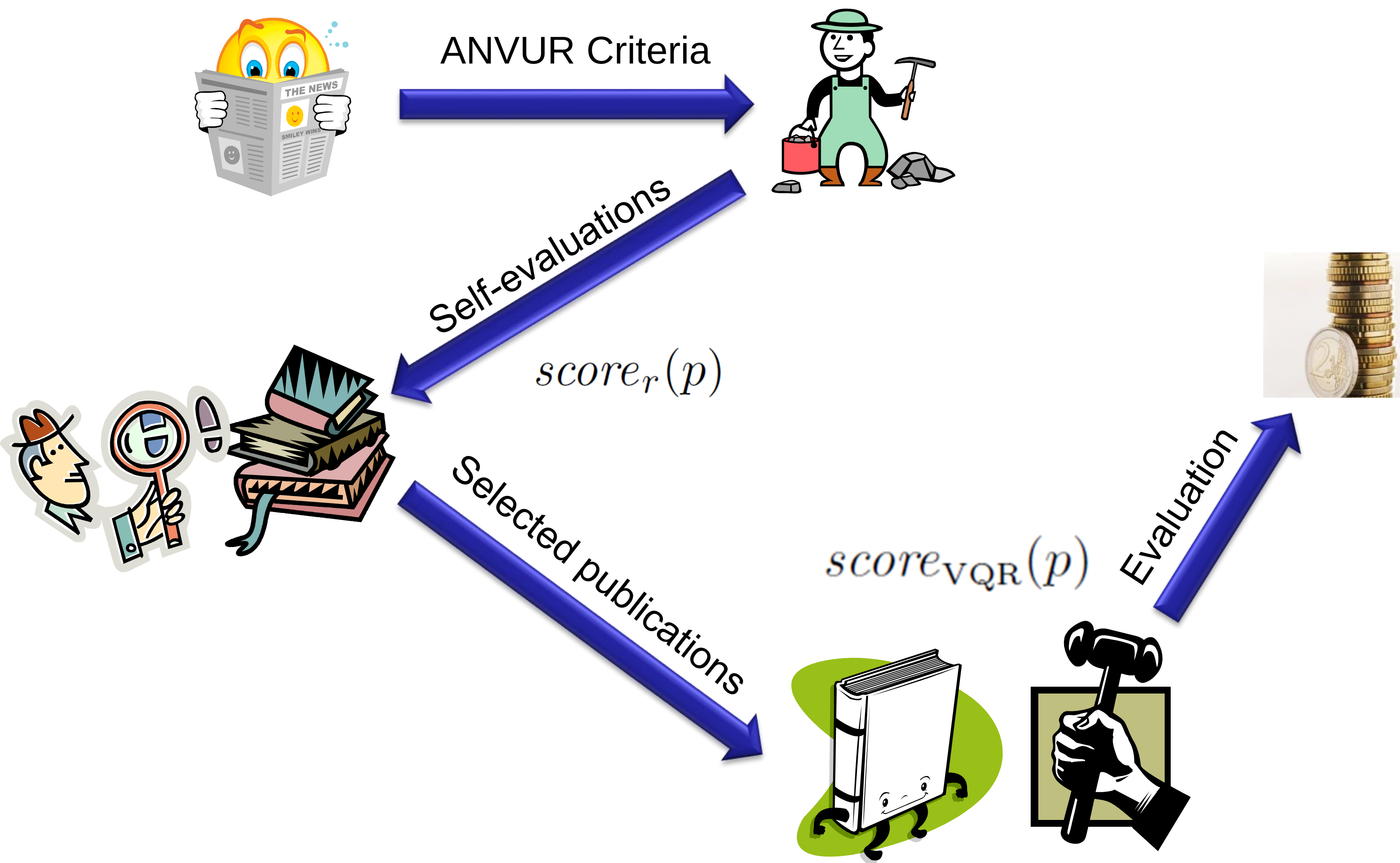

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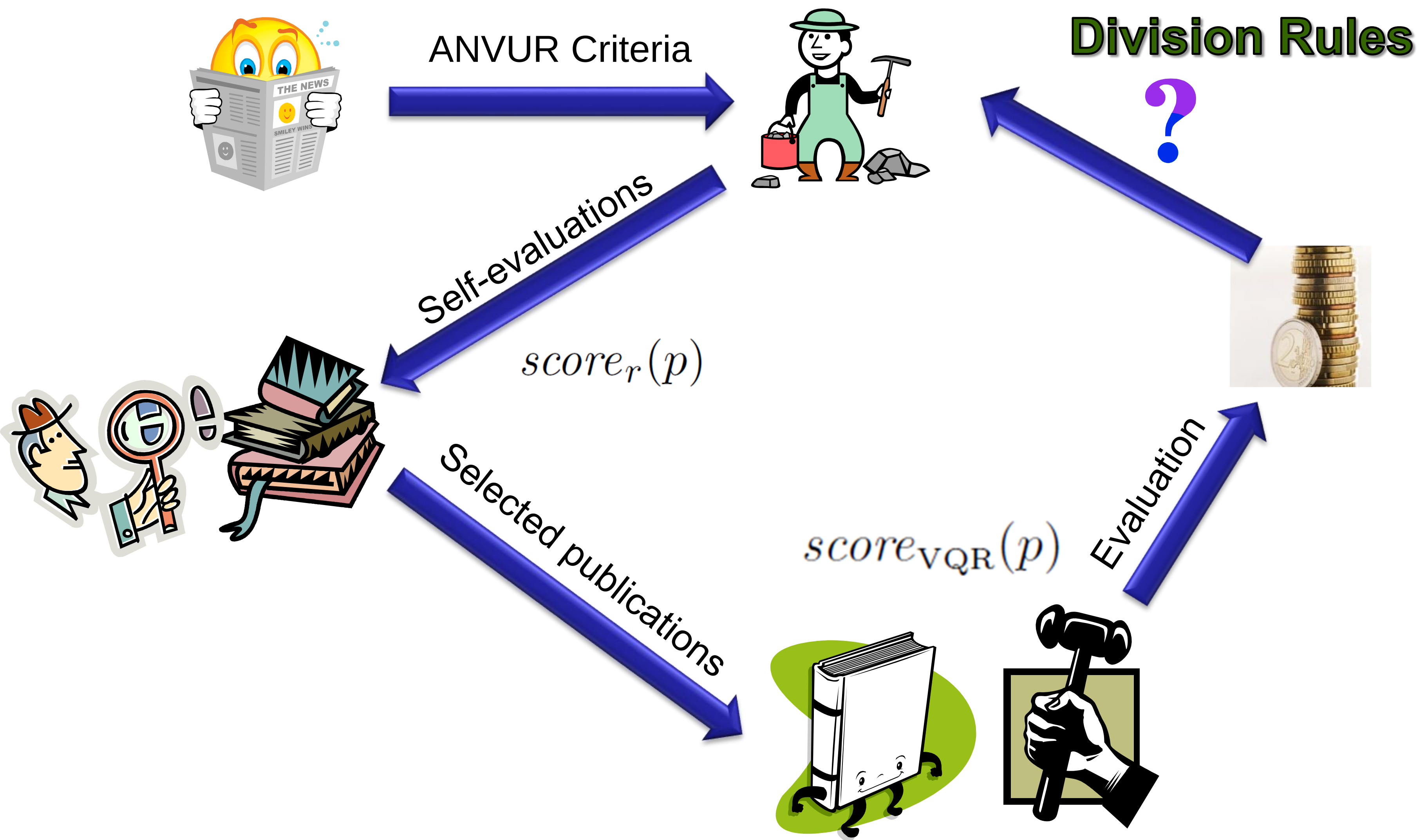

Issues

- Allocation Problem
- Valuations are declared
- The program is meant to evaluate the structures…
  - …but outcomes are used to evaluate researchers, too

Global Evaluation

Individual Evaluations

Desiderata for Division Rules

(P1) "budget-balance": *A division rule* γ *must completely distribute the VQR score of* R *over all its members, i.e.,* $\sum_{r \in \mathcal{R}} \gamma_r(\psi^*) = score_{VQR}(R)$.

