North-Holland
Microprocessing and Microprogramming 29 (1990) 1-13

Dependability analysis of fault-tolerant systems: a literature survey

Andrea Bobbio

Istituto Elettrotecnico Nazionale Galileo Ferraris, Strada delle Cacce 91, 10135 Torino, Italy

Systems with high reliability requirements are designed with a great degree of fault tolerance. These systems use extensive redundancy, have complex recovery management techniques, and are highly reconfigurable. There are two classes of problems that arise in modelling the dependability of complex fault tolerant architectures: the construction of a comprehensive model of the system, and the solution of the model once formulated.

This paper surveys the literature on the research work available in the area of dependability modelling, with particular emphasis on the modelling techniques, the adopted numerical methodologies and the implemented software tools.

Keywords: Dependability analysis, Reliability, Performability, Fault-tolerance, Complex systems, Modelling and analysis.

Andrea Bobbio graduated in nuclear engineering from Politecnico di Torino in 1969. In 1971 he joined the Istituto Elettrotecnico Nazionale Galileo Ferraris, where he was involved in researches on degradation mechanisms in electronic devices.

In 1974 he was assigned to the staff for setting up an Italian Reliability Data Bank on electronic components in cooperation with Italian electronic industries. From 1978 he turned his attention to the modelling and analysis of the performance and reliability of complex systems, with particular emphasis to the areas of Markov models, stochastic reward models and Petri nets.

During 1984, 1986 and 1987 he has been visiting scientist in the Department of Computer Science at the Duke University, Durham NC (USA), and in 1989 in the Institute of Electronics and Telecommunication of the Technical University of Budapest.

He is author of several reliability oriented papers, and he has been principal investigator in several grants with public and private institutions.

Work partially supported by Centro Ricerche Fiat.

1. Introduction

The increasing complexity of electronic systems demands for a high degree of fault-tolerance. Classical motivation for fault-tolerance can be found in [12], and techniques for designing fault-tolerant structures are addressed to in [142]. In order to properly express the attributes of fault-tolerant (FT) systems a new terminology has recently emerged from the scientific and technical community [98]. This new terminology, with clear and widely acceptable definitions is presented in [13]: "Dependability is that property of a computer system that allows reliance to be justifiably placed on the service it delivers [13]". Quantifying the alternation between delivery of proper or improper service and quantifying the total amount of work or the performance level of the service versus time leads to the definition of distinct measures: reliability, availability, performability. These measures correspond to different perceptions of the same attribute of the system: its dependability [13]. An error is the phenomenological manifestation of a fault which is usually classified in two classes: physical fault and human-made fault. An *error* is that part of a system state which is liable to lead to a failure. A *failure* occurs when the delivered service deviates from the specified service. Achieving a dependable system calls for the utilization of a set of methods which can be classed into: fault avoidance, fault tolerance, fault removal, fault forecasting.

Quantitative evaluation of the dependability measures requires the development of suitable mathematical modelling and analysis techniques. In order to properly model the system service versus time there is a need to take into account in a single framework [157]:

- the structure of the system and the interactions between its modules
- the stochastic occurrence of a fault
- the propagation of a fault into an error and a failure

- the fault error detection and recovery mechanism
- the effect of a fault on the delivered service (the performance level).

The effectiveness of a model is related to its modelling power (ability to represent the interesting features of the real system) and to its analytical tractability (ability to evaluate properties and behaviours of the system by solving equations rather than running the real system). Dependability evaluation is of great practical importance for identifying current problems and correct them, for preventing future problems by improving design, for predicting the behaviour of the system versus time and for supporting decision in design and technical assistance.

This paper intends to provide an extensive survey on the literature available in the area of dependability modelling and analysis of complex systems. In Section 2, the techniques for modelling the occurrence of a fault, the recovery from a fault and the effect of a fault on the performance of the system are considered. In particular, the need for a combined evaluation of performance and reliability measures is emphasized. Section 3 reviews the numerical techniques proposed to solve complex dependability models both in transient and in steady state. Three issues are particularly addressed: the largeness of the model, the stiffness, and the computation of more detailed measures of effectiveness.

2. Modelling techniques

Pioneering work in the area of modelling and analysis of FT systems was carried out by Bouricious et al. at IBM [32,31]. The basic architecture considered by the authors was a standby redundant sparing system with the incorporate ability to recover from a fault by inserting a sparing module. The effect of the recovery strategy was captured into a single parameter called the *coverage* factor. The coverage was formally defined as the probability of surviving a failure without irreparable damage. The authors showed the remarkable influence of the coverage on the reliability features of the system. Several subsequent papers [10,22,148,157,5,60] have studied the effect of the coverage on redundant recongifurable systems.

Since then, two main lines of research have been pursued. The first one attempts to provide a detailed representation of the fault handling mechanism (FHM) and to incorporate the FHM into a general modelling framework for the whole system [154,70,159]: the second one investigates the effect of a fault, and of the consequent reconfiguration [163,94,121], on the performance level of the system. In parallel with the refinement of the modelling techniques, there has been a need to define new and more significant dependability measures. While the classical reliability and availability measures [150,15] are calculated at a given instant of time or in steady state and can thus be classified as *instantaneous* measures, the development of the dependability theory has led to the definition of *cumulative* measures, that reflect system's operational characteristics over a finite time interval [73]. Since cumulative measures are defined over a finite horizon, their computation requires the transient analysis of the underlying stochastic process.

2.1 Fault handling model for FT systems

In order to construct a general dependability model three sets of inputs are necessary [158]: the system structure, the fault-occurrence behaviour, and the fault-handling behaviour. The first category of inputs provides information regarding the set of components or modules, their interconnections and conditions under which the system will fail. Usual approaches for representing the system structure at this level are: block diagrams [15], fault-trees [15,145], Petri nets [4,14,19,110], directed graphs [139].

The second category of inputs describes the stochastic occurrence of faults in a system. For numerical reasons, the fault-occurrence model is usually described in terms of a discrete-state continuous-time homogeneous Markov chain. The markovian assumption implies that all the random times appearing in the model are exponentially distributed. To overcome this limitation Semi-Markov models [41,131] or regenerative processes [49,20,21] have been considered.

The third category of inputs models the behaviour of the system when a fault occurs and is called the fault-handling model. The information gathered into the fault-handling model refers to the class of the fault, the possible propagation of the fault into errors and final failures, and the way in which the system reacts to the fault. Basically, a fault can be dormant or active, an error latent or detected and a failure benign or catastrophic. Exits from the fault-handling model can be usually categorized into three types: transient restoration (as a consequence of an intermittent fault or a benign failure), successful recovery, or fatal failure.

