Waiting Times in Simulated Stock Markets

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1 Introduction

Stock market is pretended to be a complex system where small variation in the environment could dramatically affect the outcomes. This research focus on our own opinion that Continuous Double Auction (CDA) seems to show the same behavior in waiting time distribution even under different setups and different order flows.

To investigate this idea we used an agent based model able to reproduce both the Milan stock exchange and the NYSE orders ranking. Into the model operated a wide bunch of fully similar agents that simply toss random decisions about: i) buying or selling, ii) the share quantity going to be bought or sold and iii) the requested price. To create different orders flows several decision routine have been used.

During the research several different market scenarios have been experimented through the interaction of one thousand agents for one hundred days each time. Each experiment has been repeated using different pseudo random generation seed to confirm the obtained figures. The observation of such data confirms that the waiting time distribution seem to be fully independent by both different setups and order flows, so it appears to be only related to the continuous double action mechanism.

2 Book setups and Orders Flows Modeling

Many regulated markets implement trading via the continuous double auction (CDA), through which buyers and sellers send their orders at random times. These orders, including price and quantity (volume), are collected in a book. The limit order book is designed for the continuous double auctions mechanism. In CDA, any order may be submitted in any moment during the trading period. If at any time there are open bids and asks that are compatible in terms of price and quantity of good, a trade is executed immediately. Then some information about the trade are broadcast to all participants (price, volume...) depending on local rules.

Depending on market regulations, traders may submit various kinds of orders. The most common are limit orders: orders to buy a stated amount of a security at or below a given price, or to sell it at or above a given price (the limit price) and market orders: orders to buy or sell a stated quantity of a security at the most advantageous price obtainable after the order is represented in the Trading Crowd.

2.1 The Limit Order Book

The limit order book of a specific stock, at a specific instant in time t, can be described as follows:

$$\beta_n \leq \ldots \beta_3 \leq \beta_2 \leq \beta_1 < \alpha_1 \leq \alpha_2 \leq \alpha_3 \ldots \leq \alpha_m \tag{1}$$

where the β_i represent buy limit orders (bids) and the α_i sell limit orders (offers or asks); those limit orders are characterized by a limit price, a volume and a time of arrival in the book, and are all waiting in queue to be (potentially) executed. The highest bid β_1, also called best bid, and the lowest offer α_1, or best offer, define the spread $\alpha_1 - \beta_1$. Orders queued in the book are usually sorted by price, time of arrival and volume, with variations from market to market – see Hasbrouck *et al.* (1993) for a more detailed description. β_1 will be executed, only if the book receives a market sell order or a limit sell order with a offering price lower than β_1 ($\beta_1 \geq \alpha_x$). In this case, a trade is generated and the new market price becomes β_1.

But, can changes in rules/architectures really affect market performances?

Some researchers (Bottazzi *et al.*, 2005; Pellizzari and Forno, 2005; LiCalzi and Pellizzari, 2006) compared price dynamics of different market protocols (Walrasian market, batch auction, continuous double auction and dealership) in agent-based artificial exchanges.

We introduce a comparison of rules for incoming orders flows ranking in the book, according to Price-Time-Quantity rule and Price-Quantity-Time rule[3].

The Price-Quantity-Rule (herefore “Nyse” or PQT) is used in New York Stock Exchange[4]:

[3]This comparison was firstly overviewed in a previous work (Cappellini, 2006), founding that:

> In a homogeneous population of traders there is no concrete advantage regarding the two rules, as shown by our results for which Milan is a little quick in order execution. However in real world orders are strategically placed for every level of prices, that means to be quicker (PTQ) or bigger (PQT) than the average of traders.

[4]`http://rules.nyse.com/`

Rule 72. Priority and Precedence of Bids and Offers

I. Bids. – Where bids are made at the same price, the priority and precedence shall be determined as follows:

Priority of first bid

(a) Except as provided in paragraph (b) below, when a bid is clearly established as the first made at a particular price, the maker shall be entitled to priority and shall have precedence on the next sale at that price, up to the number of shares of stock or principal amount of bonds specified in the bid, irrespective of the number of shares of stock or principal amount of bonds specified in such bid.

[...]

The Price-Time-Quantity rule (herefore "Milan" or PTQ) is applied in various order-driven market like the Italian Stock Exchange[5]:

Article 4.1.4

The orders for each instrument shall be automatically ranked on the book according to price – in order of decreasing price if to buy and increasing price if to sell – and, where the price is the same, according to entry time. Modified orders shall lose their time priority if the modification implies an increase in the quantity or a change in the price.

[...]

2.2 Orders Flows Modeling

In stock market simulations, the basic price decision mechanism is usually very simple, the agents:

1. know only the last executed price;
2. choose randomly the buy or sell side;
3. fix their limit price and quantity.

