Rational Closure for Description Logics of Typicality

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Résumé

Nous définissons la notion de fermeture rationnelle dans le contexte des logiques de description étendues avec un opérateur de typicalité. Notre point de départ est le langage $\mathcal{ALC} + \mathbf{T}$, une extension de $\mathcal{ALC}$ avec un opérateur de typicalité $\mathbf{T}$: intuitivement le concept $\mathbf{T}(C)$ sélectionne les instances "les plus normales" d'un concept de C. La sémantique que nous considérons est basée sur les modèles rationnels. Mais nous restreignons ultérieurement la sémantique aux modèles rationnels *minimaux*, c'est à dire aux modèles qui minimisent le rang des éléments du domaine. Nous montrons que cette sémantique capture exactement une notion de fermeture rationnelle qui est une extension naturelle aux logiques de description de la construction originaire de Lehmann et Magidor pour des bases des connaissances propositionnelles. Nous étendons également la notion de fermeture rationnelle à la composante ABox d'une base de connaissances. Nous fournissons un algorithme de complexité ExpTime pour le calcul de la fermeture rationnelle d'une Abox et nous montrons qu'il est correct et complet par rapport à la sémantique des modèles minimaux.

Abstract

We define the notion of rational closure in the context of Description Logics extended with a typicality operator. We start from $\mathcal{ALC} + \mathbf{T}$, an extension of $\mathcal{ALC}$ with a typicality operator $\mathbf{T}$: intuitively the concept $\mathbf{T}(C)$ is meant to select the "most normal" instances of a concept C. The semantics we consider is based on rational model. But we further restrict the semantics to minimal models, that is to say, to models that minimise the rank of domain elements. We show that this semantics captures exactly a notion of rational closure which is a natural extension to Description Logics of Lehmann and Magidor's original one. We also extend the notion of rational closure to the Abox component of a knowledge base. We provide an ExpTime algorithm for computing the rational closure of an Abox and we show that it is sound and complete with respect to the minimal model semantics.

1 Introduction

Recently, in the domain of Description Logics (DLs) a large amount of work has been done in order to extend the basic formalism with nonmonotonic reasoning features. The aim of these extensions is to reason about prototypical properties of individuals or classes of individuals. In these extensions one can represent for instance knowledge expressing the fact that the heart is usually positioned in the left-hand side of the chest, with the exception of people with *situs inversus*, that have the heart positioned in the right-hand side. Also, one can infer that an individual enjoys all the typical properties of the classes it belongs to. So, for instance, in the absence of information that someone has situs inversus, one would assume that it has the heart positioned in the left-hand side. A further objective of these extensions is to allow to reason about defeasibile properties and inheritance with exceptions. As another example, consider the standard penguin example, in which typical birds fly, however penguins are birds that do not fly. Nonmonotonic extensions of DLs allow to attribute to an individual the typical properties of the most specific class it belongs to. In this example, when knowing that Tweety is a bird, one would conclude that it flies, whereas when discovering that it is also a penguin, the previous inference is retracted, and the fact that Tweety does not fly is concluded.

In the literature of DLs, several proposals have appeared [21, 2, 1, 7, 15, 5, 13, 3, 17, 8, 20]. However, finding a solution to the problem of extending DLs for reasoning about prototypical properties seems far from being solved.

In this paper, we introduce a general framework for nonmonotonic reasoning in DLs based on (i) the use of a typicality operator $\mathbf{T}$; (ii) a minimal model mechanism (in the spirit of circumscription). The typicality operator **T**, introduced in [9], allows to directly express typical properties such as $\mathbf{T}(HeartPosition) \sqsubseteq \mathit{Left}$, $\mathbf{T}(Bird) \sqsubseteq \mathit{Fly}$, and $\mathbf{T}(Penguin) \sqsubseteq \neg \mathit{Fly}$, whose intuitive meaning is that normally, the heart is positioned in the left-hand side of the chest, that typical birds fly, whereas penguins do not. The **T** operator is intended to enjoy the well-established properties of preferential semantics, described by Kraus Lehmann and Magidor (henceforth KLM) in their seminal work [16, 18]. KLM proposed an axiomatic approach to nonmonotonic reasoning, and individuated two systems, preferential logic **P** and rational logic **R**, and their corresponding semantics. It is commonly accepted that the systems **P** and **R** express the core properties of nonmonotonic reasoning.

In [13, 11] nonmonotonic extensions of DLs based on the **T** operator have been proposed. In these extensions, the semantics of **T** is based on preferential logic **P**. Nonmonotonic inference is obtained by restricting entailment to *minimal models*, where minimal models are those that minimise the truth of formulas of a special kind. In this work, we present an alternative and more general approach. First, in our framework the semantics underlying the **T** operator is not fixed once for all : although we consider here only KLM's **P** or **R** as underlying semantics, in principle one might choose any other underlying semantics for **T** based on a modal preference relation. Moreover and more importantly, we adopt a minimal model semantics, where, as a difference with the previous approach, the notion of minimal model is completely independent from the language and is determined only by the relational structure of models.

The semantic approach to nonmonotonic reasoning in DLs presented in this work is an extension of the one described in [14] within a propositional context. We then propose a rational closure construction for DL extended with the **T** operator as an algorithmic counterpart of our minimal model semantics, whenever the underlying logic for **T** is KLM logic **R**. *Rational closure* is a well-established notion introduced in [18] as a nonmonotonic mechanism built on the top of **R** in order to perform some further truthful nonmonotonic inferences that are not supported by **R** alone. We extend it to DLs in a natural way, so that, in turn, we can see our minimal model semantics as a semantical reconstruction of rational closure.