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(P3) "implementability": *A division rule* γ *must be indifferent w.r.t. the scores (possibly cheats) declared for unverified products, that is, for products not occurring in the selected allocation.*

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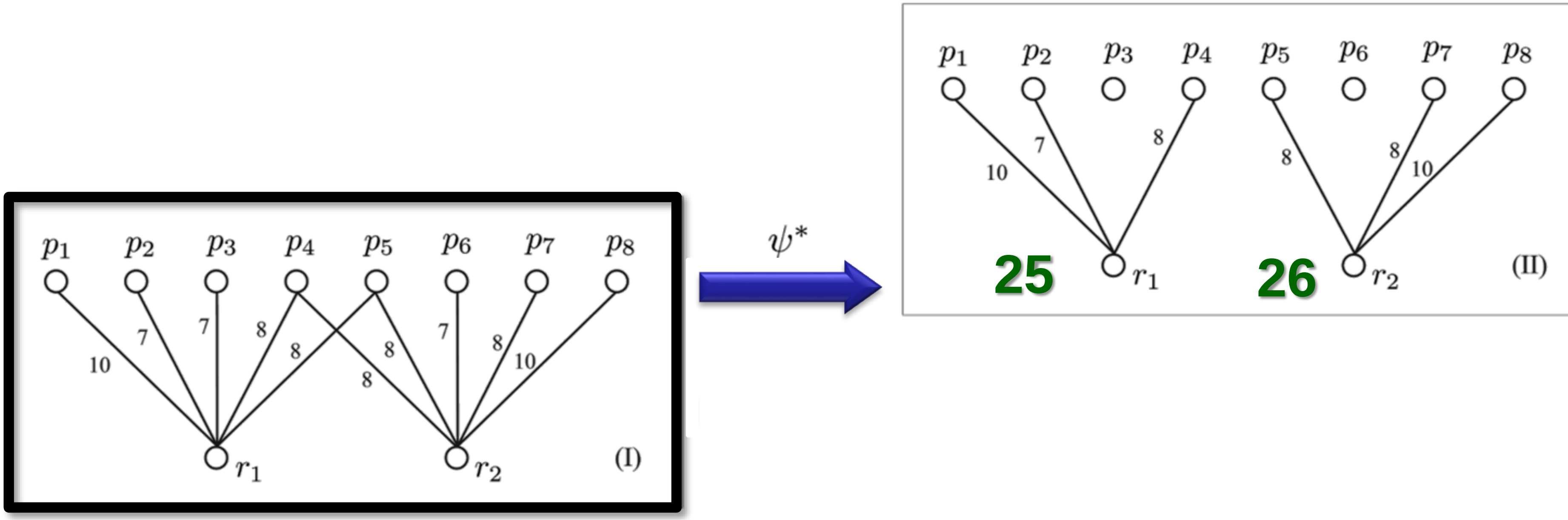
Are they good division rules?

The last one is clearly not implementable, because it depends on publications without any evaluation by ANVUR. What about the others?

Division Rule: `proj`



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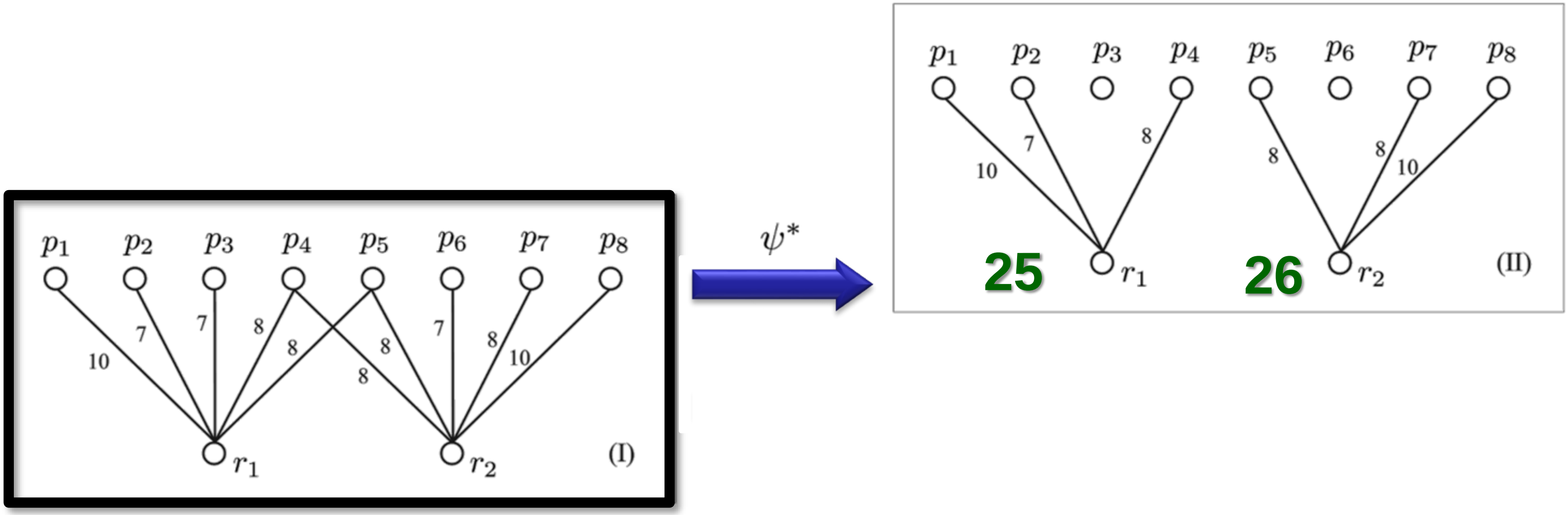

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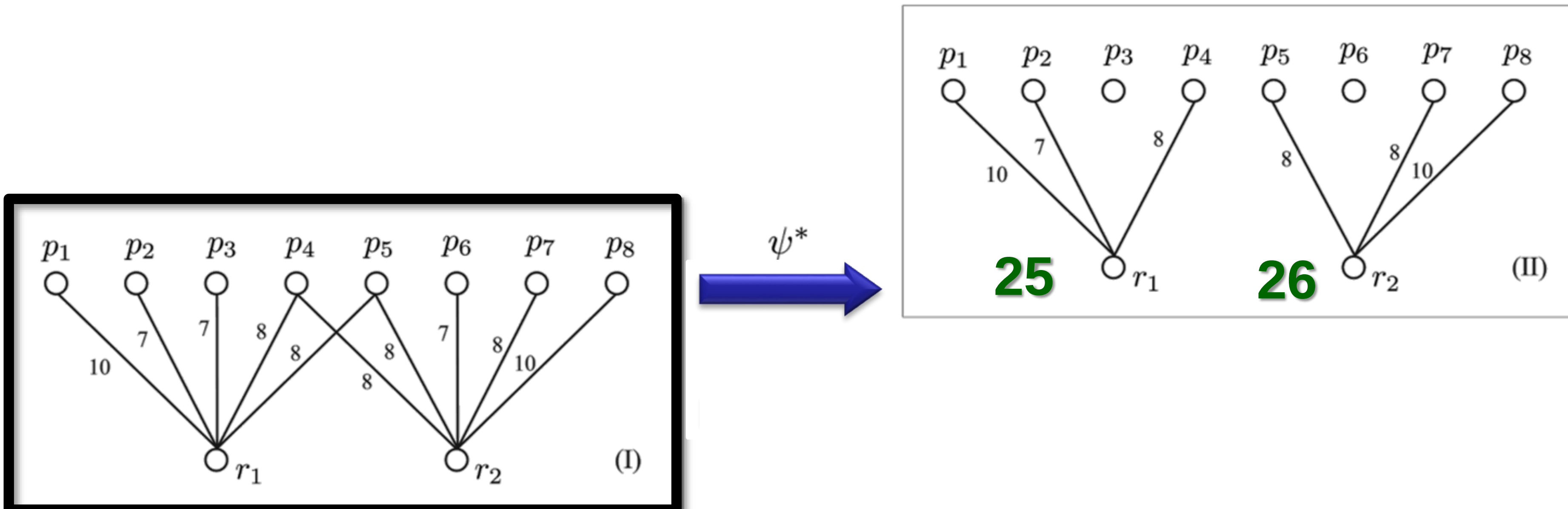

