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MODELLING WEAR-OUT BY MULTISTATE HOMOGENEOUS MARKOV PROCESSES

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In this paper we study the possibility of analysing the reliability of systems, whose components experience wear-out, by representing each component by a Multistate Homogeneous Markov Model (MHMM). It is shown that this technique realizes a good compromise between the ability of correctly representing the features of the system to be modelled and the possibility of algorithmically determining the properties of the model. Some examples are reported.

1. INTRODUCTION

The success of any model depends on two factors: its modelling power and its decision power. Modelling power refers to the ability of correctly representing the system to be modelled. Decision power refers to the possibility of algorithmically determining the properties of the model. These two factors are generally in conflict.

Since reliability analysts are required to evaluate reliability measures pertinent to the system, they need to resort to models with high decision power but consequently low modelling power. We refer typically to models based on the assumption that components are s-independent with exponentially distributed failure/repair times.

However a very common physical situation in which these assumptions do not hold true is the presence of wear, that is the presence of component degradation which induces a failure time distribution belonging to the class IFR (increasing failure rate) or IFRA (increasing failure rate average) as discussed by Barlow and Proschan in [1]. Many approaches have appeared in the literature for modelling component wear-out, but they seem to have poor decision power when applied to systems, since they become algorithmically intractable as soon as the system complexity grows up.

As a compromise between modelling power and decision power we propose in this paper an approach based on the representation of the failure (or repair) process of each component by a Multistate Homogeneous Markov Model (MHMM). This approach, which is a generalization of the well known stage device [2, 3], seems to be well suited in wear-out modelling since it yields a better approximation of the behaviour of each component by increasing the number of the states of the model, while at the same time the overall system is still modelled by an homogeneous Markov process. Thus the MHMM can be viewed as a proposal for extending the modelling power of the standard two-state Markov approach while retaining most of its decision power.

Even if the stage device has been already applied to reliability problems [3, 4] we concentrate in this paper on its capability in wear-out modelling. In Sect. 2 we discuss its ability as a mathematical artifice for approximating non exponential probability distributions (IFR or IFRA) in comparison with other approaches. In Sect. 3 we discuss how to handle the MHMM as a modelling tool for representing actual systems with s-dependent components subject to wear and in the presence of different maintenance policies.

2. APPROXIMATION OF WEAR-OUT DISTRIBUTIONS BY MHMM's

The mathematical properties of the MHMM's have been deeply examined in [4] [5][6], thus in this section we recall only some basic definitions.

As known, the stage device consists in splitting the transition from one condition (say a working condition) to another condition (say a failed condition) in more than two states connected in series or in parallel. The MHMM generalizes this device by dropping the hypothesis of parallel-series connection, and can be defined as follows:

An N - MHMM is an N - state time continuous homogeneous Markov process characterized by its time - independent transition rate matrix Λ and by its initial probability vector R. The set of the N states is partitioned into two mutually exclusive subsets U and D such that the modelled system is up or down if the corresponding process is in U or D respectively.

If the D subset is absorbing (system without repair) the probability of being in D, as a function of time, is the cumulative distribution function (Cdf) of the passage time from U to D, i.e. the failure time Cdf; on the other hand, if D is not absorbing (system with repair), the probability of being in D is the system unavailability.

From a numerical point of view it has been found convenient to limit the study to Markov processes described by acyclic transition graphs, for which, after a suitable state ordering, the transition rate matrix assumes a triangular form (TMHMM). The Cdf of a generic N-TMHMM has a rational Laplace transform with non-positive real poles coincident with the diagonal entries of Λ.

It has been proved in [6] that an N-TMHMM can be transformed into canonical forms with the minimum number of parameters (i.e. the minimum number of non-zero entries of Λ). Even if there are at least three different canonical forms the most useful in wear-out problems is that reported in Fig. 1, in which

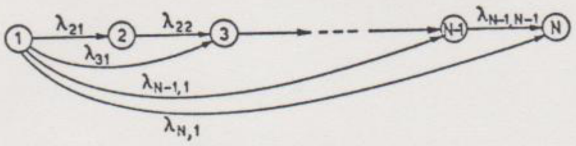


Fig. 1. Canonical form of an N-TMHMM.

(being N the down state) transitions are allowed from each state to the subsequent one and from the first state to all the others; moreover the diagonal entries of Λ must be ordered in the sequence $-\lambda_{11} \geqslant -\lambda_{N-1,N-1} \geqslant -\lambda_{N-2,N-2} \geqslant \cdots \geqslant -\lambda_{22} > 0$.