Usually the order price at time t (o_t) is defined as a variation of the last executed price in the market (p_{t-1}).

The orders price can be defined as a Gaussian,

$$o_t = N(p_{t-1}, 0.2) \tag{2}$$

or as a multiplicative process

$$o_t = p_{t-1} \cdot \xi \tag{3}$$

(where ξ can be $N(1, 0.2)$ or $U[0.5; 1.5]$),
or additive

[5] `http://www.borsaitaliana.it/documenti/regolamenti/`

$$o_t = p_{t-1} + \xi \tag{4}$$

(where ξ can be $U[-1;1]$ or $N(0, 0.2)$),
or exponential

$$o_t = p_{t-1} \cdot e^{\delta \cdot \xi} \tag{5}$$

where δ is the price deviation, 0.02 and $\xi = N(0, 1)$.

The order quantity q_t can be modeled according to a Gaussian, $N(2, 50)$, or an uniform distribution, $U[1; 100]$, or can be simply constant and equal to 1.

For this work, we explore all the combination of prices and quantity decisions, having 18 scenarios to be applied to the two different book mechanisms.

3 The simulation framework: SumWEB

The Agent Based Modeling (ABM) paradigm Testfatsion (2003) focuses on the central role of the agent, the representation in silicon of a real operator able to perform an own and autonomous behavior. Artificial agents may be endowed with the ability to compute and modify their own strategies to adapt them to the current conditions of the environment. Following the ABM approach, phenomena observed at an aggregate level could be reproduced through the interaction among simple entities performed by running the simulation. ABM has become the main instrument to deal with complexity, so it is strongly eligible as a tool to investigate matters related to the evolution and adaptation of autonomous behaviors.

SumWEB (SUM Web Economic Behaviour)[6] is basically an extension of Pietro Terna's Terna (2000b,a) simulation SUM (Surprising (Un)realistic Market).

The main feature of the software is the realism of its implementation. The large part of the coded mechanisms were directly inspired by MTA (*Mercato Telematico Azionario*, the Italian Stock Exchange), that's an order driven book. So SUM and SumWEB have a tick per tick realistic formation price mechanism (a continuous double auction CDA), with characteristics as opening and closing auctions, market and limit price orders.

Then it can manage more than one single stock, and different financial instruments (futures and market indexes). This means that it can be used to understand rules and regulation of real markets dealing with their microfoundations, or to act as policy maker developing and new rules tester.

The Sum model can host several type of artificial agents:

[6] `http://eco83.econ.unito.it/sumweb/` described in Cappellini (2003) and Cappellini (2005)

- Random. They are the simplest agents, that choose randomly their actions. They represent a crowd of little investors;
- Financial Technique. They use schemata and rules such as traders. Eg. They can arbitrage a future against the underlying assets or use stop-loss/take-profit strategy;
- Simple Cognitive. They perform simple behavior observing the market operations. They can imitate the last movement or ask for suggestion to other agents;
- Social. Agents increase their knowledge by "informal" chatting into the group and, after pondering, raise beliefs to be used both acting in the market and spreading to others.
- Minded. They have a cognitive-like and adaptable structure. They are principally genetic algorithms, neural networks and classifier systems. They evolve themselves during every run;
- Avatar, the interface for humans. They "capture" humans' orders bringing them into the book.

As technological overview of the software we can observe that it is a SWARM[7] Langton *et al.* (1995) based Objective-C simulation.

4 Results

We run several simulations for each scenario, composed by different decisions mechanisms in term of quantity or price, and by different market rules. The price realizations are quite heterogeneous, as shown by box plot (figure 1) of closing prices for different scenarios.

All the realizations show the same shapes in waiting (pending) time distribution (see Scalas *et al.* (2006); Raberto *et al.* (2002); Scalas *et al.* (2004)) under different market conditions.

We define tick time as the arrival time in book of every order (incremented by one at every event), so the pending time[8] is the amount of time an order waits in the book until its execution.

Here we explore the result of 1000 agents operating for 100 days of 12 "turns", into a stock market that applies the enqueing rules of the NYSE

[7] `http://www.swarm.org`

[8] The waiting times could be defined as follows:

- intertrade time, among executed orders;
- arrival time, among orders submissions;
- pending time, permanence in book until their execution.

The tick times is an internal measure of the simulation, regarding events, such as every agent actions or book events (executions, submissions, etc.). In this work we use the orders cardinality, or the progressive number of each order as they were received by the book.