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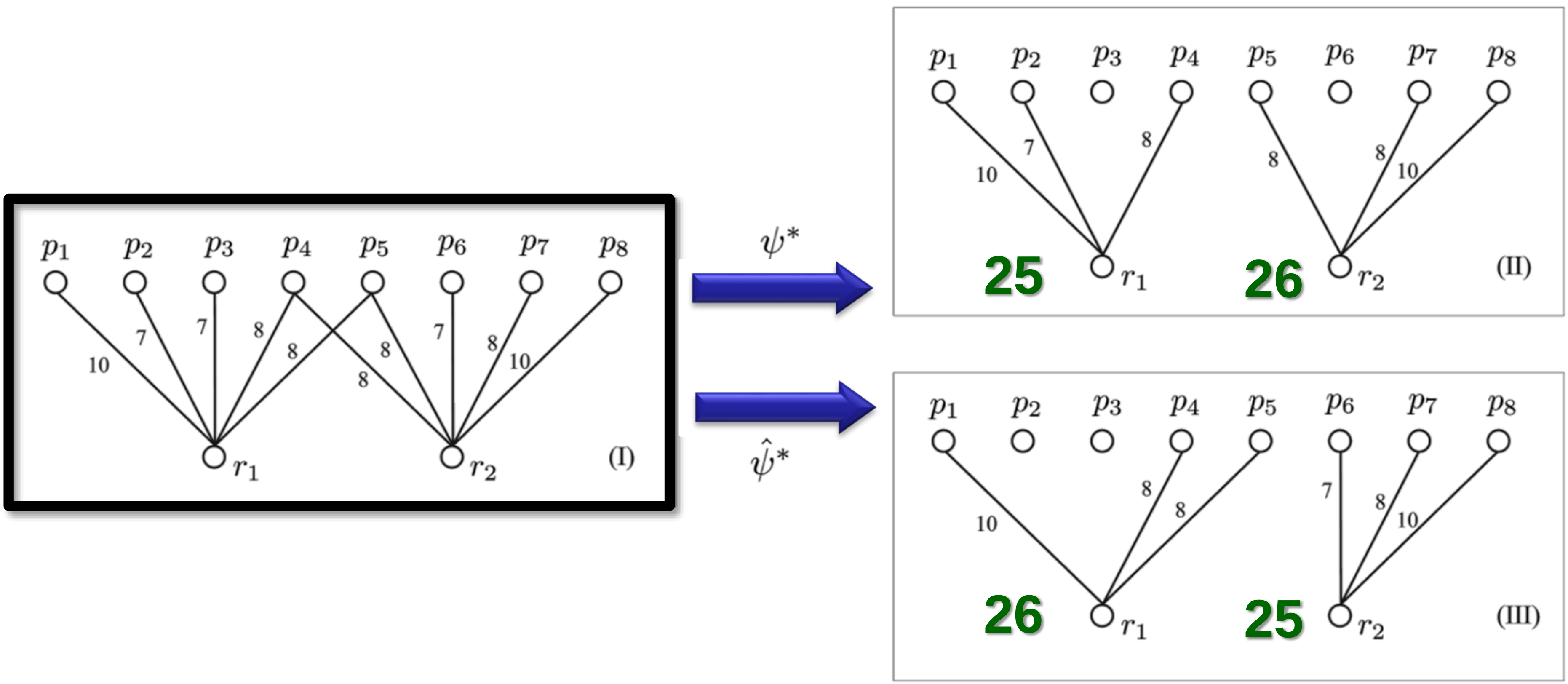

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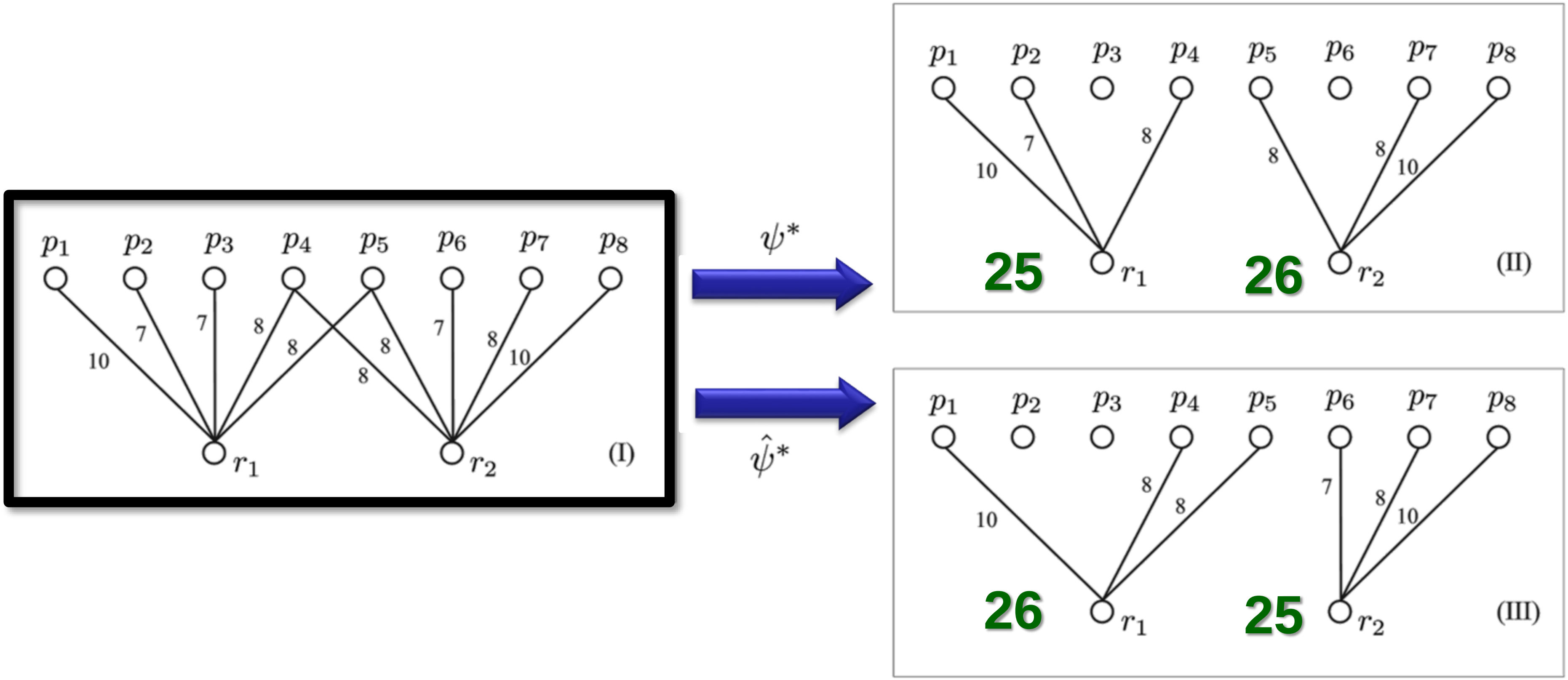