Due to the high sensitivity of the dependability measures with respect to the fault-handling model, extensive research work has been devoted to characterize the type of the fault, the propagation of the fault into error and failure, and the final outcome from the fault-handling model by means of discrete-state representations. Papers dealing with the modelling of fault-handling mechanisms can be found in [154,70,100,148,149,158,5,45]. A Petri net approach to the fault-handling model has been presented in [157]. In [141] the use of retry techniques for recovery from transient faults has been analysed. A discussion and a comparison among various techniques for modelling the coverage in a FT system is reported in [60].

The combination of the three mentioned categories of inputs into a single final model generates two main problems: the explosion of the state space, and the potential appearance of numerical instabilities caused by the presence in the combined model of time constants (the fault-occurrence rates and the fault-handling rates) which differ by many orders of magnitude. Computer packages developed with the intent of coping with these problems are briefly mentioned in paragraph 3.4.

2.2 *Performance oriented reliability analysis*

Fault-tolerance may assume different forms. A way to achieve high reliability coupled with high performance is to allow the system to be gracefully degrading. Graceful degradation [30] means that, upon occurrence of a fault, the system attempts to reconfigure into an operating state with one fewer active module: a successful reconfiguration keeps the system operational but with reduced performance capabilities so that the delivered service degrades. In this case, the most appropriate and significant measure is the ability of the system to provide a given amount of work in a given time taking into account failures and repairs. The total work accumulated by the system up to a given time is a random variable called *performability*. Performability analysis merges the traditional fields of performance evaluation and reliability.

The classical methods of the steady-state (fault-free) performance evaluation [83] overestimate system capacity as a function of time. On the other hand, classical reliability theory is based on the assumption that each component, and the system as a whole, can be modelled by a binary variable [15] representing two possible states: functional or non-functional. This assumption implies that the state space of the system can be univocally partitioned into two mutually exclusive subsets of states, one containing the up states, the other containing the down states. The classical reliability measures are defined on this binary partitioning of states.

The new methodology, proposed to face up the unified analysis of performance and reliability for FT systems, consists in modelling the variation of the system configuration versus time with a discrete state stochastic process, and associating to each state a non-negative real constant representing the effective working capacity (or performance level) of the system in that state. The stochastic process is referred to as the *structure-state* process and the associated constant is referred to as the *reward rate*. The structure-state process, together with the reward assigned to each state form the Stochastic Reward Model [133].

Markov and Semi-Markov reward models have been the subject of an extensive literature [106,41,92,93,84], but only recently they have received attention as algorithmically feasible tools in the dependability analysis of FT or degradable systems.

The idea of introducing a performance index (or reward rate) measuring the effective computation capacity of a system in each configurational state was first proposed in [17]. A formal definition of the performability (with the proposal of this neologism) has been given in [109].

Two different views in performability analysis can be envisaged:

- a *system oriented view* [17,69,108,56,87] in which the aim of the analysis is to determine the ability of the system to provide a given amount of service at an acceptable accomplishment level; in the framework of Stochastic Reward Models, this view implies the evaluation of the distribution function of the random variable defined as the total accumulated reward up to a given time

- a *task oriented view* [68,36,37] in which the aim is to evaluate the distribution function of the completion time of a task that requires an assigned amount of work; in this case, the interaction of the task in progress with the system when a fault occurs and a recovery procedure is started plays a very important role [68,96] on the execution of the task.

An unified approach to the above points of view has been elaborated by Kulkarni et al. in [96,95]. In this approach, the system is modelled by a Reward Stochastic Model, where two possible mechanisms of preemption and recovery between the task in execution and the system have been considered:

- When a change of state occurs in the structure-state process, the system keeps memory of the work already done and the task in progress is resumed in the new state. This policy is called *preemptive resume (prs)*.
- When a change of state occurs in the structure-state process, the system cannot keep track of the past, and the work already done is lost; the task in progress must be restarted from scratch in the new state. This policy is called *preemptive repeat identical (pri)* if the repeated task has the same work requirement of the original preempted task, and *preemptive repeat different (prd)* if the work requirement is different, but sampled from the same distribution.

Research work in the area of performability analysis is aimed at evaluating the distribution function of the accumulated reward up to a given time (*performability*) or of the task completion time, when the structure-state process is a Markov or a Semi-Markov process and under various combinations of preemption policies. Closed form solutions for the distribution functions have been obtained in the Laplace transform domain: recent work can be found in [9,56,75,86,87,155,62,77].

The ability of evaluating the distribution function of cumulative measures defined over the structure-state process has stimulated the definition of new quantities that more precisely characterize the behaviour of the system over a finite horizon. An interesting cumulative measure is the interval availability [73,76,63,62,57], defined as the proportion of time spent in the operating states during a finite time interval. A further enlargment of the classical reliability theory consists in analysing the system lifetime, or the repair strategy, when the system breakdown depends on the total time spent in the set of the down states [64,137,147,74].

2.3 Petri Nets

Petri Nets (PN) are a graphical tool for the formal description of the logical interaction or of the flow of activities in complex systems [1,128]. With respect to other more popular graphical techniques, PN are particularly suited to model in a natural way situations like concurrency, conflict, blocking, synchronization and sequentiality. A recent tutorial paper, with an extended bibliography, on the classical theory of PN can be found in [118].

The classical PNs do not convey any notion of time; in order to make the model suitable for the quantitative analysis of the time behaviour of systems a class of extended models called *Stochastic Petri Net (SPN)* has been formulated. The basic idea in SPN [120,115,3,66] is to associate to each PN-transition a random variable representing the amount of time that must elapse before an enabled transition can fire. When all the random variables are exponentially distributed, the SPN can be mapped into a homogeneous Markov chain, so that the dynamic of the system can be evaluated by solving the corresponding Markov chain equations. When the random variables associated to the PN-transitions are generally distributed, the semantics of the model becomes more complex. In order to establish a procedure for univocally mapping the PN into a stochastic point process, further specifications should be given. The set of these specifications forms the PN-*execution policy* [2].

Examples of the use of SPN for modelling FT degradable systems have been considered in a number of papers: [19,4,157,59,110,6,7]. In particular, [157] and [59] propose PN models for representing complex fault-occurrence/fault-handling behaviours. The use of SPN as a tool for the quantitative analysis of stochastic systems has been surveyed in [23].

2.4 Fault trees

The representation of the occurrence of a particular event in a system (the top event) by means of a logical tree is still one of the most important and diffused technique in reliability engineering and safety. When the top event is a critical condition for the system and the basic leaves initiating the tree are non-operational states for the components, the logical tree is referred to as the *fault tree*. The construction and analysis of fault trees is described in many excellent reliability textbooks as, for instance, in [15]. A bibliographic review on methods, tools and applications is given in [101].