More in details, we take $\mathcal{ALC} + \mathbf{T}$ as the underlying DL and we define a nonmonotonic inference relation on the top of it by restricting entailment to minimal models : they are those ones which minimize the *rank of domain elements* by keeping fixed the extensions of concepts and roles. We then proceed to extend in a natural way the propositional construction of rational closure to $\mathcal{ALC} + \mathbf{T}$ for inferring defeasible subsumptions from the TBox (TBox reasoning). Intuitively the rational closure construction amounts to assigning a *rank* (a level of exceptionality) to every concept ; this rank is used to evaluate defeasible inclusions of the form $\mathbf{T}(C) \sqsubseteq D$: the inclusion is supported by the rational closure whenever the rank of C is strictly smaller than the one of $C \sqcap \neg D$. Our goal is to link the rational closure of a TBox to its minimal model semantics, but in general it is not possible. The reason is that the minimal model semantics is not tight enough to support the inferences provided by the rational closure. However we can obtain an exact correspondence between the two if we further restrict the minimal model semantics to *canonical models* : these are models that satisfy each intersection $(C_1 \sqcap \ldots \sqcap C_n)$ of concepts drawn from the KB that is satisfiable with respect to the TBox.

We then tackle the problem of extending the rational closure to ABox reasoning : we would like to ascribe defeasible properties to individuals. The idea is to maximize the typicality of an individual : the more is "typical", the more it inherits the defeasible properties of the classes it belongs too (being a typical member of them). We obtain this by minimizing its rank (that is, its level of exceptionality), however, because of the interaction between individuals (due to roles) it is not possible to assign a unique minimal rank to each individual and alternative minimal ranks must be considered. We end up with a kind of *skeptical* inference with respect the ABox. We prove that it is sound and complete with respect to the minimal model semantics restricted to canonical models.

The rational closure construction that we propose has not just a theoretical interest and a simple minimal model semantics, we show that it is also *feasible* since its complexity is "only" EXPTIME in the size of the knowledge base (and the query), thus not worse than the underlying monotonic logic. In this respect it is less complex than other approaches to nonmonotonic reasoning in DLs [13, 2] and comparable in complexity with the approaches in [5, 4, 20], and thus a good candidate to define effective nonmonotonic extensions of DLs.

2 The operator $\mathbf{T}$ and the General Semantics

Let us briefly recall the DLs $\mathcal{ALC} + \mathbf{T}$ and $\mathcal{ALC}^{\mathbf{R}}\mathbf{T}$ introduced in [9, 10], respectively. The intuitive idea is to extend the standard $\mathcal{ALC}$ allowing concepts of the form $\mathbf{T}(C)$, where C does not mention **T**, whose intuitive meaning is that $\mathbf{T}(C)$ selects the *typical* instances of a concept C. We can therefore distinguish between the properties that hold for all instances of concept C ($C \sqsubseteq D$), and those that only hold for the typical instances of C ($\mathbf{T}(C) \sqsubseteq D$) that we call $\mathbf{T}$-inclusions, where C is a concept not mentioning $\mathbf{T}$. Formally, the language is defined as follows.

Definition 2.1 (Knowledge Base) *We consider an alphabet of concept names $\mathcal{C}$, of role names $\mathcal{R}$, and of individual constants $\mathcal{O}$. Given $A \in \mathcal{C}$ and $R \in \mathcal{R}$, we define $C_R := A \mid \top \mid \bot \mid \neg C_R \mid C_R \sqcap C_R \mid C_R \sqcup C_R \mid \forall R.C_R \mid \exists R.C_R$, and $C_L := C_R \mid \mathbf{T}(C_R)$. A KB is a pair (TBox, ABox). TBox contains a finite set of concept inclusions $C_L \sqsubseteq C_R$. ABox contains assertions of the form $C_L(a)$ and $R(a,b)$, where $a, b \in \mathcal{O}$.*

The semantics of $\mathcal{ALC}+\mathbf{T}$ and $\mathcal{ALC}^{\mathbf{R}}\mathbf{T}$ is defined respectively in terms of preferential and rational [1] models : ordinary models of $\mathcal{ALC}$ are equipped by a *preference relation* $<$ on the domain, whose intuitive meaning is to compare the "typicality" of domain elements, that is to say $x < y$ means that x is more typical than y. Typical members of a concept C, that is members of $\mathbf{T}(C)$, are the members x of C that are minimal with respect to this preference relation (s.t. there is no other member of C more typical than x). Preferential models, in which the preference relation $<$ is irreflexive and transitive, characterize the logic $\mathcal{ALC} + \mathbf{T}$, whereas the more restricted class of rational models, so that $<$ is further assumed to be modular, characterizes $\mathcal{ALC}^{\mathbf{R}}\mathbf{T}$ [2].

Definition 2.2 (Semantics of $\mathcal{ALC} + \mathbf{T}$) *A model $\mathcal{M}$ of $\mathcal{ALC} + \mathbf{T}$ is any structure $\langle \Delta, <, I \rangle$ where : Δ is the domain ; $<$ is an irreflexive and transitive relation over Δ that satisfies the following* Smoothness Condition *: for all $S \subseteq \Delta$, for all $x \in S$, either $x \in Min_<(S)$ or $\exists y \in Min_<(S)$ such that $y < x$, where $Min_<(S) = \{u : u \in S$ and $\nexists z \in S$ s.t. $z < u\}$; I is the extension function that maps each concept C to $C^I \subseteq \Delta$, and each role R to $R^I \subseteq \Delta^I \times \Delta^I$. For concepts of $\mathcal{ALC}$, C^I is defined in the usual way. For the $\mathbf{T}$ operator, we have $(\mathbf{T}(C))^I = Min_<(C^I)$.*

Definition 2.3 (Semantics of $\mathcal{ALC}^{\mathbf{R}}\mathbf{T}$) *A model $\mathcal{M}$ of $\mathcal{ALC}^{\mathbf{R}}\mathbf{T}$ is an $\mathcal{ALC}+\mathbf{T}$ model as in Definition 2.2 in which $<$ is further assumed to be* modular *: for all $x, y, z \in \Delta$, if $x < y$ then either $x < z$ or $z < y$.*