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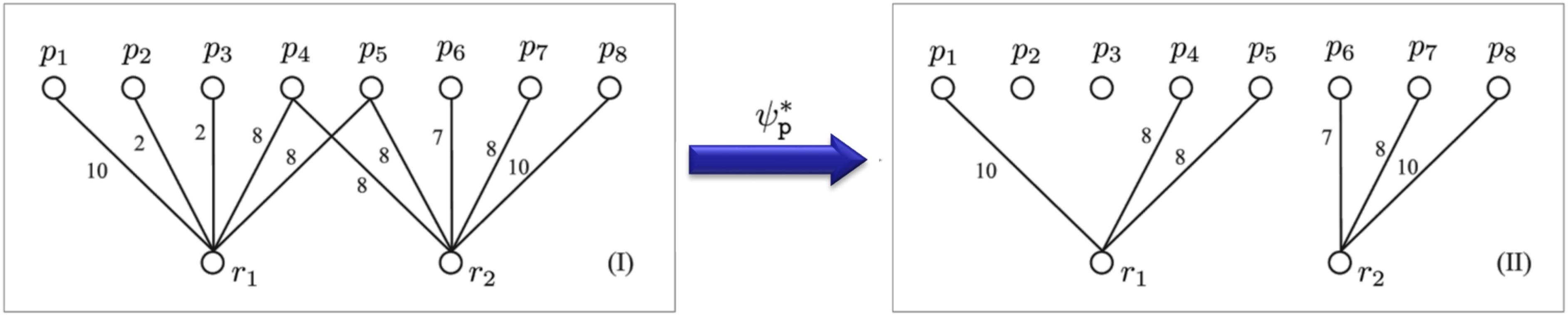
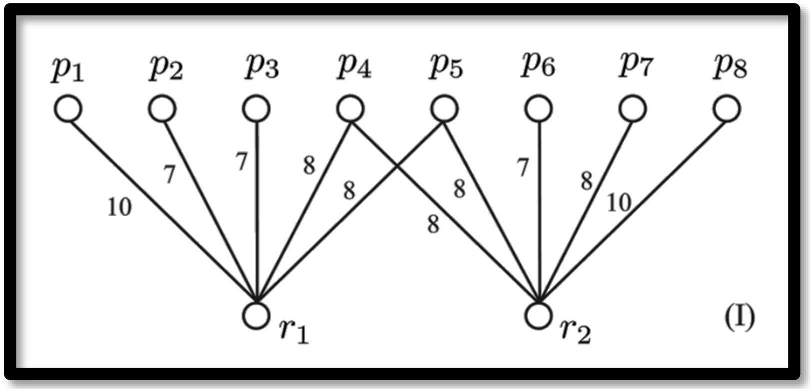
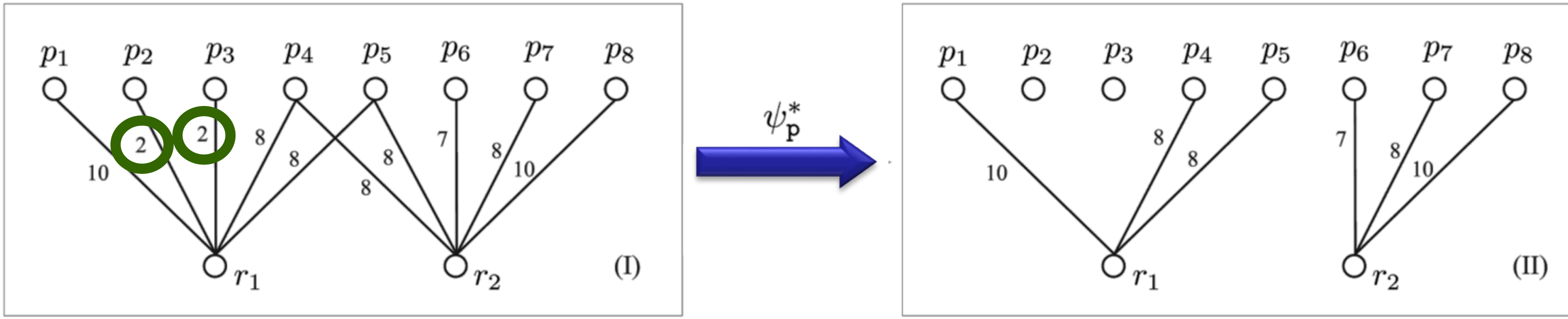
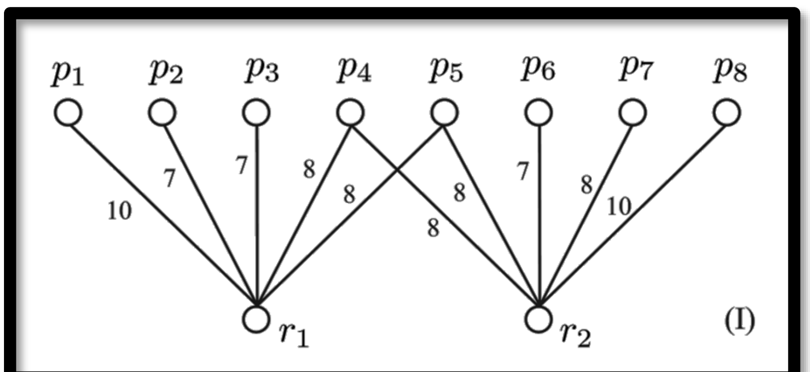
Strategic Manipulations: proj