The basic idea behind the methods for both qualitative and quantitative analysis of fault trees is the reduction of the top event in terms of basic events using boolean algebra [145]. Recent results along this line are reported in [116,144,145]. The probability of the top event can be evaluated either by means of exact algorithms [126], or by approximate algorithms [113,143]. In the latter case the tightness of the approximation is of primary concern [143]. Moreover, the lack of pertinent failure rate data for the basic events has stimulated studies on the propagation of the uncertainty along the tree [44,138,105]. A representation of fault trees in terms of Petri nets has been described [85], where some results from the PN theory are exploited in the fault trees analysis.

The straightforward area of application of fault trees is the analysis of very large systems of binary independent components. Several extensions have been recently proposed aimed at making the technique suited for modelling the dependability of FT systems. Fault trees with multiple components (components represented by more than two states operating at possibly different performance levels) have been discussed in [162]. Common cause failures (uncovered failures in FT systems can be categorized as a particular type of common cause) have been investigated in [116], while in [58] the coverage models are explicitly included in the fault tree analysis.

3. Numerical techniques in dependability analysis

While fault trees are solved by resorting to combinatorial techniques and Boolean algebra, the modelling techniques surveyed in 2.1, 2.2 and 2.3 lead to the formulation of stochastic point processes. The numerical analysis of the process provides the desired values for the dependability measures.

When the object of the analysis is the computation of the instantaneous measures or of the time averaged expected values of the cumulative measures, the numerical problem consists in evaluating the state probabilities versus time, or in steady state, of the corresponding stochastic process. For the sake of numerical tractability, the time behaviour of the model is usually assumed to be represented by a homogeneous Markov chain. Even if the numerical solution of Markov chains has been widely considered in the literature, two main problems still require research work: the model largeness and the model stiffness.

When the expected values of the dependability measures are not sufficiently accurate for the characterization of the system effectiveness, the complete distribution functions of the cumulative measures need to be estimated. These distribution functions are expressed in closed form as analytic functions in the complex space. Different solution methods have been explored in this case, and an overview is reported in paragraph 3.2.

3.1 Numerical analysis of large stiff Markov chains

We can distinguish between methods for solving steady state Markov equations and methods for solving transient Markov equations. In the first case, the problem assumes the form of a set of linear equations for which standard solution methods can be found in classical textbooks on matrix computation [153,71]. Due to the large dimension and the sparsity of the transition rate matrix, iterative methods (like Gauss-Seidel or Successive Over Relaxation – SOR) are usually preferred. Peculiar studies devoted to the steady state solution of Markov chains are given in [119,80,35,97].

For what concerns the transient analysis, the problem consists in solving a set of first-order differential equations. These equations are often sparse and stiff. Stiffness [112] arises when the model contains time constants very short with respect to the integration interval. In dependability modelling of FT systems, stiffness is caused by the need to include into a single comprehensive model events which occur in very different time scales: failures and repairs [99], fault free system operation and fault occurrence behaviour [4,108], fault handling and fault occurrence behaviour [160].

A comparison among different solution techniques for the transient analysis of Markov chains is presented in [114,134,104]. Following these results, the most promising techniques are the randomization technique [79,91,111,81,107] (for what concerns accuracy and execution time), and implicit methods for ordinary differential equations when the stiffness ratio increases [112,82,42,39,134].

In order to cope with the explosion of the state space, the following three main research lines can be identified.

3.1.1 Automatic generation of the transition rate matrix

If an algorithm can be envisaged to build up the complete transition rate matrix starting from smaller blocks only the block submatrices need to be stored. The automatic techniques for the matrix generation can be based either on graphical approaches (like block diagrams or Petri nets), or on algorithms which exploit the structure and the symmetries of the complete transition rate matrix. The Kronecker algebra for matrices [33,54] has been used for this purpose in the reliability field [8,34,102]. However, other algorithms, based on a different ordering of the state space with respect to the one induced by the Kronecker algebra, are available [127]. These automatic algorithms fall short in the presence of common cause failures (more than one component failure in a state) as pointed out in [102]. Methods to overcome this limitation are investigated in [130].

3.1.2 Aggregation/disaggregation techniques

These techniques are aimed at decomposing the original problem into subproblems in such a way that the final (either exact or approximate) solution can be obtained by solving smaller and more stable sets of equations. These methods can be applied both to the steady state as well as to the transient analysis. The fundamental principles of these algorithms have been discussed in [47,40,146,65,156]. A transition rate matrix, that contains strongly connected blocks which are weakly connected to each other, is said to be *nearly completely decomposable*. In this case, a single step aggregation algorithm for the steady state analysis has been formulated in [47,161]. The extension of the same idea to the transient analysis has been discussed in [27]. The peculiarity of this aggregation technique is that a stiff problem is decomposed into smaller non-stiff subproblems. A perturbation approach to separate different time scales in a Markov chain is discussed in [129,43,136].

Bounds for the steady state decomposition algorithm are investigated in [48]. In [152] it is proved that the elimination of the stiffness by the reduction of the fault handling model to a branch point (the coverage probability approximation) provides always conservative reliability estimations.

An approach, referred to as behavioural decomposition, based on the inspection of the physical system has been illustrated in [160]. When the transition rate matrix is defined in terms of a Kronecker product of submatrices, peculiar solution algorithms have recently appeared [102,130,156].

3.1.3 State space truncation

Since real systems are designed to have a high level of reliability or availability, they spend most of the time in states with the majority of their components operational. This observation implies that most of the probability mass is concentrated in a relatively small subset of the state space. Truncation techniques are thus intended at generating only that part of the whole state space in which the system spends most of the time so that results calculated from the truncated state space are accurate enough with respect to the exact values. In fault tree analysis the order of the cut sets provides a natural way to truncate the analysis at a preassigned accuracy level [113]. In the transient analysis of Markov chains, an algorithm to generate a reduced state space has been presented in [18]. The truncation criterion is based on the computation of the probability of exiting from the already generated state space at a given time. When the exiting probability is below a preassinged threshold level, the state space generation stops. In the case of the steady state availability analysis, a truncation algorithm is presented in [117]. This algorithm provides both a lower and an upper bound, whose computation is based on results reported in [48].

3.2 Computation of cumulative measures

Cumulative measures [73] reflect the system characteristics over a finite time interval. When the performability model is in the form of a Stochastic Reward Process, typical cumulative measures are the total accumulated reward (*performability*) and the task completion time. Cumulative measures are random variable that can be characterized through their expected values or their distribution functions [133].

