Alcune applicazioni della Teoria dei Giochi al trasporto ferroviario

Vito Fragnelli
Dipartimento di Scienze e Tecnologie Avanzate
Università del Piemonte Orientale
E-mail: vito.fragnelli@mfn.unipmn.it

Sommario

How to Share Railways Infrastructure Costs? - V. Fragnelli, I. García-Jurado, H. Norde, F. Patrone, S. Tijs

The scenario - The problem - The game theoretical model - The solution - An example

Coalition Formation and Railway Scheduling Reoptimization - E. Borgo, V. Fragnelli, M. Marchi

The problem - Current approaches - Competitive scenario - C-Solution - Examples

Università di Urbino "Carlo Bo" - 30 Marzo 2004

The scenario

EEC directive 440/91 and EC directives 18/95 and 19/95 → reorganization of the railway sector in Europe, via separation between infrastructure management and transport operations

- Track capacity allocation
  Nilsson (1995), Brewer and Plott (1996), Bassanini and Nastasi (1998), Caprara, Fischetti, Guida, Monaci, Sacco and Toth (2001), Caprara, Fischetti and Toth (2002)

- Access tariff for the railway transport operators
  based on profitability and social utility of the journey, congestion issues, number of passengers and/or goods transported, services required, infrastructure costs, etc.

The problem

Hypotheses:

- A railway path is used by different types of trains belonging to several operators
- The infrastructure costs have to be divided among these trains (joint cost allocation problem)
- The infrastructure consists of some "facilities" (track, signalling system, stations, etc.)
- Different groups of trains need these facilities at different levels (fast trains need a more sophisticated track and signalling system; for local trains station services are more important)

The infrastructure is the "sum" of different facilities, required by the trains at different levels of cost

Infrastructure costs are sum of:

- "building" costs (independent from number of users) → "airport game" (Littlechild and Thompson, 1977)
- "maintenance" costs (dependent on the number of users)

Related problems:

- bridge used by small and big cars
- glasses for a restaurant chain

The game theoretical model - I

Definition 1 *Suppose we are given k groups of players $g_1, \ldots, g_k$ with $n_1, \ldots, n_k$ players respectively and k non-negative numbers $b_1, \ldots, b_k$. The* building cost game *corresponding to $g_1, \ldots, g_k$ and $b_1, \ldots, b_k$ is the cooperative (cost) game $\langle N, c_b \rangle$ with $N = \cup_{i=1}^{k} g_i$ and cost function c_b defined by*

$$c_b(S) = b_1 + \cdots + b_{j(S)} \qquad S \subseteq N$$

where $j(S) = \max\{j \,:\, S \cap g_j \neq \emptyset\}$

Definition 2 *Suppose we are given k groups of players $g_1, \ldots, g_k$ with $n_1, \ldots, n_k$ players respectively and $k(k+1)/2$ non-negative numbers $\{\alpha_{i,j}\}_{i,j \in \{1,\ldots,k\}, j \geq i}$. The* maintenance cost game *corresponding to $g_1, \ldots, g_k$ and $\{\alpha_{i,j}\}_{i,j \in \{1,\ldots,k\}, j \geq i}$ is the cooperative (cost) game $\langle N, c_m \rangle$ with $N = \cup_{i=1}^{k} g_i$ and cost function c_m defined by*

$$c_m(S) = \sum_{i=1}^{j(S)} |S \cap g_i| A_{ij(S)} \qquad S \subseteq N$$

where $A_{ij} = \alpha_{i,i} + ... + \alpha_{i,j}$ for all $i, j \in \{1, ..., k\}$ with $j \geq i$

Interpretation:
If one player in g_i used the facility, to restore it up to level i the maintenance costs are $A_{ii} = \alpha_{i,i}$; to restore the facility up to level $i+1$, then extra maintenance costs $\alpha_{i,i+1}$ will be made. So, in order to restore the facility up to level j (with $j \geq i$) the maintenance costs are $A_{ij} = \alpha_{i,i} + \alpha_{i,i+1} + ... + \alpha_{i,j}$

The game theoretical model - II

Theorem 1 *Let* $\langle N, c_m \rangle$ *be the maintenance cost game corresponding to* $g_1, \ldots, g_k$ *and* $\{\alpha_{i,j}\}_{i,j \in \{1,\ldots,k\}, j \geq i}$. *Then the following four statements are equivalent:*

1. $\langle N, c_m \rangle$ *is concave*
2. $\langle N, c_m \rangle$ *is balanced*
3. $\sum_{i \in N} c_m(i) \geq c_m(N)$
4. $\alpha_{i,j} = 0$ *for every* $j > i$

Other theoretical properties are in Norde, Fragnelli, García-Jurado, Patrone, Tijs (2000)

Definition 3 *A* one facility infrastructure cost game *with groups of players* $g_1, \ldots, g_k$ *is the cooperative (cost) game* $\langle N, c \rangle$ *with* $N = \cup_{i=1}^{k} g_i$ *and cost function* $c = c_b + c_m$. *An* infrastructure cost game *with groups of players* $g_1, \ldots, g_k$ *is the cooperative (cost) game* $\langle N, c \rangle$ *with* $N = \cup_{i=1}^{k} g_i$ *and cost function* $c = c^1 + \ldots + c^l$ *such that, for every* $r \in \{1, \ldots, l\}$, $\langle N, c^r \rangle$ *is a one facility infrastructure cost game with groups of players* $g_{\pi^r(1)}, \ldots, g_{\pi^r(k)}$, *where* π^r *is a permutation of* $\{1, \ldots, k\}$.