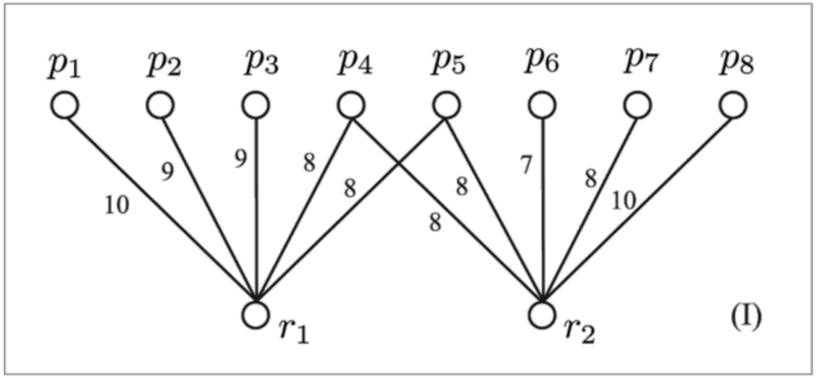
Strategic Manipulations: `proj`

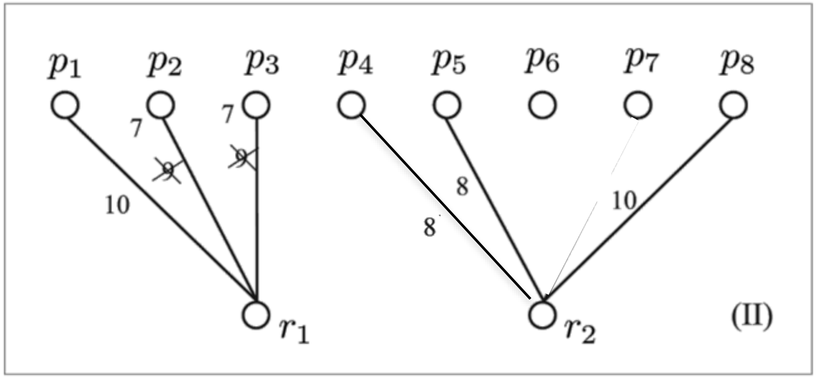


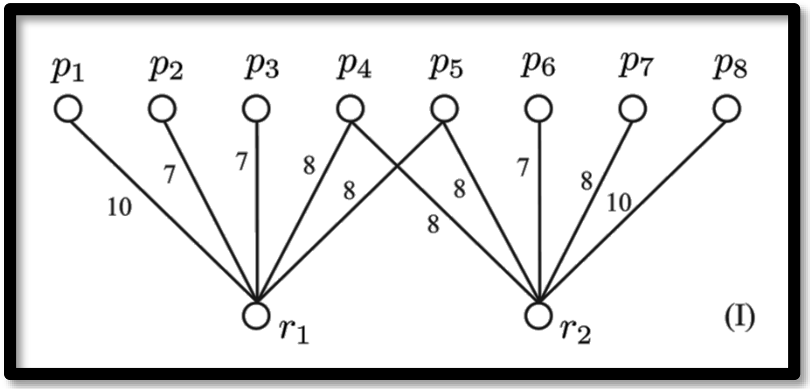


Under- estimation

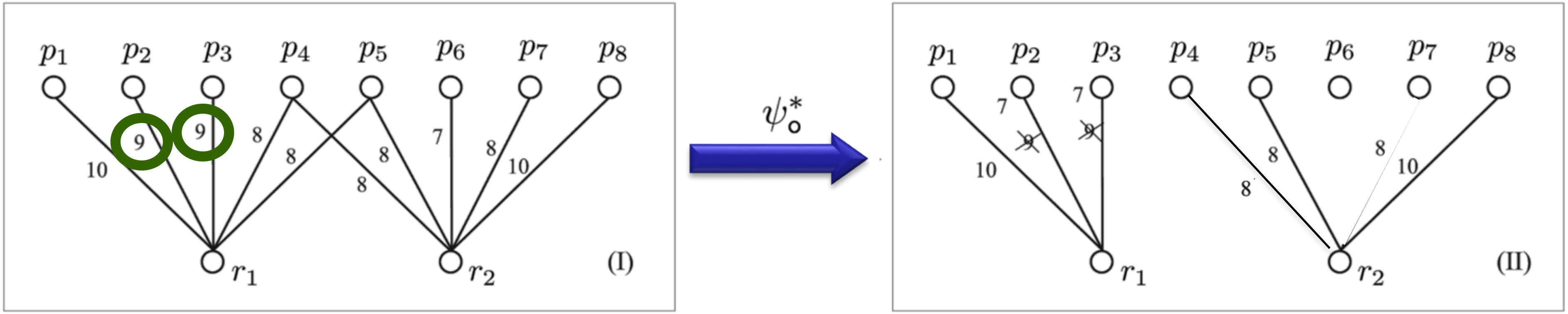
Strategic Manipulations: `owner`

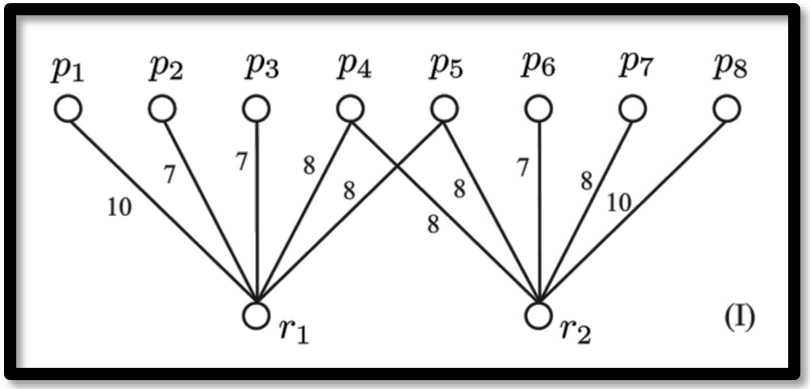




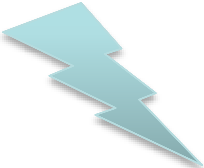


Strategic Manipulations: owner





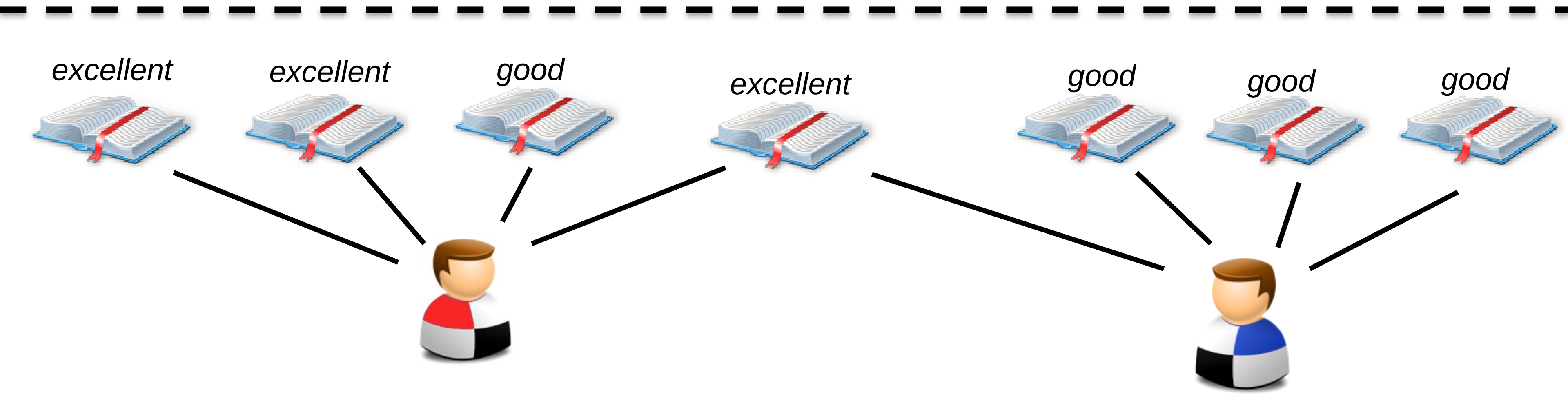
Over- estimation



The optimal solution is missed!!!