Since a generic N-TMHMM can always be transformed into the form of Fig. 1, in the sequel we will refer to canonical models only.

A worthnoting result in TMHMM theory [6] is that it is possible to approximate as closely as desired any Cdf as the number of states grows to infinity. However from an engineering point of view it is important to emphasize that a good fit to the most common Cdf's is obtained also with TMHMM's of low order. As an example we show how canonical TMHMM's of increasing order approximate the Weibull Cdf of Fig. 2, with shape

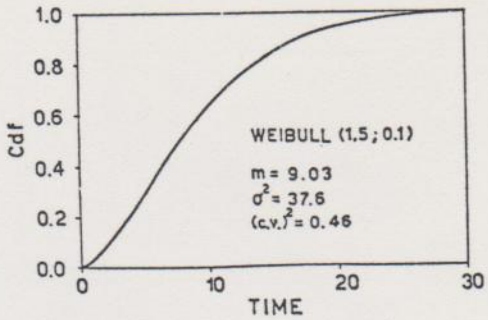


Fig. 2. Weibull distribution.

parameter 1.5 and scale parameter 0.1. The algorithm for determining the model transition rates which provide the best fit to a given Cdf has been described in [5], and is based on the minimization of the maximum absolute difference between the model Cdf and the Cdf to be approximated, i.e. on the minimization of the deviation measure adopted in the Kolmogorov-Smirnov goodness-of-fit test.

The results are presented in Table I in which the best TMHMM's of orders 3, 4 and 5 are shown together with the maximum deviation of their Cdf from that of Fig. 2. For the sake of comparison also the mean value, the variance and the squared coefficient of variation are shown in the Table.

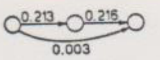
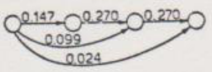
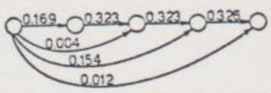
N	BEST TMHMM	MOMENTS	MAX. DEV.
3		$m = 9.17$ $\sigma^2 = 42.7$ $(c.v.)^2 = 0.50$	0.0104
4		$m = 9.07$ $\sigma^2 = 39.4$ $(c.v.)^2 = 0.48$	0.0036
5		$m = 9.03$ $\sigma^2 = 37.8$ $(c.v.)^2 = 0.46$	$6.3 \cdot 10^{-4}$

Table I. Canonical TMHMM'S which approximate the Cdf of Fig.2.

Some further examples of approximation of log-normal Cdf's have been already presented in [3] and [5]; moreover the capability of this technique in approximating multimodal Cdf's (as those which have bathtub-shaped hazard rate functions) has been studied in [5].

Other techniques for wear-out modelling have appeared in the literature (Markov degradation models, Poisson shock models, etc.) and an extensive discussion of their relation with the MHMM approach can be found in [7].

3. THE TMHMM AS A MODELLING TOOL IN WEAR-OUT PROBLEMS

A very attractive feature of the method described in the previous section is its ability in describing and analysing the performance of systems in situations in which other popular methods fail. The following examples indicate some areas in which the modelling power of TMHMM's can be successfully exploited.

3.1 System with s-independent components subject to wear

The general theory has been outlined in [4] and some further examples can be found in [8]. The problem is to model the failure time distribution of each component by a suitable TMHMM, and then to resolve the homogeneous Markov process of the whole system, taking advantage of the component s-independence (product rule for probabilities).

3.2 System with s-dependent components subject to wear

This case is of particular interest and will be treated more deeply. In the usual modelling techniques, where each component is characterized by a single up state and a single down state, the failure rate of a component can s-depend only on the working or failed condition of the other ones. And this restriction holds true also in the framework of non-homogeneous Markov processes, where non-exponential probability distributions are considered by introducing time-dependent transition rates. However in actual systems the failure rate of a component is likely to s-depend on the wear condition of the others; in particular for series - connected systems, this kind of s-dependence is the only possible one. As long as experimental data on such a kind of s-dependence are available, the TMHMM technique offers a tool for modelling this situation.
As a very simple example, consider a system consisting of two identical components connected in series. Each component is described by a 3-state series model (second order gamma distribution), where states 1 and 2 are working and state 3 is failed, and the transition graph of the system is the one depicted in Fig. 3 where the dashed line encircles the states corresponding to system failure. It is then clear that in case of s-independence $a' = a$ and $b' = b$,