Definition 2.4 (Model satisfying a Knowledge Base) *Given a model $\mathcal{M}$, I is extended to assign a distinct element [3] a^I of the domain Δ to each individual constant a of $\mathcal{O}$. $\mathcal{M}$ satisfies a knowledge base K=(TBox,ABox), if it satisfies both its TBox and its ABox, where : - $\mathcal{M}$ satisfies TBox if for all inclusions $C \sqsubseteq D$ in TBox, it holds $C^I \subseteq D^I$; - $\mathcal{M}$ satisfies ABox if : (i) for all $C(a)$ in ABox, $a^I \in C^I$, (ii) for all aRb in ABox, $(a^I, b^I) \in R^I$.*

In [9] it has been shown that reasoning in $\mathcal{ALC} + \mathbf{T}$ is ExpTime complete, that is to say adding the $\mathbf{T}$ operator does not affect the complexity of the underlying DL $\mathcal{ALC}$. We are able to extend the same result also for $\mathcal{ALC}^{\mathbf{R}}\mathbf{T}$ (we omit the proof due to space limitations) :

Theorem 2.5 (Complexity of $\mathcal{ALC}^{\mathbf{R}}\mathbf{T}$) *Reasoning in $\mathcal{ALC}^{\mathbf{R}}\mathbf{T}$ is* ExpTime *complete.*

From now on, we restrict our attention to $\mathcal{ALC}^{\mathbf{R}}\mathbf{T}$ and to finite models. Given a knowledge base K and an inclusion $C_L \sqsubseteq C_R$, we say that it is derivable from K (we write $K \models_{\mathcal{ALC}^{\mathbf{R}}\mathbf{T}} C_L \sqsubseteq C_R$) if $C_L^I \subseteq C_R^I$ holds in all models $\mathcal{M} = \langle \Delta, <, I \rangle$ satisfying K.

Definition 2.6 (Rank of a domain element) *The rank $k_{\mathcal{M}}$ of a domain element x in $\mathcal{M}$ is the length of the longest chain $x_0 < \ldots < x$ from x to a minimal x_0 (such that for no x' it holds that $x' < x_0$).*

Finite $\mathcal{ALC}^{\mathbf{R}}\mathbf{T}$ models can be equivalently defined by postulating the existence of a function $k : \Delta \rightarrow \mathbb{N}$, and then letting $x < y$ iff $k(x) < k(y)$.

Definition 2.7 (Rank $k_{\mathcal{M}}$ of a concept) *Given a model $\mathcal{M} = \langle \Delta, <, I \rangle$, the rank $k_{\mathcal{M}}(C_R)$ of a concept C_R in $\mathcal{M}$ is $i = min\{k_{\mathcal{M}}(x) : x \in C_R^I\}$. If $C_R^I = \emptyset$, then C_R has no rank and we write $k_{\mathcal{M}}(C_R) = \infty$.*

It is immediate to verify that :

Proposition 2.8 *For any $\mathcal{M} = \langle \Delta, <, I \rangle$, we have that $\mathcal{M}$ satisfies $\mathbf{T}(C) \sqsubseteq D$ iff $k_{\mathcal{M}}(C \sqcap D) < k_{\mathcal{M}}(C \sqcap \neg D)$.*

As already mentioned, although the typicality operator $\mathbf{T}$ itself is nonmonotonic (i.e. $\mathbf{T}(C) \sqsubseteq D$ does not imply $\mathbf{T}(C \sqcap E) \sqsubseteq D$), the logics $\mathcal{ALC} + \mathbf{T}$ and $\mathcal{ALC}^{\mathbf{R}}\mathbf{T}$ are monotonic : what is inferred from K can still be inferred from any K' with $K \subseteq K'$. In order to define a nonmonotonic entailment we introduce the second ingredient of our minimal model semantics. As in [13], we strengthen the semantics by restricting entailment to a class of minimal (or preferred) models, more precisely to models that minimize *the rank of worlds*. Informally, given two models of K, one in which a given x has rank 2 (because for instance $z < y < x$) , and another in which it has rank 1 (because only $y < x$), we would prefer the latter, as in this model x is "more normal" than in the former. We call the new logic $\mathcal{ALC}^{\mathbf{R}}_{min}\mathbf{T}$.

Let us define the notion of *query*. Intuitively, a query is either an inclusion relation or an assertion of the ABox, and we want to check whether it is entailed from a given KB.

1. We use the expression "rational model" rather than "ranked model" which is also used in the literature in order to avoid any confusion with the notion of rank used in rational closure.

2. One may think of considering a more complex semantics with several preference relations. We briefly discuss this possibility in the Conclusions

3. We assume the well-established *unique name assumption*.

Definition 2.9 (Query) *A* query *F is either an assertion $C_L(a)$ or an inclusion relation $C_L \sqsubseteq C_R$. Given a model $\mathcal{M} = \langle \Delta, <, I \rangle$, a query $F = C_L(a)$ holds in $\mathcal{M}$ if $a^I \in C_L^I$, whereas a query $F = C_L \sqsubseteq C_R$ holds in $\mathcal{M}$ if $C_L^I \subseteq C_R^I$.*

In analogy with circumscription, there are mainly two ways of comparing models with the same domain : 1) by keeping the valuation function fixed (only comparing $\mathcal{M}$ and $\mathcal{M}'$ if I and I' in the two models respectively coincide) ; 2) by also comparing $\mathcal{M}$ and $\mathcal{M}'$ in case $I \neq I'$. In this work we consider the semantics resulting from the first alternative, whereas we leave the study of the other one for future work (see Section 6 below). The semantics we introduce is a *fixed interpretations minimal semantics*, for short $FIMS$.

Definition 2.10 ($FIMS$) *Given $\mathcal{M} = \langle \Delta, <, I \rangle$ and $\mathcal{M}' = \langle \Delta', <', I' \rangle$ we say that $\mathcal{M}$ is preferred to $\mathcal{M}'$ ($\mathcal{M} <_{FIMS} \mathcal{M}'$) if $\Delta = \Delta'$, $I = I'$, and for all $x \in \Delta$, $k_{\mathcal{M}}(x) \leq k_{\mathcal{M}'}(x)$ whereas there exists $y \in \Delta$ such that $k_{\mathcal{M}}(y) < k_{\mathcal{M}'}(y)$.*

Given a knowledge base K, we say that $\mathcal{M}$ is a minimal model of K with respect to $<_{FIMS}$ if it is a model satisfying K and there is no $\mathcal{M}'$ model satisfying K such that $\mathcal{M}' <_{FIMS} \mathcal{M}$.