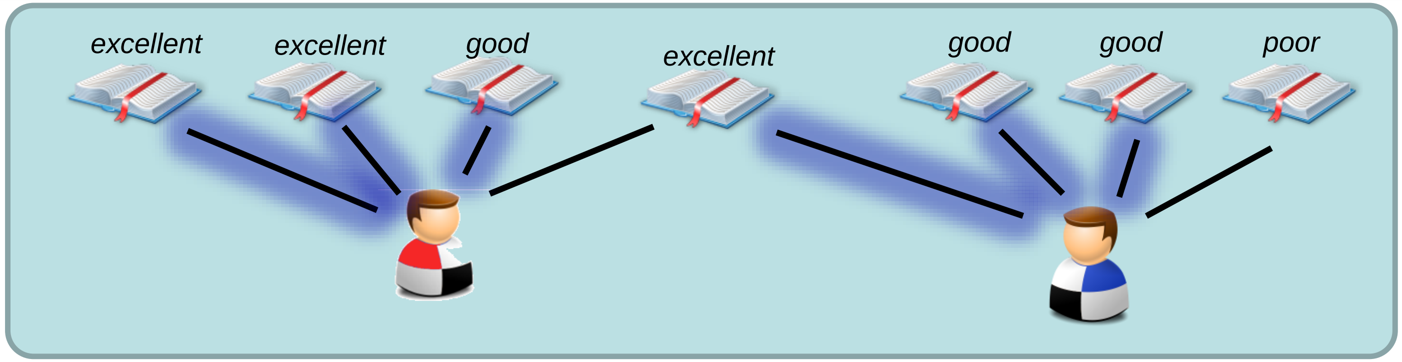
A Closer Look

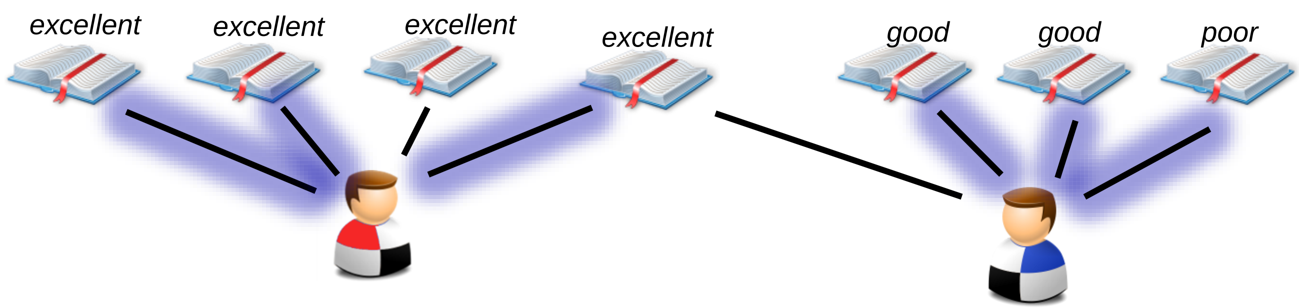
Efficiency VS Fairness

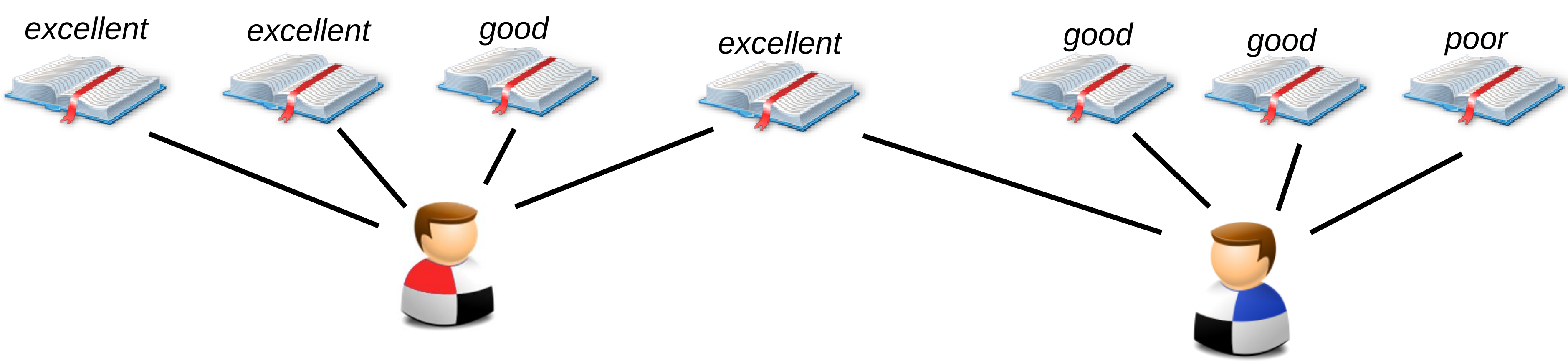


A Closer Look

Optimal Allocation

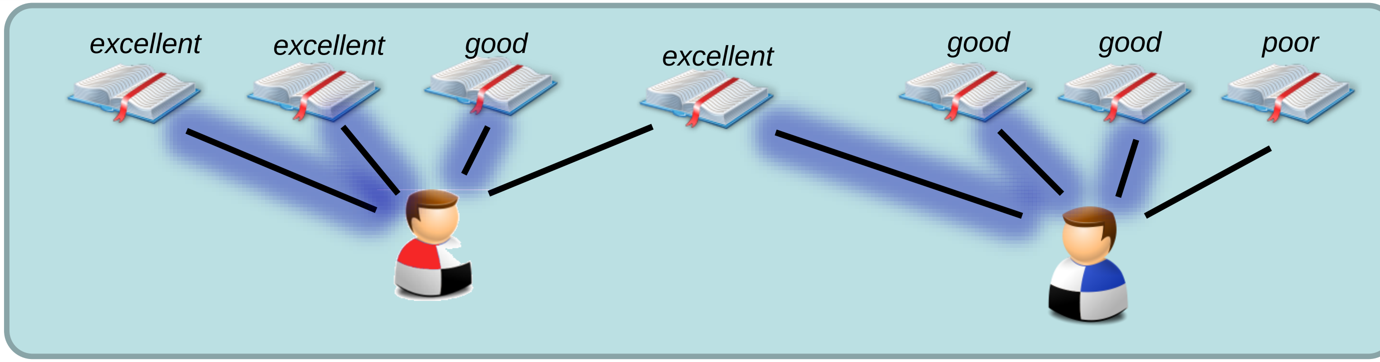


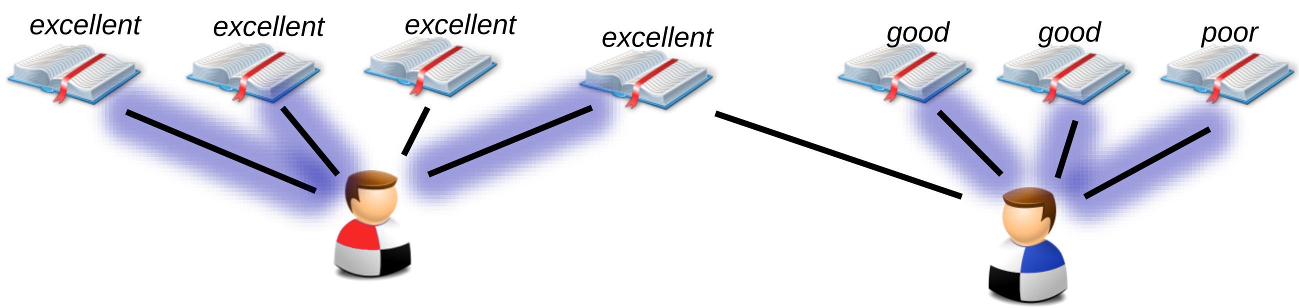




A Closer Look

Optimal Allocation

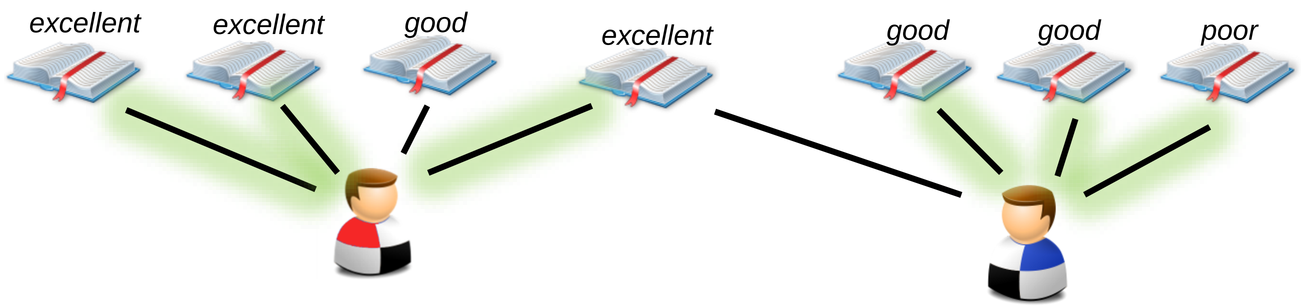




- «Penalizing»  is not fair!
- Unless it is clear that no penalization will occur,  will act «strategically»

The Story….

- ANVUR did not specify a division rule
- Reserchers considered `proj` as «the rule»
- Researchers submitted (rated) only the minimum number of publications required (by default 3), thus implicitly under-estimating all their other products
- To avoid overlapping submissions, «agreements» have been made



- Conflicts resolved «strategically», «hierarchically», …

The optimum has been missed!

No fairness at all!

