1 Introduction

Recently, several stylized facts of financial markets, such as the power-law distribution of price increments, traded volumes, and volatilities, have been reported [1–5]. Critical phenomena in the physical systems have been the most interesting fields, and in these fields the twin concepts of scaling and universality have been proved to be important in a number of scientific fields [6]. Following these arguments, several works have been performed to financial systems and they have also reported some reasonable results [7–9]. In those works, the authors argued that the analogy with thermodynamic systems at critical point can be led to the explanation of the two-phase behavior of financial markets.

In the minority games, the volatility clustering is detected. To investigate the possible relationship between volatility clustering and the two-phase phenomena, Zheng et al [10] have analyzed the two-phase phenomena with the German DAX and compare it with those in minority games and herding models. They concluded that the volatility clustering and the two-phase phenomena are independent characteristics of financial dynamics. Matia and Yamasaki [11] showed that the scaling of trading volumes was to be a key factor in the emergence of two-phase behavior of volume imbalance conditional to the local intensity. The purpose of this paper is to examine mainly the two-phase phenomena for the Korean treasury bond (KTB) futures and
its correlated Brownian walk. As done in Ref. [10], we analyze the two-phase phenomena with the volatility instead of the volume imbalance [9].

2 Volatilites and Cumulative Distributions

The logarithmic increment is represented in units of ticks as
\[ G(t) = \log y(t + \Delta) - \log y(t) \] (1)

where \( y(t) \) denotes the futures quotation at time \( t \), and \( \Delta \) is the size of window, over which the logarithmic increment and the volatility are defined. The volatility of logarithmic increments is described as
\[ r(t) = \langle |\log y(t'' + 1)/y(t'') - \log y(t'' + 1)/y(t'')|\rangle \] (2)

Here the bracket denotes the average over the interval \([t, t + \Delta]\). The window size \( \Delta \) is set to be 1 min. The window size has no effect on the two phase behavior of the time series. To guarantee a large number of statistics under analysis, the statistics can be calculated using the overlapping window.

The detrended fluctuation analysis helps us to detect an inherent correlation in time series, and the detrended fluctuation is defined as
\[ F(s) = \left[ \frac{1}{N_s} \sum_{\nu=1}^{N_s} F^2(\nu, s) \right]^{1/2} \sim s^\alpha, \] (3)

where the root mean square fluctuation is determined by
\[ F^2(\nu, s) = \frac{1}{s} \sum_{k=1}^{s} [Y((\nu - 1)s + k) - Z_\nu(k)]^2 \] (4)

for each segment \( \nu \), where \( \nu = 1, \cdots, N_s \). Here \( Y(i) \) is the integrated time series or profile and \( Z_\nu(k) \) is the fitting polynomial in segment \( \nu \).

3 Numerical results and conclusions

We analyze the two-phase behavior of the derivative security from the data set of tick-by-tick recorded KTB412 futures quotations which was traded for six months from 1st July of 2004. This tick data consists of about 14,000 quotations. In general, since the trade of contracts becomes active as getting close to the maturity, the quotations are concentrated in the period of 3 months close to the maturity of contracts.

For a cumulative distribution \( P(G) \sim G^{-\mu} \), the values of the scaling exponent \( \mu \) are approximately 2.3 for the logarithmic return with \( \Delta = 1 \) min
Fig. 1. Plot of (a) the detrended fluctuation analysis and (b) the correlation function; the KTB412 and its correlated geometric Brownian walk for $\Delta = 1$ min.
Fig. 2. $P(G|r^*)$ of (a) the KTB412 and (b) its correlated geometric Brownian walk. These are conditional to the volatility. $r^*$ is used as a scale of real volatility and the scale is determined by dividing the range of volatilities into 100 intervals. A large $r$ is excluded since the corresponding data size is too small to get a reliable result.
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and 2.1 for the volatility of logarithmic increments with $\Delta = 10$ min in the KTB412, respectively.

Next we calculate the autocorrelation function and the detrended fluctuation analysis on the KTB412 and its correlated Brownian walk in order to check the existence of higher-order correlation. This analysis is performed on the volatility with $\Delta = 1$ min. Fig. 1(a) shows that the KTB412 and its correlated Brownian walk have, respectively, an correlation, while there is no correlation for shuffled KTB412 and the Brownian walk [12]. For the KTB412 and its correlated Brownian walk, we obtain $\alpha \simeq 0.75$ and $\alpha \simeq 0.91$, respectively.

Next we calculate the distribution of the logarithmic increments conditional to the volatility to examine the two phase behavior of KTB412 following the procedure taken in Ref. [10]. Figs. 2(a) and 2(b) show the distribution $P(G|r^*)$ for the KTB412 and its correlated Brownian walk. For convenience, we use the rescaled volatility, $r^* = 1, 2, \ldots$, for which a volatility range is defined as $[r_{\text{min}} + (r^* - 1) \times (r_{\text{max}} - r_{\text{min}})/100], r_{\text{min}} + r^* \times (r_{\text{max}} - r_{\text{min}})/100]$, where $r_{\text{min}} = 0$ and $r_{\text{max}} = 44.46$. For the KTB412, it appears two phase phenomena at $r^* = 5$, comparable to the German DAX [10]. The other data set of the correlated Brownian walk shows no clear distinction between distribution conditional to varying volatilities. It means that the two phase phenomena is not consistent with the volatility clustering.

In conclusion, we have examined the two-phase phenomena for the KTB412 and its correlated Brownian walk. We have confirmed that KTB412 with heavy-tailed distribution has the two-phase behavior, while the Brownian walk has no clue of it, which means that the two-phase behavior is independent of the clustering volatility. For a small volatility the price changes accumulate near zero, which implies that the market is efficient. As the volatility increases, it goes to the two-phase. According to this result, we can easily determine the state of the market if we find a reliable and novel method to estimate a volatility from small number of quotations.

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References