3.2.1 Expected values

The computation of the expected values of the cumulative measures involves the computation of the integrals of the state occupation probabilities of the associated Markov chain. The computation of the integrals is done at a cost of the same order of the cost for the computation of the instantaneous probabilities. All the numerical techniques available for the transient analysis can be modified for the computation of the integrals, as described in [135]. The extension of the decomposition technique for stiff Markov chains to the integral case has been presented in [29].

3.2.2 Distribution functions

The computation of the distribution functions of the cumulative measures requires a very involved numerical procedure [56,96], even in the simplest case in which the underlying fault-occurrence model is a Markov chain.

An algorithm for the solution of performability problems when the preemption policy is of *resume* type has been illustrated in [151]; this algorithm computes first the eigenvalues of a particular matrix and then resorts to the numerical inversion of a Laplace transform equation (for which a wide choice of different methods are available [67,51,55,89,88]). In [87] a recursive formula has been proposed for computing the moments of the accumulated reward by resorting to the spectral representation of the transition rate matrix. In [78] and [155] the first two moments of the distribution of the performability have been calculated by means of the randomization technique. An extension of the randomization technique for the computation of the distribution of the interval availability and of some performability figures has been investigated in [63] and [62], respectively.

A different and computationally more versatile approach has been proposed in [28,26]. This approach is based on the use of a family of distribution functions called Coxian [50] or Phase type (PH) distributions [122,123], which are defined as the distribution of the time till absorption of a continuous time homogeneous Markov chain. Indeed, if the work requirement of a given task is a PH random variable, the task completion time is still a PH random variable [28] and can thus be calculated by solving a homogeneous Markov chain. The continuous problem in the complex space is converted into a discrete problem in the real space. This technique is able to accommodate for any probabilistic mixture of preemption policies.

However, a great deal of research is still needed in the field of efficient numerical algorithms for the computation of the distribution function of cumulative dependability measures.

3.3 Non-Markovian models

A further important point to be mentioned is the need of extending the modelling power of the proposed methodologies by allowing the relevant random variables to be generally distributed, thus overcoming the exponential assumption. Even if Semi-Markov or regenerative processes have been investigated for this purpose, the most promising approach, from a computational point of view, seems to be the use of the class of PH distributions mentioned in the previous paragraph [123]. A non-markovian discrete-state stochastic process whose transition times are PH random variables can be converted into a homogeneous Markov chain defined over an extended state space [24]. Therefore, the efficient numerical techniques developed for Markov chains can be applied in this case.

PH distributions combine formal elegance with computational efficiency [122,53]; moreover, the PH family forms a dense set of distributions so that any distribution function can be approximated as closely as desired by a member of this family. A number of usual distributions often encountered in applied stochastic modelling (like: the Erlang distribution, the hyperexponential and, obviously, the exponential) belongs to the PH family.

Interesting properties have been proved for these distributions in the reliability field [11], and applicative examples are reported in [124,25,38]. General purpose packages incorporating the class of PH distributions have begun to appear [46,52,140].

3.4 Software tools

Examples of automated analytic models for dependability analysis, which incorporates at different levels of details some of the features examined in the previous paragraphs are: ARIES [103,125], SURF [46], CARE III [154], SAVE [72] HARP [61,16], SHARPE [140].

A critical evaluation of the mathematical fundations of some of these packages has been performed in [70]. An extensive comparison among available software tools (non restricted to the ones mentioned above) is documented in [90]. The characteristics considered in [90] are: the area of application, the required input specifications, the stochastic model on which the package is based, the adopted solution technique and the obtained output measures.

4. Conclusion

A survey of the literature dealing with the modelling and analysis techniques for the evaluation of the dependability of FT or degradable complex system has been reported.

As a final comment it should be stressed that the theoretical activity related to the development of dependability models is necessary for coping with the increasing complexity of new FT systems; however, this activity to be successful and practically applicable must proceed together with the actual implementation of FT systems and with the knowledge of experimental field data regarding the performance features and the failure rates of the modules integrated in the FT realization.

References

[1] T. Agerwala, Putting Petri nets to work, *Computer* (December, 1979) 85–94.

[2] M. Ajmone Marsan, G. Balbo, A. Bobbio, G. Chiola, G. Conte and A. Cumani, The effect of execution policies on the semantics and analysis of stochastic Petri nets, *IEEE Trans. Software Engrg.* SE-15 (1989) 832–846.

[3] M. Ajmone Marsan, G. Balbo and G. Conte, A class of generalized stochastic Petri nets for the performance evaluation of multiprocessor systems, *ACM Trans. Computer Systems* 2 (1984) 93–122.

[4] M. Ajmone Marsan, A. Bobbio, G. Conte and A. Cumani, Performance analysis of degradable multiprocessor systems using generalized stochastic Petri nets, *IEEE Computer Society Newsletters* 6, SI-1 (1984) 47–54.

[5] H.H. Amer and E.J. McCluskey, Calculation of coverage parameters, *IEEE Trans. Reliability* R-36 (1987) 194–198.

[6] H.H. Ammar, Y.F. Huang and Ruey-Wen Liu, Hierarchical models for system reliability, maintainability and availability, *IEEE Trans. Circuits and Systems* CAS-34 (1987) 629–638.

[7] H.H. Ammar and S.M.R. Islam, Time scale decomposition of a class of generalized stochastic Petri net models, *IEEE Trans. Software Engrg.* SE-15 (1989) 809–820.

[8] V. Amoia, G. De Micheli and M. Santomauro, Computer oriented formulation of transition rate matrices via Kronecker algebra, *IEEE Trans. Reliability* R-30 (1981) 123–132.

[9] J. Arlat and J.C. Laprie, Performance-related dependability evaluation of supercomputer systems, in: *Proc. 13-th Internat. Symp. Fault-Tolerant Computing* (1983) 276–283.

[10] T.F. Arnold, The concept of coverage and its effect on the reliability model of a repairable system, *IEEE Trans. Computers* C-22 (1973) 251–254.

[11] D. Assaf and B. Levikson, Closure of phase type distributions under operations arising in reliability theory, *Ann. Probab.* 10 (1982) 265–269.

[12] A. Avizienis, Fault-tolerance, the survival attribute of digital systems, *Proc. IEEE* 66 (1978) 1109–1125.

[13] A. Avizienis and J.C. Laprie, Dependable computing: from concepts to design diversity, *Proc. IEEE* 74 (1986) 629–638.