A one facility infrastructure cost game is the sum of a building cost game plus a maintenance cost game with the same groups of players ordered in the same way

An infrastructure cost game is the sum of a finite set of one facility infrastructure cost games with the same groups of players, not necessarily in the same order

The solution - I

Shapley value

$$\phi_i(v) = \frac{1}{n!} \sum_{\pi} (v(P(\pi; i) \cup \{i\}) - v(P(\pi; i))$$

where $P(\pi; i)$ are the predecessors of i in permutation π

Properties:

- additive solution
- infrastructure access tariff based on the Shapley value can be computed very easily for a player $i \in g_s$, $s = 1, ..., k$:

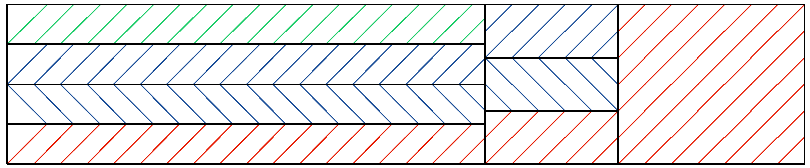
$$\begin{aligned} \phi_i(c_b) &= \sum_{l=1,\dots,s} \frac{b_l}{G_{lk}} \\ \phi_i(c_m) &= \alpha_{s,s} + \sum_{l=s+1,\dots,k} \alpha_{s,l} \frac{G_{lk}}{G_{lk}+1} + \sum_{l=2,\dots,s} \sum_{j=1,\dots,l-1} \alpha_{j,l} \frac{|g_j|}{(G_{lk})(G_{lk}+1)} \end{aligned}$$

where $G_{lk} = \left| \bigcup_{h=l,\dots,k} g_h \right|$

The solution - II

$N = g_1 \cup g_2 \cup g_3; g_1 = \{1\}; g_2 = \{2,3\}; g_3 = \{4\}$

Building cost game

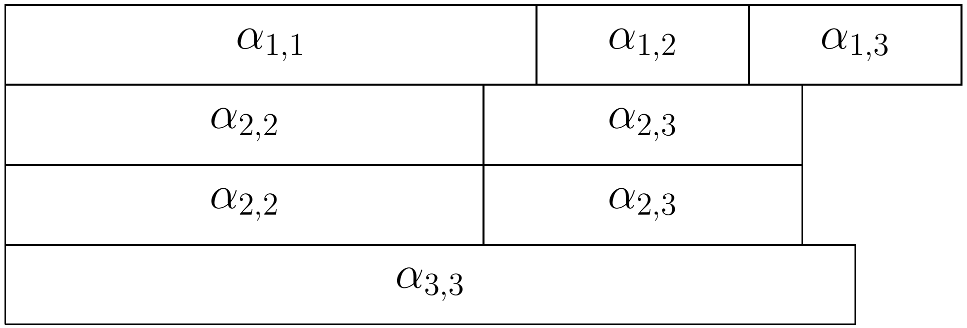


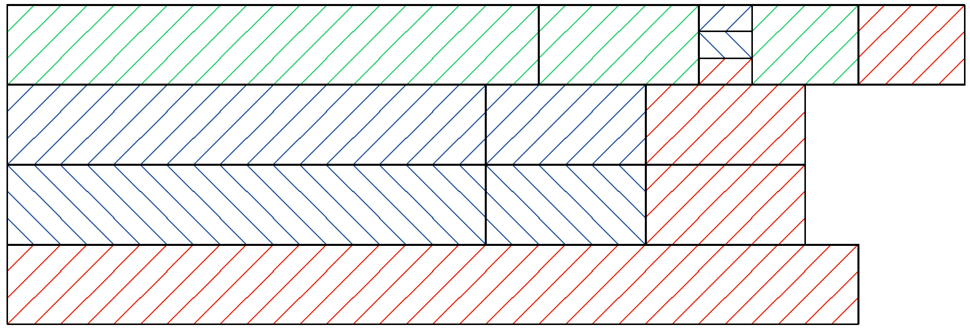
$\phi_1(c_b) = \frac{1}{4}b_1$

$\phi_2(c_b) = \phi_3(c_b) = \frac{1}{4}b_1 + \frac{1}{3}b_2$

$\phi_4(c_b) = \frac{1}{4}b_1 + \frac{1}{3}b_2 + b_3$

Maintenance cost game





$\phi_1(c_m) = \alpha_{1,1} + \frac{3}{4}\alpha_{1,2} + \frac{1}{2}\alpha_{1,3}$

$\phi_2(c_m) = \phi_3(c_m) = \alpha_{2,2} + \frac{1}{2}\alpha_{2,3} + \frac{1}{3} \times \frac{1}{4}\alpha_{1,2}$