Next, we extend the notion of minimal model by also taking into account the individuals named in the ABox.

Definition 2.11 (Model minimally satisfying K) *Given K=(TBox,ABox), let $\mathcal{M} = \langle \Delta, <, I \rangle$ and $\mathcal{M}' = \langle \Delta', <', I' \rangle$ be two models of K which are minimal w.r.t. Definition 2.10. We say that $\mathcal{M}$ is preferred to $\mathcal{M}'$ with respect to ABox ($\mathcal{M} <_{ABox} \mathcal{M}'$) if for all individual constants a occurring in ABox, $k_{\mathcal{M}}(a^I) \leq k_{\mathcal{M}}(a^{I'})$ and there is at least one individual constant b occurring in ABox such that $k_{\mathcal{M}}(b^I) < k_{\mathcal{M}}(b^{I'})$. $\mathcal{M}$ minimally satisfies K in case there is no $\mathcal{M}'$ satisfying K such that $\mathcal{M}' <_{ABox} \mathcal{M}$.*

We say that K minimally entails a query F ($K \models_{min} F$) if F holds in all models that minimally satisfy K.

3 A Semantical Reconstruction of Rational Closure in DLs

In this section we provide a definition of the well known rational closure, described in [18], in the context of Description Logics. We then provide a semantic characterization of it within the semantics described in the previous section.

Definition 3.1 (Exceptional concept) *Let K be a DL knowledge base and C a concept. C is said to be* exceptional *for K iff $K \models_{\mathcal{ALC}^{\mathbf{R}}\mathbf{T}} \mathbf{T}(\top) \sqsubseteq \neg C$.*

Let us now extend Lehmann and Magidor's definition of rational closure to a DL knowledge base. First, we remember that the **T** operator satisfies a set of postulates that are essentially a reformulation of KLM axioms of rational logic **R** : in this respect, in [9] it is shown that the **T**-assertion $\mathbf{T}(C) \sqsubseteq D$ is equivalent to the conditional assertion $C \mid\sim D$ of KLM logic **R**. We say that a **T**-inclusion $\mathbf{T}(C) \sqsubseteq D$ is exceptional for K if C is exceptional for K. The set of **T**-inclusions which are exceptional for K will be denoted as $\mathcal{E}(K)$. Also in this case, it is possible to define a sequence of non-increasing subsets of K $E_0 \supseteq E_1, \ldots$ by letting $E_0 = K$ and, for $i > 0$, $E_i = \mathcal{E}(E_{i-1}) \cup \{C \sqsubseteq D \in K$ s.t. **T** does not occur in $C\}$. Observe that, being K finite, there is a $n \geq 0$ such that for all $m > n$, $E_m = E_n$ or $E_m = \emptyset$.

Definition 3.2 (Exceptionality rank of a concept) *A concept C has* rank *i (denoted by $rank(C) = i$) for K iff i is the least natural number for which C is not exceptional for E_i. If C is exceptional for all E_i then $rank(C) = \infty$, and we say that C has no rank.*

The notion of rank of a formula allows to define the rational closure of the TBox of a knowledge base K.

Definition 3.3 (Rational closure of TBox) *Let K=(TBox,ABox) be DL knowledge base. We define the rational closure $\overline{TBox}$ of TBox of K where*

$$\overline{TBox} = \{\mathbf{T}(C) \sqsubseteq D \mid \textit{either } rank(C) < rank(C \sqcap \neg D) \textit{ or } rank(C) = \infty\} \cup \{C \sqsubseteq D \mid K \models_{\mathcal{ALC}} C \sqsubseteq D\}$$

It is worth noticing that Definition 3.3 takes into account the monotonic logical consequences $C \sqsubseteq D$ with respect to $\mathcal{ALC}$. This is due to the fact that the language here is richer than that considered by Lehmann and Magidor, who only considers the set of conditionals $C \mid\sim D$ that, as said above, correspond to **T**-inclusions $\mathbf{T}(C) \sqsubseteq D$. The above Definition 3.3 also takes into account classical inclusions $C \sqsubseteq D$ that belong to our language.

In the following we show that the minimal model semantics defined in the previous section can be used to provide a semantical characterization of rational closure.

First of all, we can observe that $FIMS$ as it is cannot capture the rational closure of a TBox. For instance, consider the knowledge base K =(TBox,$\emptyset$) of the penguin example, where TBox contains the following inclusions : $Penguin \sqsubseteq Bird$, $\mathbf{T}(Bird) \sqsubseteq Fly$, $\mathbf{T}(Penguin) \sqsubseteq \neg Fly$. We observe that $K \not\models_{FIMS} \mathbf{T}(Penguin \sqcap Black) \sqsubseteq \neg Fly$. Indeed in $FIMS$ there can be a model $\mathcal{M} = \langle \Delta, <, I \rangle$ in which $\Delta = \{x, y, z\}$, $Penguin^I = \{x, y\}$, $Bird^I = \{x, y, z\}$, $Fly^I = \{x, z\}$, $Black^I = \{x\}$, and $z < y < x$. $\mathcal{M}$ is a model of K, and it is minimal with respect to $FIMS$ (indeed it is not possible to lower the rank of x nor of y nor of z unless we falsify K). Furthermore, x is a typical black penguin in $\mathcal{M}$ (since there is no other black penguin preferred to it) that flies. On the contrary, it can be verified that $\mathbf{T}(Penguin \sqcap Black) \sqsubseteq \neg Fly \in \overline{TBox}$. Things change if we consider the minimal models semantics applied to models that contain a domain element for *each combination of concepts consistent with* K. We call these models *canonical models*. In the example, if we restrict our attention to models $\mathcal{M} = \langle \Delta, <, I \rangle$ that also contain a $w \in \Delta$ which is a black penguin that does not fly, that is to say $w \in Penguin^I$, $w \in Bird^I$, $w \in Black^I$, and $w \notin Fly^I$ and can therefore be assumed to be a typical penguin, we are able to conclude that typically black penguins do not fly, as in rational closure. Indeed, in all minimal models of K that also contain w with $w \in Penguin^I$, $w \in Bird^I$, $w \in Black^I$, and $w \notin Fly^I$, it holds that $\mathbf{T}(Penguin \sqcap Black) \sqsubseteq \neg Fly$.