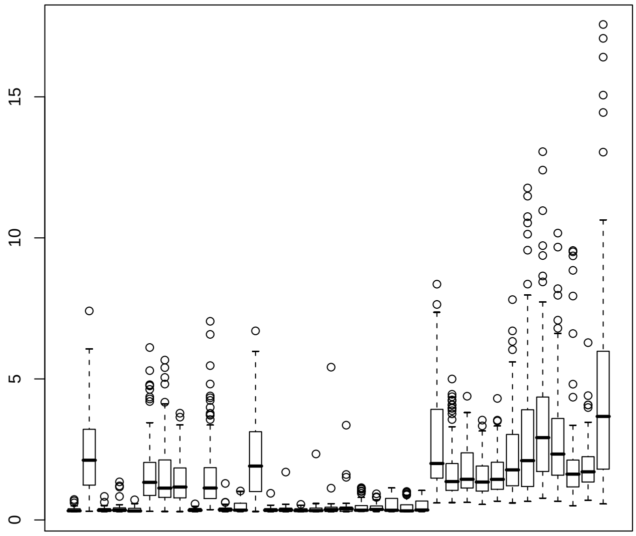

Fig. 1. Closing Prices box plots for several realizations

(PQT). The agents use a multiplicative price driven by an uniform distribution of prices (eq. 3) and a uniform distribution for quantity.

We first analyze the complete series of pending times ("absolute") during continuous trading. The main result is a good exponential fitting, draw in red in figure 2. The α^9 exponent is -2.383e-04.

Through a linear regression on the log-log plot, we estimate a power law fitting for Bids and Asks series according to the standard equation: $y = Ax^{\alpha}$. The α^{10} exponents are -1.36148 for Bids and -1.51508 for Asks. It was quite interesting fit the tails of Asks and Bids distribution with an exponential, which *alpha* are -2.023e-04 (Bids) and -2.921e-04 (Asks)[11]. These exponents were confirmed by empirical analysis in Challet and Stinchcombe (2001), who found a typical value in -1.5 ± 0.1.

[9] $y = Ae^{\alpha x}$ The A parameter is -3.748, R-squared 0.9429. The p-values are 99% siginificant for both parameters.

[10] The A parameter is 0.3026 for Bids and 0.88721 for Asks, R-squared are 0.9723 and 0.9859, respectively. The p-values are 99% siginificant.

[11] R-squared 0.6347 and 0.7315, intercept -3.774 and -3.218

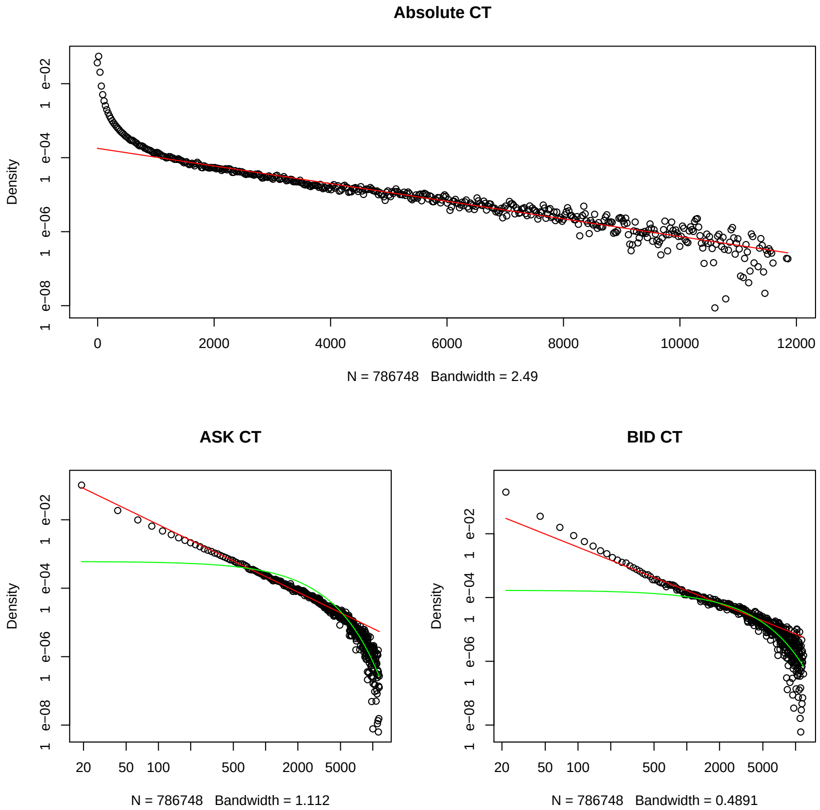

Fig. 2. Pending Time probability distribution for PQT market having orders with multiplicative-uniform prices and uniform quantities

5 Criticism

We found very similar shapes for all scenarios, but we would lie to underline two limitations: the poorness of traders population and the market rules not yet fully explored.