$\phi_4(c_m) = \alpha_{3,3} + \frac{1}{3} \times \frac{1}{4}\alpha_{1,2} + \frac{1}{2}\alpha_{1,3} + 2 \times \frac{1}{2}\alpha_{2,3}$

An example - I

Data taken from Baumgartner (1997) - Fees are in swiss francs
The costs for one facility can be decomposed into:

- a fixed part (in the sense that it does not depend on the number of players), that corresponds to the building cost game associated with this facility
- a variable part (in the sense that it is proportional to the number of players), that corresponds to the maintenance cost game part

For each kilometer of track, there are two kind of costs: *renewal costs (RWC)* and *repairing costs (RPC)*
Accordingly the track is divided into two facilities: "track renewal" and "track repairing"

Renewal costs per kilometer and per year can be approximated by the following formula:

$$RWC = 0.001125X + 11,250$$

where X measures the "number" of trains, expressed in yearly TGCK (Tons Gross and Complete per Kilometer); we assume that all of the trains are of the same weight

Repairing costs per kilometer and per year can be given by the formulas:

$$RPC_s = 0.001X + 10,000 \qquad \text{for slow trains}$$
$$RPC_f = 0.00125X + 12,500 \qquad \text{for fast trains}$$

An example - II

One kilometer of line used by a total weight of 10^7 TGCK (about 20,000 trains)

15,000 slow trains and 5,000 fast trains

$\langle N, c\rangle$ is given by:

- $N = g_1 \cup g_2$, where g_1 is the set of slow trains ($n_1 = 15{,}000$) and g_2 is the set of fast trains ($n_2 = 5{,}000$)
- $c = c^1 + c^2$, where c^1 and c^2 are one facility infrastructure cost games both having the groups of players ordered as g_1, g_2, characterized by the following parameters:

c^1	$b_1^1 = 11,250$	$b_2^1 = 0$	$\alpha_{11}^1 = 0.5625$	$\alpha_{12}^1 = 0$	$\alpha_{22}^1 = 0.5625$
c^2	$b_1^2 = 10,000$	$b_2^2 = 2,500$	$\alpha_{11}^2 = 0.5$	$\alpha_{12}^2 = 0.125$	$\alpha_{22}^2 = 0.625$

For a slow and a fast train respectively, we have:

- $\phi_s(c) = \dfrac{b_1^1}{n_1+n_2} + \alpha_{11}^1 + \dfrac{b_1^2}{n_1+n_2} + \alpha_{11}^2 + \alpha_{12}^2 \dfrac{n_2}{n_2+1} = 2.25$
- $\phi_f(c) = \dfrac{b_1^1}{n_1+n_2} + \alpha_{22}^1 + \dfrac{b_1^2}{n_1+n_2} + \dfrac{b_2^2}{n_2} + \alpha_{22}^2 + \alpha_{12}^2 \dfrac{n_1}{n_2(n_2+1)} = 2.75$

References

Bassanini A, Nastasi A (1998) A Market Based Model for Railroad Capacity Allocation. Proceedings of TRISTAN III Conference, Vol. 1, San Juan, Puerto Rico, USA.

Baumgartner JP (1997) Ordine di grandezza di alcuni costi nelle ferrovie. Ingegneria Ferroviaria 7:459-469.

Brewer PJ, Plott CR (1996) A Binary Conflict Ascending Price (BICAP) Mechanism for the Decentralized Allocation of the Right to Use Railroad Tracks. International Journal of Industrial Organization 14:857-886.

Caprara A, Fischetti M, Toth P (2002) Modeling and Solving the Train Timetabling Problem. Operations Research 50:851-861.

Caprara A, Fischetti M, Guida PL, Monaci M, Sacco G, Toth P (2001) Solution of Real-World Train Timetabling Problems. Proceedings of the 34-th Hawaii International Conference on System Sciences (HICSS 34), IEEE Computer Society Press.

Littlechild SC, Thompson GF (1977) Aircraft Landing Fees: A Game Theory Approach. Bell Journal of Economics 8:186-204.

Nilsson JE (1995) Allocation of Track Capacity. CTS Working Paper 1995:1, Centre for Research in Transportation and Society, Dalarna University College, Sweden.

Norde H, Fragnelli V, García-Jurado I, Patrone F, Tijs S (2002) Balancedness of Infrastructure Cost Games. European Journal of Operations Research 136:635-654.

The problem

A good timetable is a difficult task, as it have to take into account the service request of the users, the technical characteristics both of the network and of the trains and the costs for managing the system

It is a very large problem
In Italy:

- 16,000 Km. of tracks
- 3,500 stations
- 30,000 trains per day (technically)

Current approaches

- Manual scheduling
- Mathematical programming problem (Hooghiemstra, Kroon, Odijk, Salomon and Zwaneveld, 1998)

INPUT	- topology of the network (lines and stations)
	- technical characteristics of the tracks and of the trains
	- demand for transportation
	- utility functions of the trains
	- headway times between train (safety)
OUTPUT	- leaving time and travel time of each train
	- utility of the trains