From now on, we restrict our attention to *canonical minimal models*. First, we define a set of concepts $\mathcal{S}$ closed under negation and subconcepts. We assume that all concepts in K and in the query F belong to $\mathcal{S}$.

In order to define canonical minimal models, we consider the set of all consistent sets of concepts $\{C_1, C_2, \dots, C_n\} \subseteq \mathcal{S}$ that are consistent with K, i.e., s.t. $K \not\models_{\mathcal{ALC}} C_1 \sqcap C_2 \sqcap \dots \sqcap C_n \sqsubseteq \bot$.

Definition 3.4 (Canonical minimal model w.r.t. $\mathcal{S}$) *Given K and a query F, a model $\mathcal{M} = \langle \Delta, <, I \rangle$ minimally satisfying K is canonical w.r.t. $\mathcal{S}$ if it contains at least a domain element $x \in \Delta$ s.t. $x \in C^I$ for each combination C in $\mathcal{S}$ consistent with K.*

We can prove the following results :

Proposition 3.5 *Let $\mathcal{M}$ be a minimal canonical model of K. For all concepts $C \in \mathcal{S}$, it holds that $rank(C) = k_{\mathcal{M}}(C)$.*

The proof can be done by induction on the rank of concept C.

Theorem 3.6 *Given K, we have that $C \sqsubseteq D \in \overline{TBox}$ if and only if $C \sqsubseteq D$ holds in all canonical minimal models with respect to K and $C \sqsubseteq D$.*

This theorem directly follows from Proposition 3.5.

4 Rational Closure Over the ABox

In this section we extend the notion of rational closure defined in the previous section in order to take into account the individual constants in the ABox. We therefore address the question : what does the rational closure of a knowledge base K allow us to infer about a specific individual constant a occurring in the ABox of K ? We propose the algorithm below to answer this question and we show that it corresponds to what is entailed by the minimal model semantics presented in the previous section. The idea of the algorithm is that of considering all the possible minimal consistent assignments of ranks to the individuals explicitly named in the ABox. Each assignment adds some properties to named individuals which can be used to infer new conclusions. We adopt a skeptical view of considering only those conclusions which hold for all assignments. The equivalence with the semantics shows that the minimal entailment captures a skeptical approach when reasoning about the ABox.

Definition 4.1 (Rational closure of ABox) • *Let $a_1, \dots, a_m$ be the individuals explicitly named in the ABox. Let $k_1, k_2, \dots, k_h$ all the possible rank assignments (ranging from 1 to n) to the individuals occurring in ABox.*
• *We find the consistent k_j with ($\overline{TBox}$, ABox), where :*
- for all a_i in ABox, we define $\mu_i^j = \{(\neg C \sqcup D)(a_i)$ s.t. $C, D \in \mathcal{S}$, $\mathbf{T}(C) \sqsubseteq D$ in $\overline{TBox}$, and $k_j(a_i) \leq rank(C)\} \cup \{(\neg C \sqcup D)(a_i)$ s.t. $C \sqsubseteq D$ in TBox $\}$;
- let $\mu^j = \mu_1^j \cup \dots \cup \mu_n^j$ for all $\mu_1^j \dots \mu_n^j$ just calculated ;
- k_j is consistent *with ($\overline{TBox}$, ABox) if ABox $\cup \mu^j$ is consistent in $\mathcal{ALC}$.*
• *We consider the* minimal consistent *k_j i.e. those for which there is no k_i consistent with ($\overline{TBox}$, ABox) s.t. for all a_i, $k_i(a_i) \leq k_j(a_i)$ and for a b, $k_i(b) < k_j(b)$.*
• *We define the rational closure of ABox, denoted as $\overline{ABox}$, the set of all assertions derivable in $\mathcal{ALC}$ from ABox $\cup \mu^j$ for all minimal consistent rank assignments k_j, i.e :*

$$\overline{ABox} = \bigcap_{k_j} \{C(a) : \text{ABox} \cup \mu^j \models_{\mathcal{ALC}} C(a)\}$$

Example 4.2 Consider the standard penguin example. Let K = (TBox, ABox), where TBox = $\{\mathbf{T}(B) \sqsubseteq F, \mathbf{T}(P) \sqsubseteq \neg F, P \sqsubseteq B\}$, and ABox = $\{P(i), B(j)\}$.

Computing the ranking of concepts we get that $rank(B) = 0$, $rank(P) = 1$, $rank(B \sqcap \neg F) = 1$, $rank(P \sqcap F) = 2$. It is easy to see that a rank assignment k_0 with $k_0(i) = 0$ is inconsistent with K as μ_i^0 would contain $(\neg P \sqcup B)(i)$, $(\neg B \sqcup F)(i)$, $(\neg P \sqcup \neg F)(i)$ and $P(i)$. Thus we are left with only two ranks k_1 and k_2 with respectively $k_1(i) = 1, k_1(j) = 0$ and $k_2(i) = k_2(j) = 1$.

The set μ^1 contains, among the others, $(\neg P \sqcup \neg F)(i)$, $(\neg B \sqcup F)(j)$. It is tedious but easy to check that $K \cup \mu^1$ is consistent and it is the only minimal consistent one (being k_1 preferred to k_2, thus both $\neg F(i)$ and $F(j)$ belong to $\overline{ABox}$.