[14] J.M. Ayache, P. Azema, and M. Diaz, Towards fault-tolerant real-time systems using Petri nets, in: *Proc. 1-st European Workshop Theory and Application of Petri Nets* (1980).

[15] R.E. Barlow and F. Proschan, *Statistical Theory of Reliability and Life Testing* (Holt, Rinehart and Winston, New York, 1975).

[16] S.J. Bavuso, J.B. Dugan, K.S. Trivedi, E.M. Rothmann and W.E. Smith, Analysis of typical fault-tolerant architectures using HARP, *IEEE Trans. Reliability* R-36 (1987) 176–185.

[17] M.D. Beaudry, Performance-related reliability measures for computing systems, *IEEE Trans. Computers* C-27 (1978) 540–547.

[18] K. Begain, G. Farkas, L. Jereb and M. Telek, Step-by-step state space generation for the reliability of complex markovian systems, in: *Proc. 7-th Symp. Reliability in Electronics (RELECTRONIC '88)* (Budapest, 1988) 314-321.

[19] B. Beyaert, G. Florin, P. Lonc and S. Natkin, Evaluation of computer systems dependability using stochastic Petri nets, in: *Proc. 11-th Internat. Symp. Fault-Tolerant Computing* (1981) 79–81.

[20] A. Birolini, Some applications of regenerative stochastic processes to reliability theory, Part I: tutorial introduction, *IEEE Trans. Reliability* R-23 (1974) 186–194.

[21] A. Birolini, Some applications of regenerative stochastic processes to reliability theory, Part II: reliability and availability of 2-item redundant systems, *IEEE Trans. Reliability* R-24 (1975) 336–340.

[22] A. Bobbio, The effect of an imperfect coverage on the optimum degree of redundancy of a degradable multiprocessor system, in: *Proc. RELIABILITY '87* (Birmingham, 1987) Paper 5B/3.

[23] A. Bobbio, *System modelling with Petri nets*, Technical Report IEN-361, Ispra course JRC-ASR/88/7, September 1988, to be published.

[24] A. Bobbio and A. Cumani, Discrete state stochastic systems with phase-type distributed transition times, in: *Proc. AMSE Internat. Conf. Modelling and Simulation*, Athens 1984, 173–192.

[25] A. Bobbio and A. Cumani, A Markov approach to wear-out modelling, *Microelectron. Reliability* 23 (1983) 113–119.

[26] A. Bobbio, L. Roberti and E. Vaccarino, Computing cumulative measures in reward stochastic processes by a phase type approximation, in: V. Colombari (ed.), *Reliability Data Collection and Use in Risk and Availability Assessment* (*Proceedings of 6-th EUREDATA Conference*) (Springer, New York, 1989) 682–693.

[27] A. Bobbio and K.S. Trivedi, An aggregation technique for the transient analysis of stiff Markov chains, *IEEE Trans. Computers* C-35 (1986) 803–814.

[28] A. Bobbio and K.S. Trivedi, Computation of the distribution of the completion time when the work requirement is a PH random variable, *to be published in Stochastic Models*, 1990.

[29] A. Bobbio and K.S. Trivedi, Computing cumulative measures of stiff Markov chains using aggregation, *to be published in IEEE Trans. Computers*, 1990.

[30] B.R. Borgerson and R.F. Freitas, A reliability model for gracefully degrading and standby-sparing systems, *IEEE Trans. Computers* C-24 (1975) 517–525.

[31] W.G. Bouricius, W.C. Carter, D.C. Jessep, P.R. Schneider and A.B. Wadia, Reliability modeling for fault-tolerant computers, *IEEE Trans. Computers* C-20 (1971) 1306–1311.

[32] W.G. Bouricius, W.G. Carter and P.R. Schneider, Reliability modeling techniques for self-repairing computer systems, in: *Proc. 24th Nat. Conf. ACM* (1969) 295–309.

[33] J.W. Brewer, Kronecker products and matrix calculus in system theory, *IEEE Trans. Circuits and Systems* CAS-25 (1978) 772–781.

[34] G. Cafaro, F. Corsi and F. Vacca, Multistate Markov models and structural properties of the transition-rate matrix, *IEEE Trans. Reliability* R-35 (1986) 192–200.

[35] J.A. Carrasco, Analysis of sparse numerical methods for dependability evaluation, in: *Proc. IASTED Internat. Symp. Identification Modelling and Simulation* (1985) 437–441.

[36] X. Castillo and D.P. Siewiorek, A performance reliability model for computing systems, in: *Proc. 10th Internat. Symp. Fault-Tolerant Computing* (1980) 187–192.

[37] X. Castillo and D.P. Siewiorek, Workload, performance and reliability of digital computing systems, in: *Proc. 11th Internat. Symp. Fault-Tolerant Computing* (1981) 84–89.

[38] S. Chakravarthy, Reliability analysis of a parallel system with exponential life-times and Phase type repairs, *OR Spektrum* 5 (1983) 25–32.

[39] T.C. Chan and K.R. Jackson, The use of iterative linear equations solver for large systems of stiff IVPs for ODEs, *SIAM J. Statist. Comput.* 7 (1986) 378–417.

[40] F. Chatelin, Iterative aggregation/disaggregation methods, in: P.J. Courtois, G. Iazeolla and A. Hordijk (eds), *Mathematical Computer Performance and Reliability* (North-Holland, Amsterdam, 1984) 403–424.

[41] E. Cinlar, Markov renewal theory, *Adv. in Appl. Probab.* 1 (1969) 123–187.

[42] C.A. Clarotti and F. Picchia, MARKAN – an ad hoc designed code for integrating Markov-stiff equations, in: *Proc. 4th EUREDATA Conf.* (1983) 12.2.

[43] M. Coderch, A.S. Willsky, S.S. Sastry and D.A. Castanon, Hierarchical aggregation of singularly perturbed finite state Markov processes, *Stochastics* 8 (1983) 259–289.

[44] A.G. Colombo and R.J. Jaarsma, A powerful numerical method to combine random variables, *IEEE Trans. Reliability* R-29 (1980) 126–129.

[45] C. Constantinescu, Effect of transient faults on gracefully degrading processor arrays, *Microproc. Microprogram.* 26 (1989) 23–30.

[46] A. Costes, J.E. Doucet, C. Landrault and J.C. Laprie, SURF – a program for dependability evaluation of complex fault-tolerant computing systems, in: *Proc. 11th Internat. Symp. Fault-Tolerant Computing* (1981) 72–78.

[47] P.J. Courtois, *Decomposability: Queueing and Computer System Applications* (Academic Press, New York, 1977).

[48] P.J. Courtois and P. Semal, Bounds for the positive eigenvectors of nonnegative matrices and for their approximations, *J. ACM* 31 (1984) 804–825.