  Dutch railways (2,800 Km. of tracks, 400 stations and 5,000 trains per day; regular intervals $\longrightarrow$ one tenth of the trains)
- Graph theory (Caprara, Fischetti, Guida, Monaci, Sacco and Toth, 2001 and Caprara, Fischetti and Toth, 2002)
  They consider a network whose nodes represent the different stations in different instants and the arcs correspond to the movements of the trains between two stations or to the stops of the trains in the stations. The solution is a collection of paths from the departure station of each train to the arrival one, satisfying some feasibility constraints and minimizing a utility function related to the ideal scheduling of the trains
  Milano-Bologna, 200 trains

Competitive scenario

In Europe, after the EEC Directive 440/91, different transport operators (TOs) can operate different trains on the same infrastructure for overlapping slots so that they prefer not to communicate their strategies:

- The infrastructure manager (IM) builds up the timetable taking into account the requests, needs and utilities of different agents
  The TOs communicate the ideal departure time and a time window in which it is still profitable to schedule the departure of the train
- TOs may decide to cooperate in order to improve their utilities so they set available more details about their utility functions
  Using better information it is possible to modify their scheduling (without influencing the timetable of the other trains), increasing the profits

The utility function of a train can be affected if the final timetable does not respect the forecast about other trains: suppose that the train A considers its best choice to leave from a given station 10 minutes after the arrival of the train B; in this situation the maximum of the utility is defined according to the arrival time of train B in the previous timetable, that can be no longer valid in the new one.

Game theoretical approach - I

- each player decides to cooperate with other players not for the worth of the coalition but on the basis of its own payoff (see Hart and Kurz, 1983 and Greenberg, 1994)
- **C-Solution (Gerber, 2000) → coalition formation and numerical solution**

Definition 4 *A cooperative game without transferable utility is a couple* $G = (N, V)$*, where* $N = \{1, ..., n\}$ *is the player set and* V *is the characteristic function that maps each coalition* S *in the set of its feasible payoffs, s.t.:*

- $V(S) \subset \mathbb{R}^S$
- *V(S) is closed and non-empty*
- $V(S) = V(S) - \mathbb{R}^S_{\geq}$ *(comprehensiveness)*

Definition 5 *A bargaining problem is a couple* (F, d)*, where* F *is the feasibility set, i.e. the closed, convex, bounded and non-empty set of payoffs which players can agreed on after bargaining and* $d = (d_1, ..., d_n), d \in \mathbb{R}^n$ *is the disagreement point, i.e. the starting payoff or the minimum that players can guarantee if they do not reach an agreement.*

Game theoretical approach - II

Solutions for a bargaining problem:

- the Nash solution (1950):

$$\mathrm{N}(F,d) = argmax\left\{\prod_{i\in N}(x_i - d_i) \mid x \in F,\ x \geq d\right\}$$

- the Kalai-Smorodinsky solution (1975):

$$\mathrm{KS}(F,d) = argmax\left\{\frac{x_1 - d_1}{a_1 - d_1} = ... = \frac{x_n - d_n}{a_n - d_n} \mid x \in F,\ x \geq d\right\}$$

  where $a_i = max\{x_i \in \mathbb{R} \mid x \in F,\ x \geq d\}, i \in N$

- the Egalitarian solution (cfr. Roth, 1979):

$$\mathrm{Eg}(F,d) = argmax\{x_1 - d_1 = ... = x_n - d_n \mid x \in F,\ x \geq d\}$$

C-Solution - I

A nice characteristic of this process is that the final numerical solution specifies not only which coalitions form, but assign a payoff to each player, starting from its "power" in the other coalitions

Abstract Games (Shenoy, 1980)

- An *abstract game* is a couple (X, dom) where X is an arbitrary set and $dom \subset X \times X$ is a binary relation on X, called domination; given $x, y \in X$, x is accessible from y, or $y \longrightarrow x$, if there exist $z_1, \ldots, z_m \in X$ s.t.

$$x = z_1 \; dom \; z_2 \; dom \; \ldots \; dom \; z_{m-1} \; dom \; z_m = y$$

- An *elementary dynamic solution* is a set $S \subseteq X$ that satisfies the following conditions:
  1. $x \in S, y \in X \setminus S \Rightarrow x \not\rightarrow y$ (elements not in S are not accessible from elements of S)
  2. $x, y \in S \Rightarrow x \leftrightarrow y$ (two elements of S are accessible one another)
- A *dynamic solution* is a set P that is union of elementary dynamic solutions
- For dynamic solutions the following theorem holds: If X is finite, then P is non-empty

C-Solution - II

Assumptions

- An NTU-game is a "menu" of pure bargaining games $H^S = \{(F^S, d^S)\}$ for every coalition S, where F^S is the feasible region and d^S is the disagreement point
- Each player can join to one coalition
- The players bargain over utility distribution in each game

The players have to face two intertwined decision problems:

- which coalition to form
- which payoff vector to choose from the bargaining region for the members of each coalition

C-Solution - III
Tools - I

- A *bargaining function* is a function $\phi^S : H^S \to \mathbb{R}^N_S$ that assigns a payoff vector as a solution for each bargaining problem for S, and satisfies the following properties:

$$\begin{array}{ll} Feasibility: & \phi^S(F^S, d^S) \in F^S \\ Individual\ rationality: & \phi^S(F^S, d^S) \geq d^S \\ Pareto\ Optimality: & x \in \mathbb{R}^N_S, x > \phi^S(F^S, d^S) \Rightarrow x \notin F^S \end{array}$$

- A *payoff configuration* is a couple $(P, x) \in \bigcup_{P \in \Pi}(\{P\} \times F_V(P))$ where $P \in \Pi$ is a coalition structure, i.e. a partition of the player set N, and $F_V(P)$ is the set: $\{x \in \mathbb{R}^N \mid x \in V(S), \forall\ S \in P\}$.