Example 4.3 This example shows the need of considering multiple ranks of individual constants : normally computer science courses (CS) are taught only by academic members (A), whereas business courses (B) are taught only by consultants (C), consultants and academics are disjointed, this gives the following TBox : $\mathbf{T}(CS) \sqsubseteq \forall taught.A$, $\mathbf{T}(B) \sqsubseteq \forall taught.C$, $C \sqsubseteq \neg A$. Suppose the ABox contains : $CS(c1)$, $B(c2)$, $taught(c1, joe)$, $taught(c2, joe)$ and let K= (TBox, ABox). Computing rational closure of TBox, we get that all atomic concepts have rank 0. Any rank assignment k_i with $k_i(c1) = k_i(c2) = 0$, is inconsistent with K since the respective μ^i will contain both $(\neg CS \sqcup \forall taught.A)(c1)$ and $(\neg B \sqcup \forall taught.C)(c2)$, from which both $C(joe)$ and $A(joe)$ follow, which gives an inconsistency.

There are two minimal consistent ranks : k_1, such that $k_1(joe) = 0, k_1(c1) = 0, k_1(c2) = 1$, and k_2, such that $k_2(joe) = 0, k_2(c1) = 1, k_2(c2) = 0$. We have that ABox $\cup \mu^1 \models A(joe)$ and ABox $\cup \mu^2 \models C(joe)$. According to the skeptical definition of $\overline{ABox}$ neither $A(joe)$, nor $C(joe)$ belongs to $\overline{ABox}$, however $(A \sqcup C)(joe)$ belongs to $\overline{ABox}$.

Theorem 4.4 (Soundness of $\overline{ABox}$) *Given K=(TBox, ABox), for all a individual constant in ABox, we have that if $C(a) \in \overline{ABox}$ then $C(a)$ holds in all minimal canonical models of K.*

Proof. [Fact 0] For any minimal canonical model $\mathcal{M}$ of K= (TBox, ABox) there is a minimal rank assignment k_j consistent with respect to ($\overline{TBox}$, ABox), such that for all a in ABox and all C : if ABox $\cup \mu^j \models_{\mathcal{ALC}} C(a)$ then $C(a)$ holds in $\mathcal{M}$. This can be proven as follows. Let $\mathcal{M}$ be a minimal canonical model of K. Let k_j be the rank assignment corresponding to $\mathcal{M}$: s.t. for all a_i in ABox $k_j(a_i) = k_{\mathcal{M}}({a_i}^I)$. Obviously k_j is minimal. Furthermore, $\mathcal{M} \models$ ABox $\cup \mu^j$. Indeed, $\mathcal{M} \models$ ABox by hypothesis. To show that $\mathcal{M} \models \mu^j$ we reason as follows : for all a_i let $(\neg C \sqcup D)(a_i) \in \mu^j_i$. If ${a_i}^I \in (\neg C)^I$ clearly $(\neg C \sqcup D)(a_i)$ holds in $\mathcal{M}$. On the other hand, if ${a_i}^I \in (C)^I$: by hypothesis $rank(C) \geq k_j(a_i)$ hence by the correspondence between rank of a formula in the rational closure and in minimal canonical models (see Proposition 3.5) also $k_{\mathcal{M}}(C) \geq k_{\mathcal{M}}({a_i}^I)$, but since ${a_i}^I \in (C)^I$, $k_{\mathcal{M}}(C) = k_{\mathcal{M}}({a_i}^I)$, therefore ${a_i}^I \in (\mathbf{T}(C))^I$. By definition of μ_i, and since by Theorem 3.6, $\mathcal{M} \models \overline{TBox}$, $D(a_i)$ holds in $\mathcal{M}$ and therefore also ${a_i}^I \in (\neg C \sqcup D)^I$. Hence, if ABox $\cup \mu^j \models_{\mathcal{ALC}} C(a_i)$ then $C(a_i)$ holds in $\mathcal{M}$.

Let $C(a) \in \overline{ABox}$, and suppose for a contradiction that there is a minimal canonical model $\mathcal{M}$ of K s.t. $C(a)$ does not hold in $\mathcal{M}$. By Fact 0 there must be a k_j s.t. ABox $\cup \mu^j \not\models_{\mathcal{ALC}} C(a)$, but this contradicts the fact that $C(a) \in \overline{ABox}$. Therefore $C(a)$ must hold in all minimal canonical models of K. ■

Theorem 4.5 (Completeness of $\overline{ABox}$) *Given K=(TBox, ABox), for all a individual constant in ABox, we have that if $C(a)$ holds in all minimal canonical models of K then $C(a) \in \overline{ABox}$.*

Proof. We show the contrapositive. Suppose $C(a) \notin \overline{ABox}$, i.e. there is a minimal k_j consistent with ($\overline{TBox}$, ABox) s.t. ABox $\cup \mu^j \not\models_{\mathcal{ALC}} C(a)$. We build a minimal canonical model $\mathcal{M} = \langle \Delta, < I \rangle$ of K such that $C(a_i)$ does not hold in $\mathcal{M}$ as follows. Let $\Delta = \Delta_0 \cup \Delta_1$ where $\Delta_0 = \{\{C_1, \ldots C_k\} \subseteq \mathcal{S} : \{C_1, \ldots C_k\}$ is maximal and consistent with $K\}$ and $\Delta_1 = \{a_i : a_i$ in ABox $\}$. We define the rank $k_{\mathcal{M}}$ of each domain element as follows : $k_{\mathcal{M}}(\{C_1, \ldots C_k\}) = rank(C_1 \sqcap \ldots \sqcap C_k)$, and $k_{\mathcal{M}}(a_i) = k_j(a_i)$. We then define $<$ in the obvious way : $x < y$ iff $k_{\mathcal{M}}(x) < k_{\mathcal{M}}(y)$.