[49] D.R. Cox, *Renewal Theory* (Methuen, London, 1962).
[50] D.R. Cox, A use of complex probabilities in the theory of stochastic processes, *Proc. Cambridge Phylosophical Soc.* 51 (1955) 313-319.
[51] K.S. Crump, Numerical inversion of Laplace transforms using a Fourier series approximation, *J. ACM* 23 (1976) 89-96.
[52] A. Cumani, Esp – A package for the evaluation of stochastic Petri nets with phase-type distributed transition times, in: *Proc. Internat. Workshop Timed Petri Nets* (IEEE Computer Society Press no. 674, Torino, Italy, 1985) 144-151.
[53] A. Cumani, On the canonical representation of homogeneous Markov processes modelling failure-time distributions, *Microelectron. Reliability* 22 (1982) 583-602.
[54] M. Davio, Kronecker products and shuffle algebra, *IEEE Trans. Computers* C-30 (1981) 116-125.
[55] F.R. De Hoog, J.H. Knight and A.N. Stokes, An improved method for numerical inversion of Laplace transform, *SIAM J. Scientific Statistical Computing* 3 (1982) 357-366.
[56] L. Donatiello and B.R. Iyer, Analysis of a composite performance reliability measure for fault tolerant systems, *J. ACM* 34 (1987) 179-199.
[57] L. Donatiello and B.R. Iyer, Closed-form solution for system availability distribution, *IEEE Trans. Reliability* R-36 (1987) 45-47.
[58] J. Bechta Dugan, Fault tree and imperfect coverage, *IEEE Trans. Reliability* 38 (1989) 177-185.
[59] J. Bechta Dugan, K. Trivedi, R. Geist and V.F. Nicola, Extended stochastic Petri nets: applications and analysis, in: *Proc. PERFORMANCE '84*, Paris, 1984.
[60] J. Bechta Dugan and K.S. Trivedi, Coverage modeling for dependability analysis of fault-tolerant systems, *IEEE Trans. Computers* 38 (1989) 775-787.
[61] J. Bechta Dugan, K.S. Trivedi, M.K. Smotherman and R.M. Geist, The Hybrid Automated Reliability Predictor (HARP), *AIAA J. Guidance, Control Dynamics* 9 (1986) 319-331.
[62] E. De Souza e Silva and H.R. Gail, Calculating availability and performability measures of repairable computer systems using randomization, *J. ACM* 36 (1989) 171-193.
[63] E. De Souza e Silva and H.R. Gail, Calculating cumulative operational time distributions of repairable computer systems, *IEEE Trans. Computers* C-35 (1986) 322-332.
[64] A. Von Ellenrieder and A. Levine, The probability of an excessive non-functioning interval, *Oper. Res.* 14 (1966) 835-840.
[65] B.N. Feinberg and S.S. Chiu, A method to calculate steady-state distributions of large Markov chains by aggregating states, *Oper. Res.* 35 (1987) 282-290.
[66] G. Florin and S. Natkin, Les reseaux de Petri stochastiques, *Technique Science Informatique* 4 (1985) 143-160.
[67] D.P. Gaver, Observing stochastic processes and approximate transform inversion, *Oper. Res.* 14 (1966) 444-459.
[68] D.P. Gaver, A waiting line with interrupted service, including priorities, *J. Royal Stat. Soc.* B24 (1962) 73-90.
[69] F.A. Gay and M.L. Ketelsen, Performance evaluation for gracefully degrading systems, in: *Proc. 9th Internat. Symp. Fault-Tolerant Computing* (1979) 51-58.
[70] R.M. Geist and K.S. Trivedi, Ultrahigh reliability prediction for fault-tolerant computer systems, *IEEE Trans. Computers* C-32 (1983) 1118-1127.
[71] G.H. Golub and C.F. Van Loan, *Matrix Computation* (Johns Hopkins University Press, Baltimore, 1983).
[72] A. Goyal and S. Lavenberg, Modeling and analysis of computer system availability, *IBM J. Res. Develop.* 31 (1987) 651-664.
[73] A. Goyal, S. Lavenberg and K.S. Trivedi, Probabilistic modeling of computer system availability, *Ann. Oper. Res.* 8 (1987) 285-306.
[74] A. Goyal, V.F. Nicola, A.N. Tantawi and K.S. Trivedi, Reliability of systems with limited repair, *IEEE Trans. Reliability* R-36 (1987) 202-207.
[75] A. Goyal and A.N. Tantawi, *Evaluation of performability for degradable computer systems*, Technical Report, IMB Research Report, RC-10529, 1984.
[76] A. Goyal and A.N. Tantawi, A measure of guaranteed availability and its numerical evaluation, *IEEE Trans. Comput.* C-37 (1988) 25-32.
[77] V. Grassi, L. Donatiello and G. Iazeolla, Performability evaluation of multicomponent fault-tolerant systems, *IEEE Trans. Reliability* R-37 (1988) 216-222.
[78] W.K. Grassmann, Means and variances of time averages in markovian environment, *Europ. J. Oper. Res.* 31 (1987) 132-139.
[79] W.K. Grassmann, Transient solution in markovian queueing systems, *Comput. Oper. Res.* 4 (1977) 47-56.
[80] W.K. Grassmann, M.I. Taksar and D.P. Heyman, Regenerative analysis and steady state distributions for Markov chains, *Oper. Res.* 33 (1985) 1107-1116.
[81] D. Gross and D. Miller, The randomization technique as a modeling tool and solution procedure for transient Markov processes, *Oper. Res.* 32 (1984) 343-361.
[82] P. Gubian and M. Santomauro, RELAN: a computer program for reliability analysis of complex systems, in: *Reliability in Electrical and Electronic Components and Systems* (North-Holland, Amsterdam, 1982) 64-67.
[83] P. Heidelberger and S.S. Lavenberg, Computer performance evaluation methodology, *IEEE Trans. Computers* C-33 (1984) 1195-1220.
[84] R.A. Howard, *Dynamic Probabilistic Systems, Vol. II: Semi-Markov and Decision Processes* (John Wiley, New York, 1971).
[85] G.S. Hura and J.W. Atwood, The use of Petri nets to analyze coherent fault-trees, *IEEE Trans. Reliability* 37 (1988) 469-474.

[86] R. Huslende, A combined evaluation of performance and reliability for degradable systems, in: *Proc. ACM/SIGMETRICS Conf.* (1981) 157–164.

[87] B.R. Iyer, L. Donatiello and P. Heidelberger, Analysis of performability for stochastic models of fault-tolerant systems, *IEEE Trans. Comput.* C-35 (1986) 902–907.