To keep the focus on market mechanisms, we decide to siplify agents behaviour using only random agent, that don't perform any market analysis or any valutation on their own portfolio. So our agent act at any turn and cannot wait for strategically issuing orders, they are not sensible to any kind of news and events, and they do not relax or sleep or simply wait as in Muchnik and Solomon (2007).

The implementation of CDA mechanism is robust and flexible. In the presented realizations still miss some details.

Orders cancellation, market suspensions and circuit breakers can increase the realism but we yet prefer to follow the simplest market implementation.

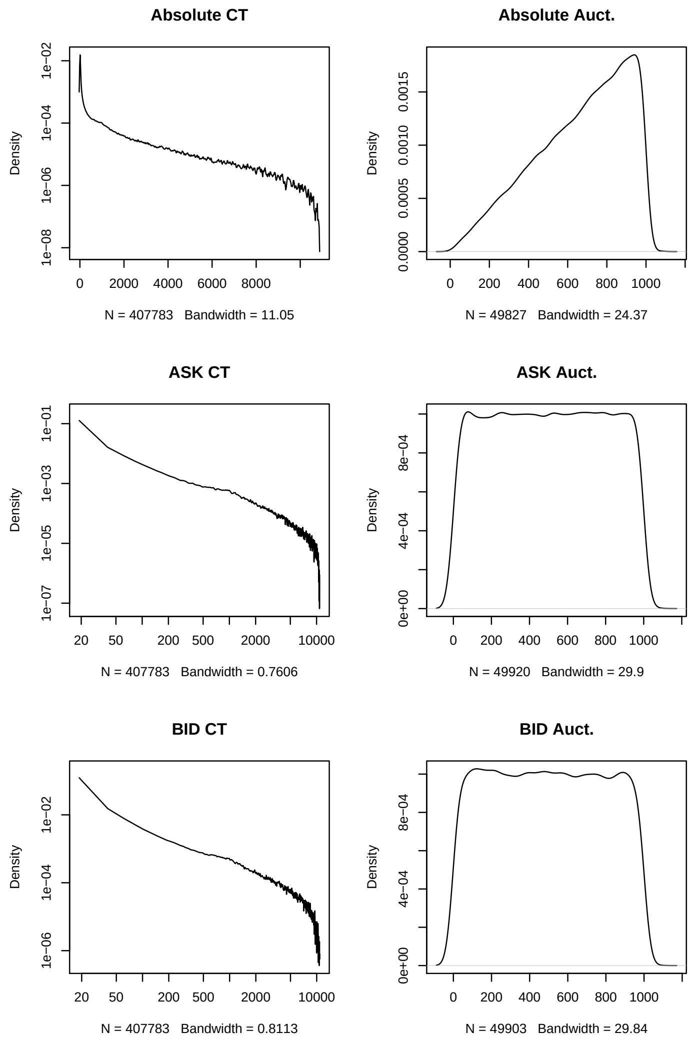

Fig. 3. Continuous Time and Auction pending times probability for PTQ market having orders with exponential prices and single quantities

As example the figure 3 shows a Milan like market (PTQ), populated by 1000 agents adopting exponential prices (eq. 5) with single quantity, for 100 days. In this simulation we add the opening and the closing auctions[12] at the beginning and at the end of all days.

During auction time the orders are collected and enqueued into the book until the auction start; then the auction price is set up and the orders are matched as far as possible according to their rank. So, we consider the ask (or bid) pending time as the time they wait until the execution (at current time t) into the book ($\tau_{ask} = t - t_{ask}$) while "absolute" time is the longest time the orders waited into the book ($\tau_{abs} = t - \min(t_{bid}; t_{ask})$).

Those results confirm us the general shapes for continuous time executed order probability distribution (exponential for absolute, and power law for ask and bid) and show very particular behaviours: triangular probability for absolute prices, and flat for asks and bids. The latest effects are due to a constant auction time at $t = 1000$ and a global uniform probability for orders to be issued in a time $[0; 1000]$.

6 Conclusions

The results witnessed the existence of some regularities that may be interpreted as the emergence of a property related to the continuous double auction mechanism. This property has shown it self to be independent from the different rules used to enqueue incoming orders in Milan stock exchange (so called PTQ) and NYSE (PQT), as well as from diverse random distribution adopted to determine the behaviors of the agents in setting up both prices and quantities.

A possible reason of this emergence could be related to the capability of the indirect interaction among the agents to clean up the waiting queues, mainly due to the prices trend. The agents are always changing their requested prices by slightly biasing current ones, such a mechanism generated a prices dynamic that ensure the large majority of the order founded a counterparty in a reasonable time span.

The results could be classified as emergent because in a priori reasoning there are not strong reason for expecting that independent random walking agents will generate a decisions time sequence that systematically would clean up the market.

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