We then define I as follows. First for all a_i in ABox we let $a_i^I = a_i$. For the interpretation of concepts we reason in two different ways for Δ_0 and Δ_1. For Δ_0, for all atomic concepts C', we let $\{C_1, \ldots, C_k\} \in C'^I$ iff $C' \in \{C_1, \ldots, C_k\}$. I then extends to boolean combinations of concepts in the usual way. It can be easily shown that for any boolean combination of concepts C', $\{C_1, \ldots, C_k\} \in C'^I$ iff $C' \in \{C_1, \ldots, C_k\}$. For Δ_1, we start by considering a model $\mathcal{M}' = \langle \Delta', <, I' \rangle$ such that $\mathcal{M}' \models$ ABox $\cup \mu^j$ and $\mathcal{M}' \not\models C(a)$. This model exists by hypothesis. For all atomic concepts C', we let $a_i \in C'^I$ in $\mathcal{M}$ iff $(a_i)^{I'} \in C'^{I'}$ in $\mathcal{M}'$. Of course for any boolean combination of concepts C', $(a_i) \in C'^I$ iff $(a_i)^{I'} \in C'^{I'}$.

In order to conclude the model's construction, for each role R, we define R^I as follows. For $X, Y \in \Delta_0$, $(X, Y) \in R^I$ iff $\{C' : \forall R.C' \in X\} \subseteq Y$. For $a_i, a_j \in \Delta_1$, $(a_i, a_j) \in R^I$ iff $((a_i)^{I'}, (a_j)^{I'}) \in R^{I'}$ in $\mathcal{M}'$. For $a_i \in \Delta_1$, $X \in \Delta_0$, $(a_i, X) \in R^I$ iff there is an $x \in \Delta'$ of $\mathcal{M}'$ such that $(a_i^{I'}, x) \in R^{I'}$ in $\mathcal{M}'$ and, for all concepts C', we have $x \in C'^{I'}$ iff $X \in C'^I$. I is extended to quantified concepts in the usual way. It can be shown that for all $X \in \Delta_0$ for all (possibly) quantified C', $X \in (C')^I$ iff $C' \in X$, and that for all a_i in Δ_1, for all quantified C', $a_i \in (C')^I$ iff $a_i \in (C')^{I'}$.

$\mathcal{M}$ satisfies ABox : for $a_i R a_j$ in ABox this holds by construction. For $C'(a_i)$, this holds since $(a_i)^{I'} \in (C')^{I'}$ in $\mathcal{M}'$, hence $(a_i)^I \in (C')^I$ in $\mathcal{M}$.

$\mathcal{M}$ satisfies TBox : for elements $X \in \Delta_0$, this can be proven as in Theorem 3.6. For Δ_1 this holds since it held in $\mathcal{M}'$. For the inclusion $C_l \sqsubseteq C_j$ this is obvious. For $\mathbf{T}(C_l) \sqsubseteq C_j$, for all a_i we reason as follows. First of all, if $k_j(a_i) >$ rank(C_l) then $a_i \notin Min_<({C_l}^I)$ and the inclusion trivially holds. On the other if $k_j(a_i) \leq$ rank(C_l), $(\neg C_l \sqcup C_j)(a_i) \in \mu^j$, and therefore $(a_i)^{I'} \in (\neg C_l \sqcup C_j)^{I'}$ in $\mathcal{M}'$, hence $(a_i)^I \in (\neg C_l \sqcup C_j)^I$ in $\mathcal{M}$, and we are done.

$C(a)$ does not hold in $\mathcal{M}$, since it does not hold in $\mathcal{M}'$. Last, $\mathcal{M}$ is minimal : if it was not so there would be $\mathcal{M}' < \mathcal{M}$. However it can be shown that we could define a $k_{j'}$ consistent with ($\overline{TBox}$, ABox) and preferred to k_j, thus contradicting the minimality of k_j, against the hypothesis. We have then built a minimal canonical model of K in which $C(a)$ does not hold. The theorem follows by contraposition. ■

Let us conclude this section by estimating the complexity of computing the rational closure of the ABox :

Theorem 4.6 *Given a knowledge base K =(TBox,ABox),* *an individual constant a and a concept C, the problem of deciding whether $C(a) \in \overline{ABox}$ is* EXPTIME*-complete.*

Proof. Let $|K|$ be the size of the knowledge base K and let the size of the query be $O(|K|)$. As the number of inclusions in the knowledge base is $O(|K|)$, then the number n of non-increasing subsets E_i in the construction of the rational closure is $O(|K|)$. Moreover, the number k of named individuals in the knowledge base is $O(|K|)$. Hence, the number k^n of different rank assignments to individuals is such that both k and n are $O(|K|)$. Observe that $k^n = 2^{Log\ k^n} = 2^{nLog\ k}$. Hence, k^n is $O(2^{nk})$, with n and k linear in $|K|$, i.e., the number of different rank assignments is exponential in $|K|$.

To evaluate the complexity of the algorithm for computing the rational closure of the ABox, observe that :
(i) For each j, the number of sets μ_i^j is k (which is linear in $|K|$). The number of inclusions in each μ_i^j is $O(|K|^2)$, as the size of $\mathcal{S}$ is $O(|K|)$ and the number of $\mathbf{T}$-inclusions $\mathbf{T}(C) \sqsubseteq D \in \overline{TBox}$, with $C, D \in \mathcal{S}$ is $O(|K|^2)$. Hence, the size of set μ^j is $O(|K|^3)$.
(ii) For each k_j, the consistency of ($\overline{TBox}$, ABox) can be verified by checking the consistency of ABox $\cup \mu^j$ in $\mathcal{ALC}$, which requires exponential time in the size of the set of formulas ABox $\cup \mu^j$ (which, as we have seen, is polynomial in the size of K). Hence, the consistency of each k_j can be verified in exponential time in the size of K.
(iii) The identification of the minimal assignments k_j among the consistent ones requires the comparison of each consistent assignment with each other (i.e. k^2 comparisons), where each comparison between k_j and $k_{j'}$ requires k steps. Hence, the identification of the minimal assignments requires k^3 steps.
(iv) To define the rational closure $\overline{ABox}$ of ABox, for each concept C occurring in K or in the query (there are $O(|K|)$ many concepts), and for each named individual a_i, we have to check if $C(a_i)$ is derivable in $\mathcal{ALC}$ from ABox $\cup \mu^j$ for all minimal consistent rank assignments k_j. As the number of different minimal consistent assignments k_j is exponential in $|K|$, this requires an exponential number of checks, each one requiring exponential time in the size of the knowledge base $|K|$. The cost of the overall algorithm is therefore exponential in the size of the knowledge base. ■