[88] R.G. Jacquot, J.W. Stedman and C.N. Rhodine, The Gaver-Stehfest algorithm for approximate inversion of Laplace transforms, *IEEE Circuit System Mag.* 4–8 (March, 1983).

[89] D.L. Jagerman, An inversion technique for the Laplace transform, *Bell System Tech. J.* 61 (October 1982) 1995–2002.

[90] A.M. Johnson and M. Malek, Survey of software tools for evaluating reliability, availability and serviceability, *ACM Comput. Surveys* 20 (1988) 227–269.

[91] J. Keilson, *Markov Chain Models – Rarity and Exponentiality* (Springer, Berlin, 1979).

[92] J. Keilson and S.S. Rao, A process with chain dependent growth rate, *J. Applied Probab.* 7 (1970) 699–711.

[93] J. Keilson and S.S. Rao, A process with chain dependent growth rate, Part II: the ruin and ergodic problems, *Adv. Applied Probab.* 3 (1971) 315–338.

[94] I. Koren and M.A. Breuer, On area and yield considerations for fault-tolerant VLSI processor arrays, *IEEE Trans. Comput.* C-33 (1984) 21–27.

[95] V.G. Kulkarni, V.F. Nicola and K. Trivedi, The completion time of a job on a multi-mode system, *Adv. Applied Probab.* 19 (1987) 932–954.

[96] V.G. Kulkarni, V.F. Nicola and K. Trivedi, On modeling the performance and reliability of multi-mode computer systems, *J. Systems Software* 6 (1986) 175–183.

[97] S. Kumar, W. Grassmann and R. Billinton, A stable algorithm to calculate steady-state probability & frequency of a Markov system, *IEEE Trans. Reliability* R-36 (1987) 58–62.

[98] J.C. Laprie, Dependable computing and fault-tolerance: concepts and terminology, in: *Proc. 15th Internat. Symp. Fault-Tolerant Computing* (1985) 2–7.

[99] J.C. Laprie, On reliability prediction of repairable redundant digital structures, *IEEE Trans. Reliability* R-25 (1976) 256–258.

[100] J.C. Laprie and K. Medhaffer-Kanoun, Dependability modelling of safety systems, in: *Proc. 10th Internat. Symp. Fault-Tolerant Computing* (1980) 245–250.

[101] W.S. Lee, D.L. Grosh, F.A. Tillman and C.H. Lie, Fault tree analysis, methods and applications – A review, *IEEE Trans. Reliability* R-34 (1985) 194–203.

[102] A. Lesanovski, Multistate Markov models for systems with dependent units, *IEEE Trans. Reliability* R-37 (1988) 505–511.

[103] S.M. Makam and A. Avizienis, ARIES 81: a reliability and life-cycle evaluation tool for fault-tolerant systems, in: *Proc. 12th Internat. Symp. Fault-Tolerant Computing* (1982) 267–274.

[104] R. Marie, Transient numerical solutions of stiff Markov chains, in: *Proc. 20th ISATA Symp.* (1989) 255–270.

[105] M. Masera, Uncertainty propagation in fault tree analyses using lognormal distributions, *IEEE Trans. Reliability* R-36 (1987) 145–149.

[106] R.A. McLean and M.F. Neuts, The integral of a step function defined on a Semi-Markov process, *SIAM J Appl. Math.* 15 (1967) 726–737.

[107] B. Melamed and M. Yadin, Randomization procedure in the computation of cumulative-time distributions over discrete state Markov processes, *Oper. Res.* 32 (1984) 926–944.

[108] J.F. Meyer, Closed form solution of performability, *IEEE Trans. Comput.* C-31 (1982) 648–657.

[109] J.F. Meyer, On evaluating the performability of degradable systems, *IEEE Trans. Comput.* C-29 (1980) 720–731.

[110] J.F. Meyer, A. Movaghar and W.H. Sanders, Stochastic activity networks: structure, behavior and application, in: *Proc. Internat. Workshop Timed Petri Nets* (IEEE Computer Society Press no. 674, Torino, Italy, 1985) 106–115.

[111] D.R. Miller, Reliability calculation using randomization for fault-tolerant computing systems, in: *Proc. 13th Internat. Symp. Fault-Tolerant Computing* (1983) 284–289.

[112] W.L. Miranker, *Numerical Methods for Stiff Equations* (Reidel, Dordrecht, 1981).

[113] M. Modarres and H. Dezfuli, A truncation methodology for evaluating large fault trees, *IEEE Trans. Reliability* R-33 (1984) 325–328.

[114] C. Moler and C.F. Van Loan, Nineteen dubious ways to compute the exponential of a matrix, *SIAM Rev.* 20 (1978) 801–835.

[115] M.K. Molloy, Performance analysis using stochastic Petri nets, *IEEE Trans. Comput.* C-31 (1982) 913–917.

[116] B.M.E. Moret and M.G. Thomason, Boolean difference techniques for time-sequence and common-cause analysis of fault-trees, *IEEE Trans. Reliability* R-33 (1984) 398–405.

[117] R.R. Muntz, E. De Souza Silva and A. Goyal, Bounding availability of repairable computer systems, *ACM Performance Evaluation Rev.* (*Proc. SIGMETRICS 1989*) 17 (1989) 29–38.

[118] T. Murata, Petri nets: properties, analysis and applications, *Proc. IEEE* 77 (1989) 541–580.

[119] J.M. Nahman, Iterative method for steady state reliability analysis of complex Markov systems, *IEEE Trans. Reliability*, R-33 (1983) 406–409.

[120] S. Natkin, *Les reseaux de Petri stochastiques et leur application a l'evaluation des systemes informatiques*, These de Docteur Ingegneur, CNAM, Paris, 1980.

[121] R. Negrini, M.G. Sami and R. Stefanelli, Fault-tolerance techniques for array structures used in supercomputing, *Computer* (February 1986) 78–87.

[122] M.F. Neuts, *Matrix Geometric Solutions in Stochastic Models* (Johns Hopkins University Press, Baltimore, 1981).

[123] M.F. Neuts, Phase type distributions: a bibliography, *Department of Systems and Industrial Engineering, University of Arizona*, Working Paper, Nr. 89-005, February, 1989.

[124] M.F. Neuts and K.S. Meier, On the use of phase type distributions in reliability modelling of systems with two components, *OR Spektrum* 2 (1981) 227-234.

[125] Y.W. Ng and A. Avizienis, A unified reliability model for fault-tolerant computers, *IEEE Trans. Comput.* C-29 (1980) 1002-1011.