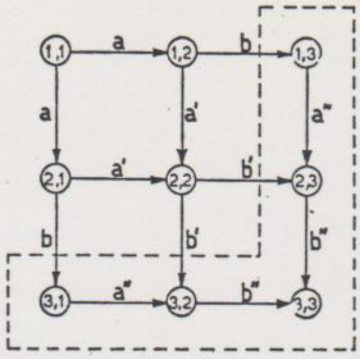


Fig. 3. Transition graph of a 2-component series system with s-dependent wear and failure.

while letting $a' > a$ and $b' > b$ allows to take into account the increased failure probability of each component due to the degraded working condition of the other. In Fig. 4 the reliability function of this system is plotted for increasing values of a', b' and fixed values of a and b.

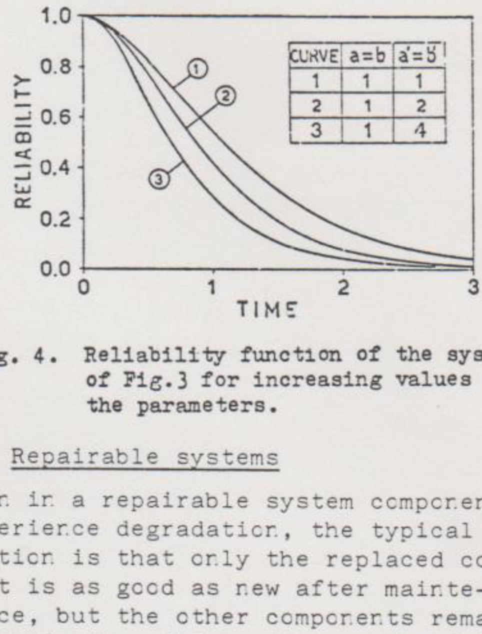


Fig. 4. Reliability function of the system of Fig.3 for increasing values of the parameters.

3.3 Repairable systems

When in a repairable system components experience degradation, the typical situation is that only the replaced component is as good as new after maintenance, but the other components remain in their degraded condition. The failure /repair process of the whole system is thus no more a renewal process, since the successive up and down times are not identically distributed but depend on the previous replacements. This problem though very simple and common in practice, is difficult to model and analyze in the framework of stochastic processes. For instance the non homogeneous Markov approach fails when applied to repairable systems, since in this framework repaired components (with IFR distribution) are intrinsically worse than old. The use of the stage device for modelling repairable systems has been first suggested by Chan and Downs in [9]. The following simple example illustrates the difference between a non homogeneous Markov model and the MHMM in the analysis of repairable systems. Consider a component whose failure time follows a second order gamma distribution:

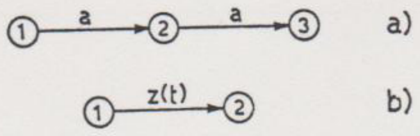


Fig. 5. **a) 3-state series TMHMM; b) equivalent 2-state nonhomogeneous model.**

the TMHMM of Fig. 5a, and the non homogeneous model of Fig. 5b are exactly equivalent if the time-dependent transition rate is given by $z(t) = a^2t/(1 + at)$. However if the component is supposed repairable (with constant repair rate μ) the corresponding models of Fig. 6a and 6b do not provide the same results: for

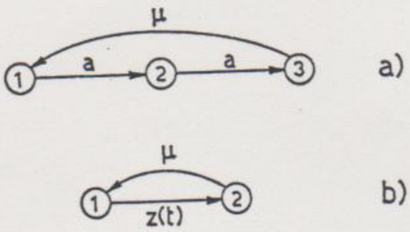


Fig. 6. **Same models as in Fig.5a) and 5b) with repair.**

instance the steady state availability is:

$$A'_s = \frac{2/a}{2/a + 1/\mu} = \frac{MUT}{MUT + MDT} \quad \text{for 6a}$$

$$A''_s = \frac{1/a}{1/a + 1/\mu} = \frac{1/z_\infty}{1/z_\infty + MDT} \quad \text{for 6b}$$