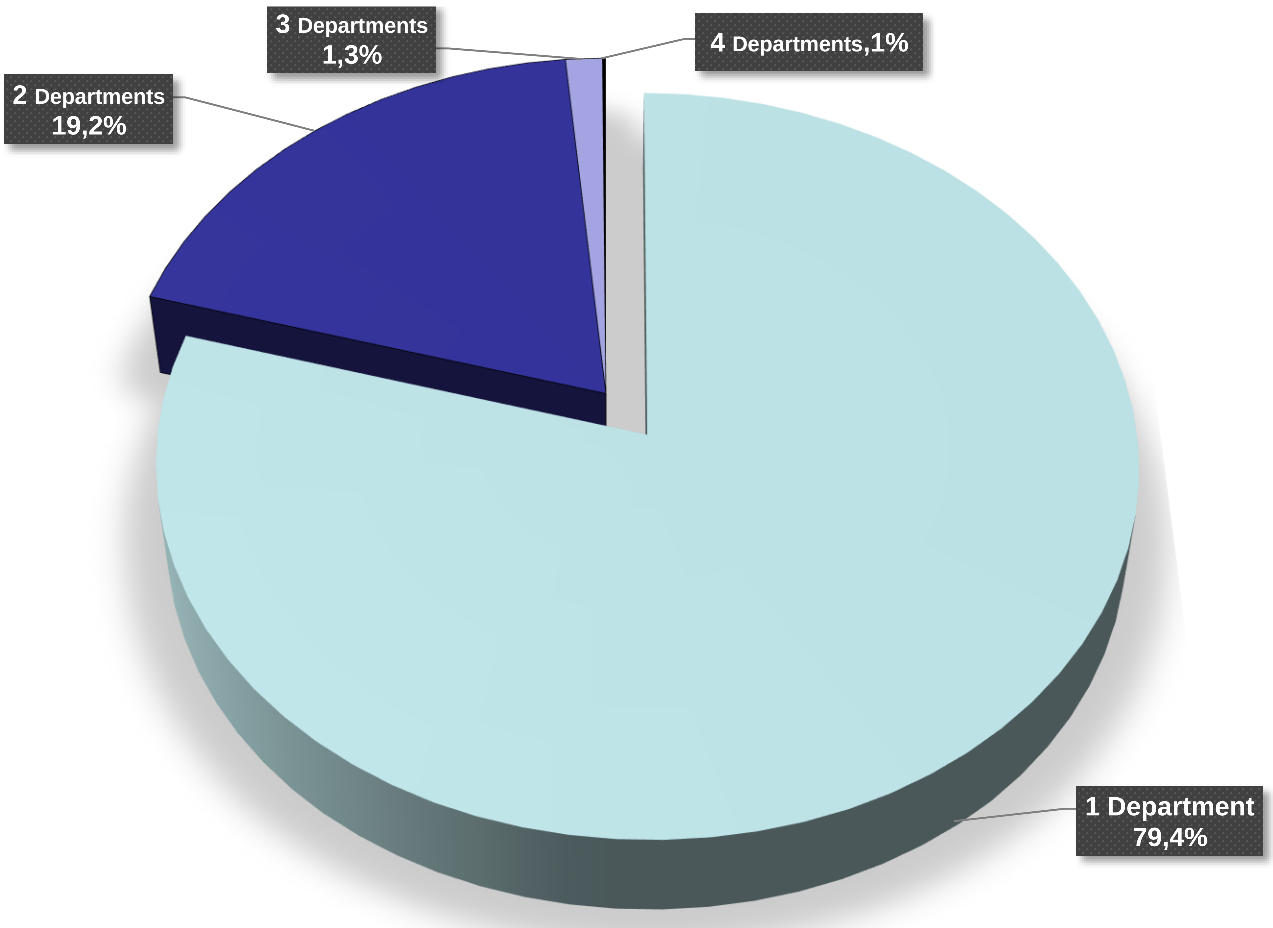
…and the reaction

- ANVUR declared that VQR has not to be used to evaluate researchers, but only structures

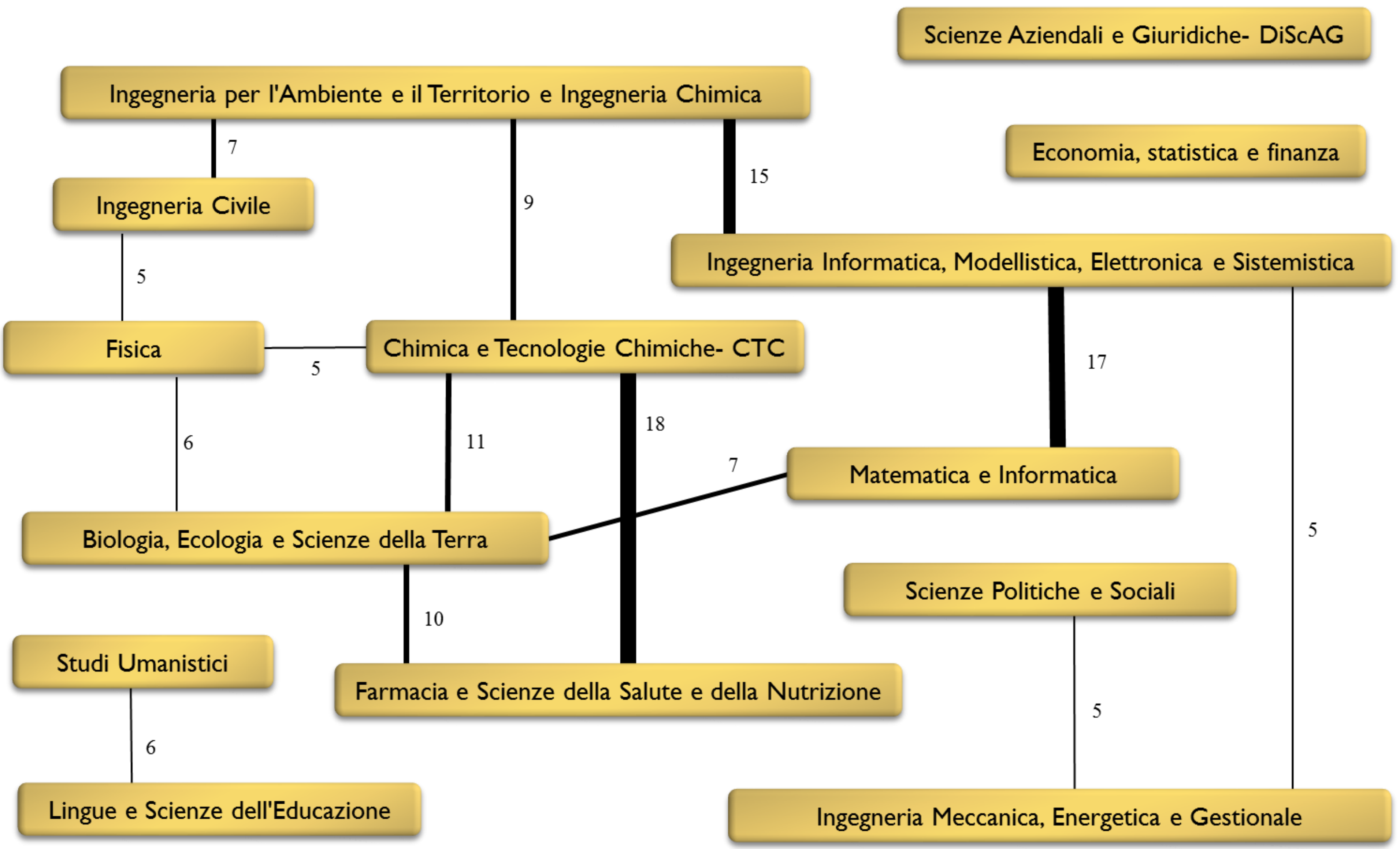
Waste of money… and even just false!

Moreover,
what about Departments?

Distribution at University of Calabria



Distribution at University of Calabria



Our Contribution (G. and Scarcello)

- Define a mechanism satisying the desirable properties
  - In fact, it is essentially the only possible one
  - Mechanism design
  - Coalitional games (Shapley value)

Wow! Let's use it.