[126] L.B. Page and J.E. Perry, An algorithm for exact fault-tree probabilities without cut sets, *IEEE Trans. Reliability* R-35 (1986) 544-559.

[127] I.A. Papazoglou and E.P. Gyftopoulos, Markov processes for reliability analyses of large systems, *IEEE Trans. Reliability* R-26 (1977) 232-237.

[128] J.L. Peterson, *Petri Net Theory and the Modeling of Systems* (Prentice Hall, Englewood Cliffs, 1981).

[129] R.G. Phillips and P.V. Kokotovic, A singular perturbation approach to modeling and control of large Markov chains, *IEEE Trans. Automatic Control* AC-26 (1981) 1087-1094.

[130] B.D. Plateau, J.M. Fourneau and K.H. Lee, A package for solving complex Markov models of parallel systems, in: *Proc. 4th Internat. Conf. on Modelling Techniques and Tools for Computer Performance Evaluation* (Palma de Mallorca, 1988) 341-360.

[131] R. Pyke, Markov renewal processes with finitely many states, *Ann. Mathemat. Stat.* 32 (1961) 1243-1259.

[132] C.V. Ramamoorthy and G.S. Ho, Performance evaluation of asynchronous concurrent systems using Petri nets, *IEEE Trans. Software Engrg.* SE-6 (1980) 440-449.

[133] A. Reibman, R. Smith and K.S. Trivedi, Markov and Markov reward model transient analysis: an overview of numerical approaches, *Europ. J. Oper. Res.* 40 (1989) 257-267.

[134] A. Reibman and K.S. Trivedi, Numerical transient analysis of Markov models, *Comput. Oper. Res.* 15 (1988) 19-36.

[135] A. Reibman and K.S. Trivedi, Transient analysis of cumulative measures of Markov chain behavior, *to be published in Stochastic Models*, 1989.

[136] J.R. Rohlicek and A.S. Willsky, The reduction of perturbed Markov generators: an algorithm exposing the role of transient states, *J. ACM* 35 (1988) 675-696.

[137] S.M. Ross and J. Schechtman, On the first time a separately maintained parallel system has been down for a fixed time, *Naval Res. Logistic Quart.* 26 (1979) 285-290.

[138] A.M. Rushdi, Uncertainty analysis of fault-tree outputs, *IEEE Trans. Reliability* R-34 (1985) 458-462.

[139] R. Sahner and K.S. Trivedi, Performance and reliability analysis using directed acyclic graphs, *IEEE Trans. Software Engrg.* SE-13 (1987) 1105-1114.

[140] R. Sahner and K.S. Trivedi, Reliability modeling using SHARPE, *IEEE Trans. Reliability* R-36 (1987) 186-193.

[141] A.M. Saleh and J.H. Patel, Transient-fault analysis for retry techniques, *IEEE Trans. Reliability* 37 (1988) 323-330.

[142] M.G. Sami and R. Stefanelli, Reconfigurable architectures for VLSI processing arrays, *Proc. IEEE* 74 (1986) 712-722.

[143] W.G. Schneeweiss, Approximate fault-tree analysis with prescribed accuracy, *IEEE Trans. Reliability* R-36 (1987) 250-254.

[144] W.G. Schneeweiss, Fault-tree analysis using a binary decision tree, *IEEE Trans. Reliability* R-34 (1985) 452-457.

[145] W.G. Schneeweiss, *Boolean Functions with Engineering Applications and Computer Programs* (Springer, New York, 1988).

[146] P.J. Schweitzer, Aggregation methods for large Markov chains, in: P.J. Courtois, G. Iazeolla and A. Hordijk (eds), *Mathematical Computer Performance and Reliability* (North-Holland, Amsterdam, 1984) 275-285.

[147] G. Shanthikumar, First failure time of dependent parallel systems with safety period, *Microelectron. Reliability* 26 (1986) 955-972.

[148] K.G. Shin and Y.H. Lee, Analysis of the impact of error detection on computer performance, in: *Proc. 10th Internat. Symp. Fault-Tolerant Computing* (1983) 356-359.

[149] K.G. Shin and Y.H. Lee, Error detection process - Model, design and its impact on computer performance, *IEEE Trans. Computers* C-33 (1984) 529-540.

[150] M.L. Shooman, *Probabilistic Reliability: An Engineering Approach* (McGraw-Hill, New York, 1968).

[151] R. Smith, K. Trivedi and A.V. Ramesh, Performability analysis: measures, an algorithm and a case study, *IEEE Trans. Comput.* C-37 (1988) 406-417.

[152] M. Smotherman, R. Geist and K. Trivedi, Provably conservative approximations to complex reliability models, *IEEE Trans. Comput.* C-35 (1986) 333-338.

[153] G.W. Stewart, *Introduction to Matrix Computation* (Academic Press, New York, 1973).

[154] J.J. Stiffler and L.A. Bryant, *CARE III, phase III Report - Mathematical description*, Technical Report, NASA Contractor Report, November 1982.

[155] U. Sumita, J.G. Shanthikumar and Y. Masuda, Analysis of fault tolerant computer systems, *Microelectron. Reliability* 27 (1987) 65-78.

[156] Y. Takahashi, *Approximate modeling of large-scale Markov chains by cross aggregation*, Technical Report 12, Faculty of Economics, Tohoku University, May 1988.

[157] K. Trivedi, J.B. Dugan, R. Geist and M. Smotherman, Modeling imperfect coverage in fault-tolerant systems, in: *Proc. 14th Internat. Symp. Fault-Tolerant Computing* (1984) 77-82.

[158] K.S. Trivedi, Modeling and analysis of fault-tolerant systems, in: *Proc. Internat. Conf. Modelling Techniques Tools for Performance Analysis* (Paris, 1984).

[159] K.S. Trivedi, Reliability evaluation of fault-tolerant sys-

tems, in: P.J. Courtois, G. Iazeolla and A. Hordijk (eds), *Mathematical Computer Performance and Reliability* (North-Holland, Amsterdam, 1984) 403–424.
[160] K.S. Trivedi and R.M. Geist, Decomposition in reliability analysis of fault-tolerant systems, *IEEE Trans. Reliability* R-32 (1983) 463–468.
[161] H. Vantilborgh, Aggregation with an error of $O(\epsilon^2)$, *J. ACM* 32 (1985), 162–190.
[162] A.P. Wood, Multistate block diagrams and fault trees, *IEEE Trans. Reliability* R-34 (1985) 236–240.
[163] S. Yalamanchili and J.K. Aggarwal, Reconfiguration strategies for parallel architectures, *Computer* 44–61 (December, 1985).