5 Related works

A number of works have considered the application of KLM logic and rational closure to DL. In particular [5] discusses the application of rational closure to DLs. The authors first describe a construction to compute rational closure in the context of propositional logic, then they adapt such a construction to the DL $\mathcal{ALC}$. As [5] extends to DLs a construction which, in the propositional case, is proved to be equivalent to Lehmann and Magidor's rational closure, it may be conjectured that their construction is equivalent to our definition of rational closure in Section 3, which is the natural extension of Lehmann and Magidor's definition. [5] keeps the ABox into account, and defines closure operations over individuals. They introduce a consequence relation $\Vdash$ among a KB and assertions, under the requirement that the TBox is unfoldable and the ABox is closed under completion rules, such as, for instance, that if $a : \exists R.C \in$ ABox, then both aRb and $b : C$ (for some individual constant b) must belong to the ABox too. Under such restrictions they are able to define a procedure to compute the rational closure of the ABox assuming that the individuals explicitly named are linearly ordered, and different orders determine different sets of consequences. They show that, for each order s, the consequence relation $\Vdash_s$ is rational and can be computed in PSPACE. In a subsequent work [6], the authors introduce an approach based on the combination of rational closure and *Defeasible Inheritance Networks* (INs).

The logic $\mathcal{ALC}^{\mathbf{R}}\mathbf{T}$ we consider as our base language is equivalent to the logic for defeasible subsumptions in DLs proposed by [3], when considered with $\mathcal{ALC}$ as the underlying DL. The idea underlying this approach is very similar to that of $\mathcal{ALC}^{\mathbf{R}}\mathbf{T}$: some objects in the domain are more typical than others. In the approach by [3], x is more typical than y if $x \geq y$. The properties of $\geq$ correspond to those of $<$ in $\mathcal{ALC}^{\mathbf{R}}\mathbf{T}$. At a syntactic level the two logics differ, so that in [3] one finds the defeasible inclusions $C \sqsubsetsim D$ instead of $\mathbf{T}(C) \sqsubseteq D$ of $\mathcal{ALC}^{\mathbf{R}}\mathbf{T}$, however it has be shown in [10] that the logic of preferential subsumption can be translated into $\mathcal{ALC}^{\mathbf{R}}\mathbf{T}$ by replacing $C \sqsubsetsim D$ with $\mathbf{T}(C) \sqsubseteq D$.

In [4] the semantics of the logic of defeasible inclusions is strengthened by a preferential semantics. Intuitively, given a TBox, the authors first introduce a preference ordering $\ll$ on the class of all subsumption relations $\sqsubsetsim$ including TBox, then they define the rational closure of TBox as the most preferred relation $\sqsubsetsim$ w.r.t. $\ll$, i.e. such that there is no other relation $\sqsubsetsim'$ such that TBox $\subseteq \sqsubsetsim'$ and $\sqsubsetsim' \ll \sqsubsetsim$. Furthermore, the authors describe an EXPTIME algorithm in order to compute the rational closure of a given TBox. However, they do not address the problem of dealing with the ABox. In [19] a plug-in for the Protégé ontology editor implementing the mentioned algorithm for computing the rational closure for a TBox for OWL ontologies is described.

6 Discussion and Conclusions

We have defined a rational closure construction for the Description Logic $\mathcal{ALC}$ extended with a typicality operator for representing "'typical properties", that is to say defeasible inclusions of classes. We have provided a declarative characterization of the rational closure in terms of a minimal model semantics based on the idea of minimizing the rank of domain objects, that is their level of “untypicality”. We have then extended the construction of rational closure to the ABox, for ascribing typical properties to individuals. and we have provided an algorithm for computing it that is sound and complete with respect to the minimal model semantics. Last, we have shown an EXPTIME upper bound for this algorithm.

This work can be the starting point to study variants or strengthening of rational closure. First, in analogy with circumscription, we can consider a stronger form of minimization where we minimize the rank of domain elements, but *we allow to vary* the extensions of concepts. This semantics allows one to obtain stronger inferences, but, as it is shown in [12] it must be suitably constrained to avoid the collapse to classical (monotonic) inference.

Moreover the semantics considered in this work is based on preferential models with one single preference relation $<$. This semantics in itself characterizes the typicality operator **T** and its restriction to minimal canonical models captures exactly rational closure. Intuitively the "global"’ preference relation among individuals compares the typicality of individuals with respect to to all possible properties. We may think of studying a variant of the current semantics where models are equipped with several preference relations, for instance indexed on different aspects (or properties). In this more-refined semantics, for two individuals x, y, we may have $x <_P y$, but $y <_P x$, that is x is more typical than y for property/aspect P, whereas it is the opposite for property/aspect Q. This sharper semantics might be useful to overcome the main weakness of rational closure, namely an exceptional class does not inherit any defeasible property of its superclasses. In the wait of further investigation, we just notice that (i) the definition of such a semantics is not straightforward : what would differentiate one preference relation from another ? What would be the dependencies between the different preference relations ? (ii) the resulting semantics would non longer correspond to the procedural construction of rational closure, thus we would have to find as well a syntactical counterpart or an algorithmic characterization of it.

Coming back to the semantics presented in this paper, there are also some computational issues that deserve to be investigated, in particular its application to so-called *low complexity* DL. To this regard, nonmonotonic extensions of *low complexity* DLs based on the **T** operator have been recently provided [11]. In future works, we aim to study the application of the proposed semantics to DLs of the $\mathcal{EL}$ and DL-Lite families, in order to define a rational closure for low complexity DLs.

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