Good solution, but we just do not want to evaluate individuals…

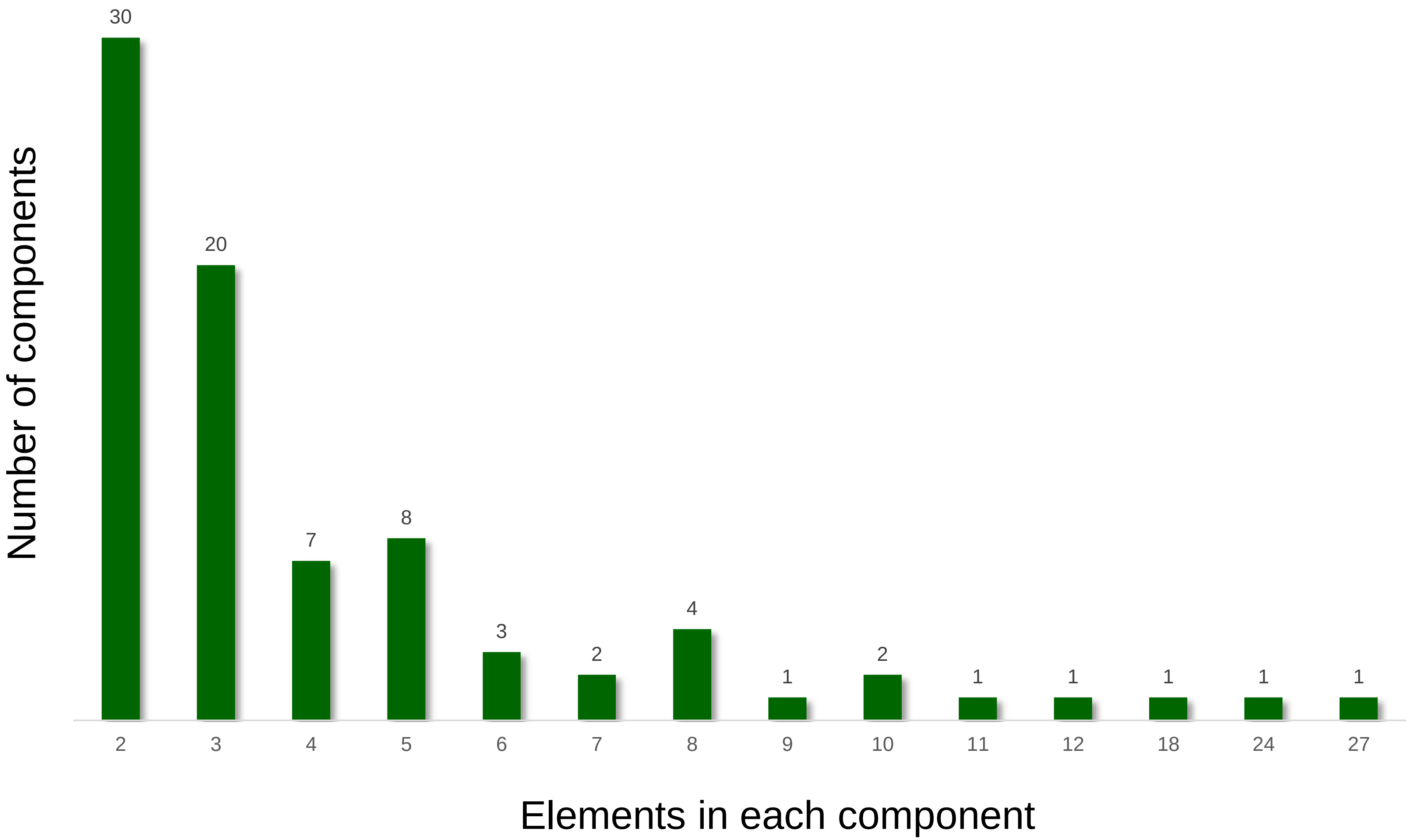
We will have a look at the paper…

Support for ANVUR

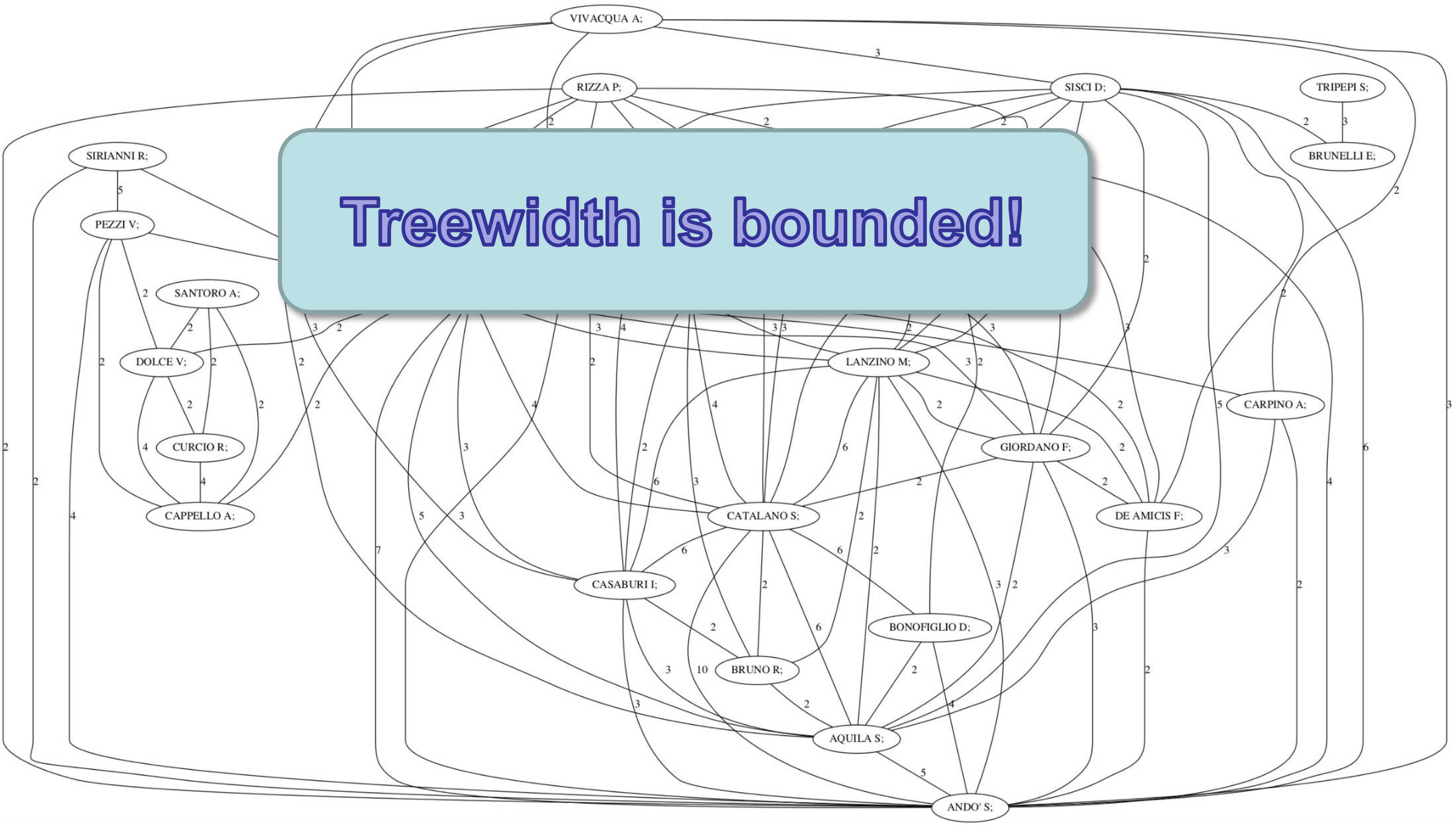
- Implementation strategies
  - Sampling
  - Structural properties

Very particular interaction graph

Components at University of Calabria



An Example Component



Treewidth is bounded!

Support for ANVUR

- Implementation strategies
  - Sampling
  - Structural properties

Very particular interaction graph

- Side results
  - Collaborations with ANVUR
  - University of Calabria uses (parts of) our findings
  - Responsible for the quality of research at University of Calabria
  - Still trying to generalize at national level….

Lessons?

GAME THEORY, AI, ...

SOCIETY

Thank you!