- The *domination* relation between two different payoff configurations $(P^1, x^1), (P^2, x^2)$ is:

$$(P^1, x^1)\ dom\ (P^2, x^2) \Leftrightarrow \exists\ R \in P^1 \mid x^1_i > x^2_i, \forall\ i \in R$$

- The set of *decisive coalitions* for a game V, or ε^V, is the set of coalitions $S, |S| \geq 2$, s.t.:

$$\exists\ y \in V(S) \mid y > \underline{x}_S$$

  where $x_S = (\underline{x}_i)_{i \in S}$ and $\underline{x}_i = sup\ \{t \in \mathbb{R} \mid te^i \in V(i)\}$

- A *reduced game* w.r.t. the coalition S, or V^{-S}, is the game:

$$V^{-S}(T) = \begin{cases} V(T) & \text{if } T \neq S \\ \{y \in \mathbb{R}^N_T \mid y \leq \underline{x}_T\} & \text{if } T = S \end{cases}$$

C-Solution - IV

Tools - II

- A *feasibility function* is a function $d_V^S : \{x \in \mathbb{R}_S^N \mid x \geq \underline{x}_S\} \longrightarrow V(S)$ that returns a feasible disagreement point if the given one is infeasible; it has to satisfy the following properties:

  1. $d_V^S(x) \geq \underline{x}_S$
  2. $d_V^S(x) = x$, if $x \in V(S)$

  A possible choice is:

$$(d_V^S(x))_i = \begin{cases} 0 & \text{if } i \notin S \\ x_i & \text{if } i \in S \text{ and } x \in V(S) \\ max\ \{\underline{x}_i, max\ \{t \in \mathbb{R} \mid (t, x_i) \in V(S)\}\} & \text{if } i \in S \text{ and } x \notin V(S) \end{cases}$$

C-Solution - V

Computation

By induction on the cardinality of the set of decisive coalitions ε^V

Initial step $|\varepsilon^V| = 0 \Rightarrow \varepsilon^V = \emptyset$

The C-Solution is $(\{\{1\}, \{2\}, \ldots, \{n\}\}, (\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_n))$

Iterative step $|\varepsilon^V| = m \geq 1$

The C-Solution is the dynamic solution of the abstract game (X, dom) where:

$$X = \left\{(P, x) \in \Pi \times \mathbb{R}^N \mid T \in P \Rightarrow \begin{cases} T \in \varepsilon^V \text{ and } x_T = \phi^T(V(T), d_V^T(y^T))\big|_T \\ \text{or} \\ T = \{i\} \text{ and } x_i = \underline{x}_i \end{cases} \right\}$$

and *dom* is the domination defined above

y^T is the average payoff of the players in the C-Solution $\{(P^1, x^1), \ldots, (P^{K(T)}, x^{K(T)})\}$ of the reduced game V^{-T}, i.e. the opportunities of the players in the other coalitions:

$$y_j^T = \begin{cases} \frac{1}{K(T)} \sum_{h=1}^{K(T)} x_j^h & \text{if } j \in T \\ 0 & \text{if } j \notin T \end{cases}$$

V^{-T} is originated according to the current ε^V, so by the inductive hypothesis the C-Solution was computed in the previous step; as X is finite, the dynamic solution is always non-empty

Examples

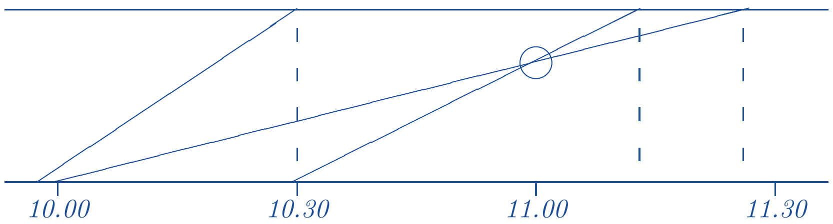
Assumptions:

- headway time = two minutes
- fixed travel time → determine only the departure time

Example 1 - Trains with transferable utilities

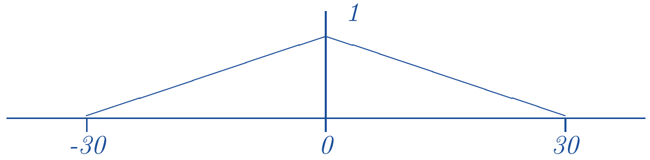
Requirement of TOs:

$$\begin{array}{llll} p_1 = 9.58 & t_1 = 32 & a_1 = 10.30 & f_1 = [-30, 30] \\ p_2 = 10.00 & t_2 = 86 & a_2 = 11.26 & f_2 = [-30, 30] \\ p_3 = 10.30 & t_3 = 43 & a_3 = 11.13 & f_3 = [-30, 30] \end{array}$$