Thus while in the model of Fig. 6a the steady state availability depends on the mean up time (MUT) and on the mean down time (MDT) (regenerative process), in the model of Fig. 6b it depends on the limit of $z(t)$ as $t \to \infty$, that is the process becomes asymptotically regenerative.
Another widely used approach for modelling repairable systems is to consider an s-independent (or unrestricted) repair policy. Under this assumption every component of the system describes an independent alternating renewal process [10] and the overall system availability can be obtained by the product rule for probabilities. For the two-component system of Fig. 3 the s-independent repair policy (with constant repair rate μ) corresponds to the transition graph of Fig. 7, and is equivalent

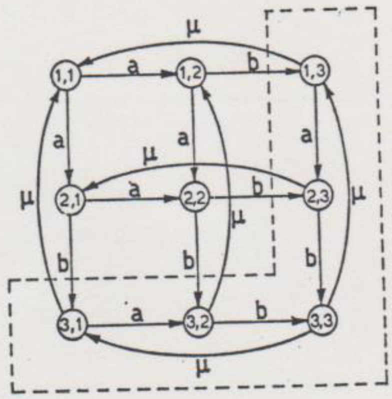


Fig. 7. **Transition graph of a 2-component series system with s-independent repair.**

to the superposition of independent alternating renewal processes.
The worthnoting feature of the schema of Fig. 7 is that the replacement of a component restores it into an "as good as new" condition without affecting the possibly degraded state of the other one (e.g. from state 3-2, the only possible repair transition is to state 1-2, where component 1 is as good as new and component 2 in its degraded state).
However the assumption of s-independent repair is often a crude simplification of the real world and the MHMM's offer a natural way for modelling more realistic maintenance policies:

i) System is switched off during maintenance.

This case is pertinent to non-redundant systems for which every component failure gives rise to a system failure, and hence the system is down during repair. As an example we report in Fig. 8 the state transition diagram which depicts this situation for the system of Fig. 7 assuming that the two components are series-connected (failed states encircled in the dashed line). Fig. 9 plots the availabilities calculated from the models of Figs. 7 and 8 for a set of parameter values. For this very simple system the difference between the two curves is not very large (and disappears either for $\mu = 0$ - istantaneous repair - or for $\mu = \infty$ - no repair at all -); however the maximum deviation is reached in steady state conditions, i.e. in the conditions most frequently used in system

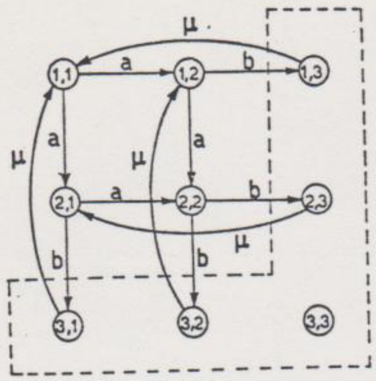


Fig. 8. Transition graph of a 2-component series system switched off during repair.

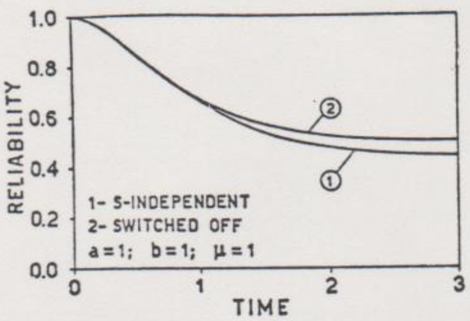


Fig. 9. Reliability functions of the systems of Fig.7 and Fig.8.

analysis; in particular the s-independent policy underestimates the actual system availability.

ii) Components are replaced at system failure.

This case is pertinent to redundant systems where the repair action is undertaken only at system failure (with single or multiple on line maintenance). For instance for the system of Fig. 10 [9],

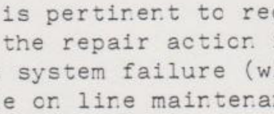


Fig.10. Three-component system.

if A fails the system is failed and repair undertaken; if B fails repair begins either when C or A fail, and the two failed components are repaired in succession in the case of single repair facility or simultaneously in the case of multiple repair facility. The complete MHMM transition graph is omitted for the sake of brevity but it could be built up quite easily following the preceding examples.

4. CONCLUSIONS

The general philosophy of the MHMM approach is that, by increasing the number of states beyond two, many practical problems can be modelled in a natural way.

The paper has explored in particular the use of MHMM's in wear-out problems and has shown that they provide a valuable tool both for approximating different kinds of wear-out distributions encountered in the literature, and for modelling systems whose components are subject to wear. In this last case some examples have been reported which indicate how to tackle practical situations that cannot be easily faced up by other methods.

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