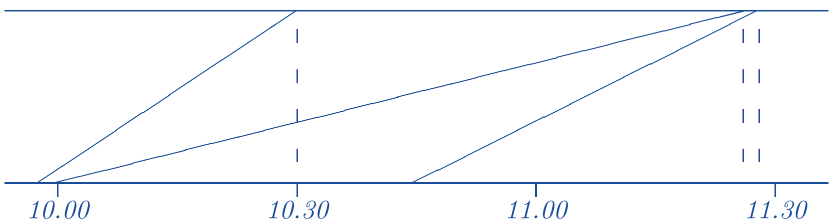
Standard utility function:

$$\hat{u}(x_i) = \begin{cases} 1 - \frac{|x_i|}{30} & \text{if } |x_i| \leq 30 \\ 0 & \text{otherwise} \end{cases}$$



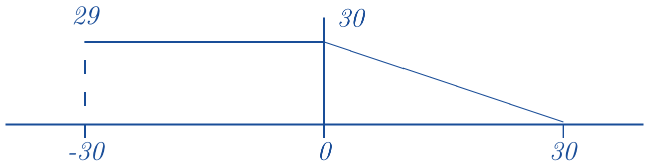
IM scheduling

$$\begin{aligned} p_1 &= 9.58 \\ p_2 &= 10.00 \\ p_3 &= 10.45 \end{aligned}$$



Real utility functions of TOs:

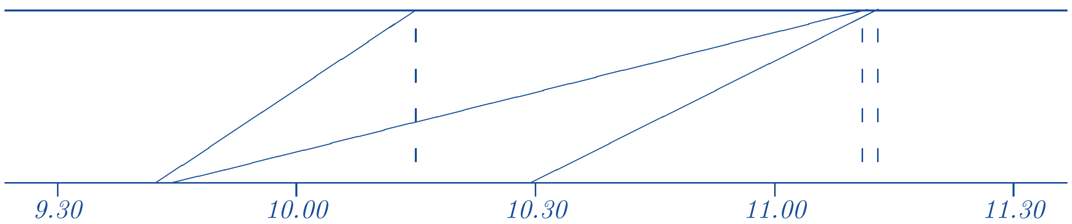
$$u_i(x_i) = \begin{cases} 30 + \frac{x_i}{30} & \text{if } -30 \le x_i \le 0 \\ 30 - x_i & \text{if } \quad 0 < x_i \le 30 \end{cases}$$



Actual payoffs: 30,30,15

New optimal scheduling:

$$p_1 = 9.43$$
$$p_2 = 9.45$$
$$p_3 = 10.30$$



Interpretation:

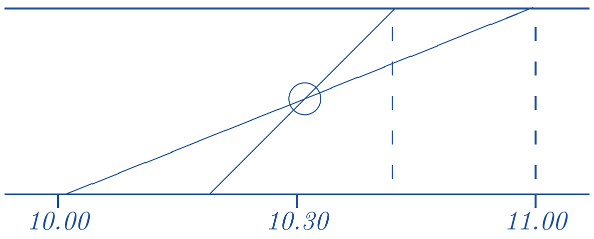
Train	*Departure*	*Utility*	*Compensation*	*Final payoff*
1	*15 minutes early*	29.50	1.16 *from player 3*	30.66
2	*15 minutes early*	29.50	7.16 *from player 3*	36.66
3	*ideal departure time*	30.00	8.83 *to players 1 and 2*	21.66

All the players have increased their payoffs

Example 2 - Trains without transferable utilities

The disagreement point is the utility in the IM scheduling

Requirements of TOs:

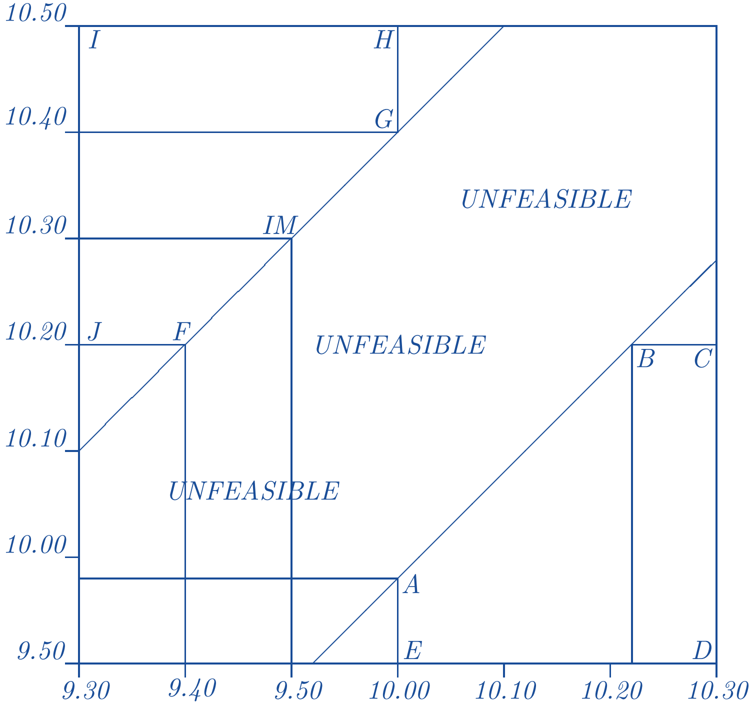


$p_1 = 10.00 \quad t_1 = 60 \quad a_1 = 11.00 \quad f_1 = [-30, 30]$

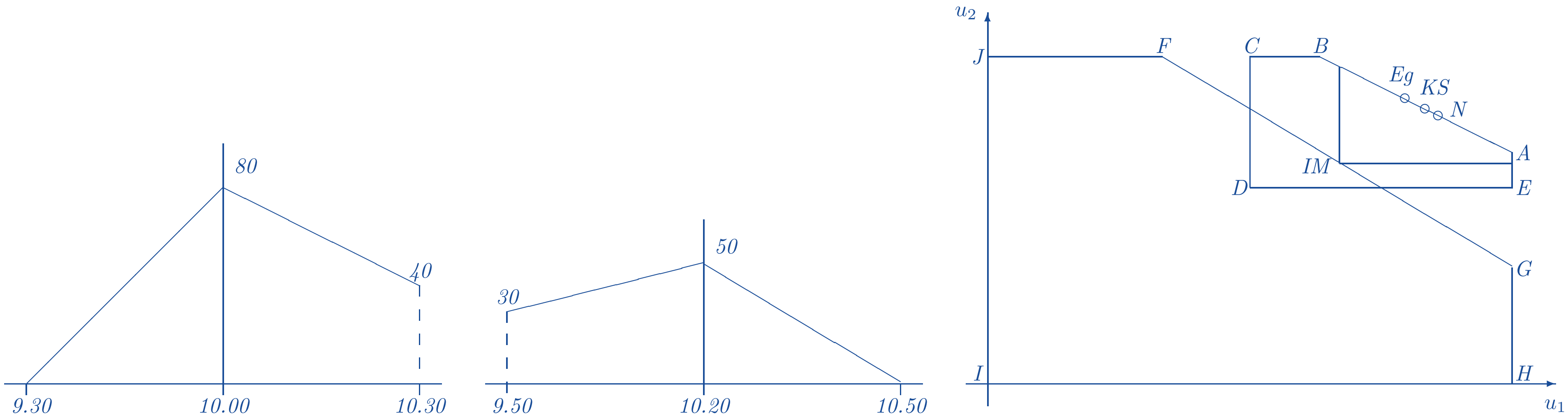
$p_2 = 10.20 \quad t_2 = 22 \quad a_2 = 10.42 \quad f_2 = [-30, 30]$

IM scheduling: $p_1 = 9.50$; $p_2 = 10.30$

Space of strategies



Real utility functions of the two TOs and related bargaining problem:

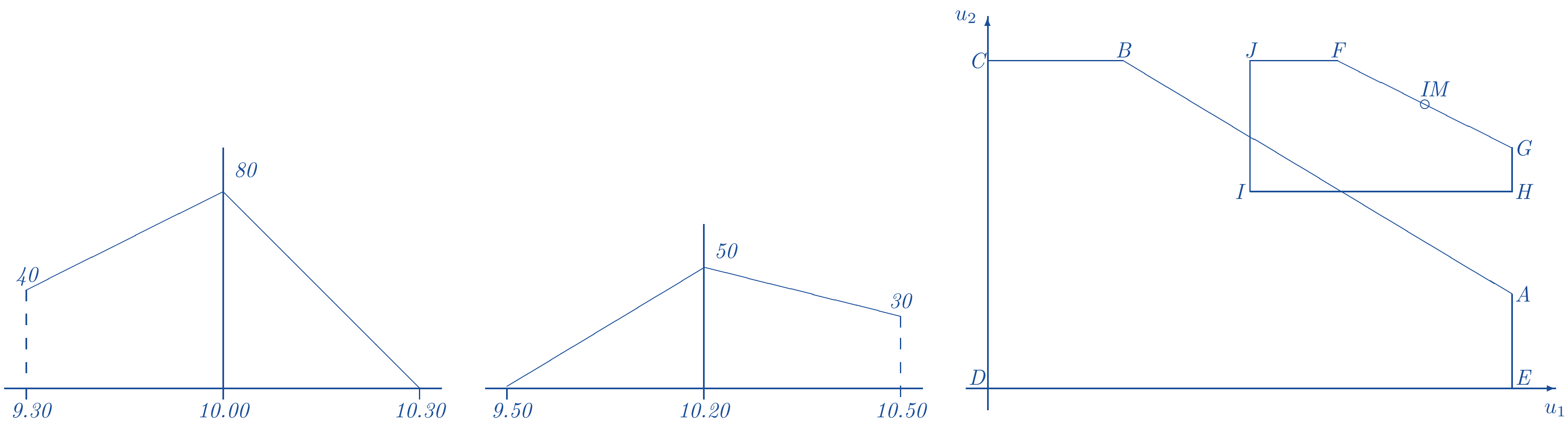


Corresponding scheduling:

$$
\begin{array}{lll}
N & p_1 = 10.08(.30) & \approx 10.08 \\
 & p_2 = 10.06(.30) & \approx 10.06 \\
KS & p_1 = 10.10.00 & = 10.10 \\
 & p_2 = 10.08.00 & = 10.08 \\
Eg & p_1 = 10.12(.20) & \approx 10.12 \\
 & p_2 = 10.10(.20) & \approx 10.10
\end{array}
$$

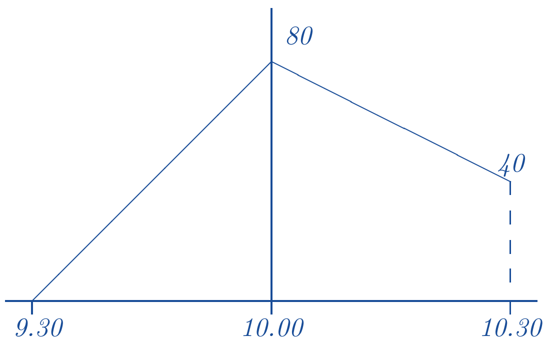
Example 3 - Pareto optimal disagreement point

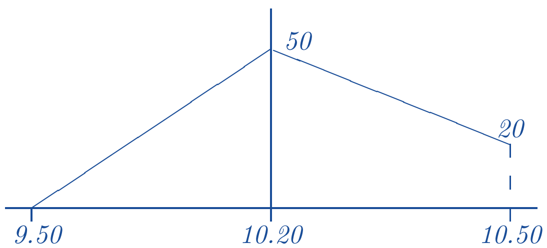
Real utility functions of the two TOs and related bargaining problem:

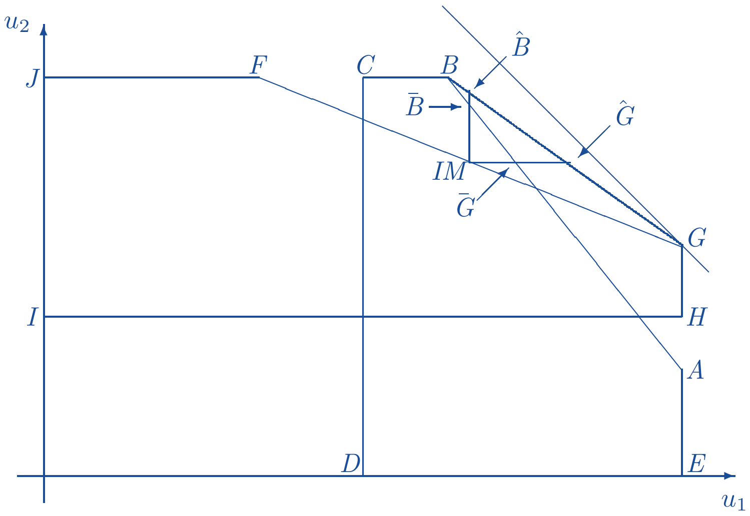


Example 4 - Non-convex bargaining region

Real utility functions of the two TOs and related bargaining problem:







In the space of strategies the segment $\hat{B}\hat{G}$ corresponds to infeasible strategies

- *correlated strategies are impossible because the timetable cannot be varied on the basis of a probabilistic event*
- *lotteries of the two (feasible) events* B *and* G *if players accept the risk of a result that can be worse of the solution of the IM*
- *side payments between the players, but in this case they choose* G *coming back to the situation of Example 1*
- *restrict to consider only the individually rational Pareto boundary $\bar{B}\bar{G}$, returning to a usual bargaining problem, as in Example 2*

References

Caprara A, Fischetti M, Toth P (2002) Modeling and Solving the Train Timetabling Problem. Operations Research 50:851-861.

Caprara A, Fischetti M, Guida PL, Monaci M, Sacco G, Toth P (2001) Solution of Real-World Train Timetabling Problems. Proceedings of the 34-th Hawaii International Conference on System Sciences (HICSS 34), IEEE Computer Society Press.

Gerber A (2000) Coalition Formation in General NTU-Games. Review of Economic Design 5:149-175.

Greenberg J (1994) Coalition Structures. in Handbook of Game Theory, vol. II (Aumann RJ, Hart S eds):1305-1338. North Holland, Amsterdam, The Netherlands.

Hart S, Kurz M (1983) Endogenous Formation of Coalitions. Econometrica 51:1047-1064.

Hooghiemstra JS, Kroon LG, Odijk MA, Salomon M, Zwaneveld PJ (1998) Decision Support Systems Support the Search for Win-Win Solutions in Railway Network Design. Technical Report, Nederlandese Spoorwegen, The Netherlands.

Kalai E, Smorodinsky M (1975) Other Solutions to Nash's Bargaining Problem. Econometrica 43:513-518.

Nash JF (1950) The Bargaining Problem. Econometrica 21:128-140.

Roth AE (1979) Axiomatic Models of Bargaining. Springer, Berlin, Germany.

Shenoy PP (1980) A Dynamic Solution Concept for Abstract Games. Journal of Optimization Theory and Applications 32:151-169.